



$$L_1 \frac{dI_1}{dt} + R_1 I_1 = u(t-5) \cdot 120 \sin(60(2\pi t))$$

$$L_1 \mathcal{L} \left\{ \frac{dI_1}{dt} \right\} + R_1 \mathcal{L} \{ I_1 \} = 120 \mathcal{L} \{ u(t-5) \sin(120\pi t) \}$$

$$L_1 \left( s \mathcal{L} \{ I_1 \} - I_1(0) \right) + R_1 \mathcal{L} \{ I_1 \} = \frac{120 e^{-5s}}{s^2 + (120\pi)^2}$$

$$\left( s + \frac{R_1}{L_1} \right) \mathcal{L} \{ I_1 \} = \frac{120}{L_1} \left( \frac{e^{-5s}}{s^2 + 14400\pi^2} \right)$$

$$\mathcal{L} \{ I_1 \} = \frac{\left( \frac{120}{L_1} e^{-5s} \right)}{\left( s + \frac{R_1}{L_1} \right) (s^2 + 14400\pi^2)}$$

$$= \frac{A}{s + \frac{R_1}{L_1}} + \frac{Bs + D}{s^2 + 14400\pi^2}$$

$$\frac{120}{L_1} e^{-5s} = \left( Bs + D \right) \left( s + \frac{R_1}{L_1} \right) + A \left( s^2 + 14400\pi^2 \right)$$

$$= Bs^2 + \left( \frac{R_1}{L_1} B + D \right) s + \frac{R_1}{L_1} D + As^2 + A \cdot 14400\pi^2$$

$$= (A+B)s^2 + \left( \frac{R_1}{L_1} B + D \right) s + \left( \frac{R_1 D}{L_1} + A \cdot 14400\pi^2 \right)$$

$$A+B=0$$

$$\frac{R_1}{L_1} B + D = 0$$

$$\frac{R_1 D}{L_1} + A \cdot 14400\pi^2 = \frac{120}{L_1} e^{-5s}$$

$$\frac{R_1 B}{L_1} + D = 0 \quad D = -\frac{R_1}{L_1} B$$

$$\frac{R_1 D}{L_1} - B \cdot 14400\pi^2 = \frac{120}{L_1} e^{-5s}$$

$$\frac{R_1}{L_1} \left( -\frac{R_1}{L_1} B \right) - B \cdot 14400\pi^2 = \frac{120}{L_1} e^{-5s}$$

$$-B \left( \frac{R_1^2}{L_1^2} + 14400\pi^2 \right) = \frac{120}{L_1} e^{-5s}$$

$$B = -\frac{\frac{120}{L_1} e^{-5s}}{\frac{R_1^2}{L_1^2} + 14400\pi^2}$$

$$A = \frac{\frac{120}{L_1} e^{-5s}}{\frac{R_1^2}{L_1^2} + 14400\pi^2}$$

$$D = +\frac{R_1}{L_1} \left( \frac{\frac{120}{L_1} e^{-5s}}{\frac{R_1^2}{L_1^2} + 14400\pi^2} \right)$$

$$I_1(t) = A L^{-1} \left\{ \frac{1}{s + \frac{R_1}{L}} \right\} +$$

$$+ B L^{-1} \left\{ \frac{s}{s^2 + (120\pi)^2} \right\} +$$

$$+ \frac{D}{120\pi} L^{-1} \left\{ \frac{120\pi}{s^2 + (120\pi)^2} \right\}$$

$$I_1(t) = A e^{-\frac{R_1}{L}t} + B \cos(120\pi t) + \frac{D}{120\pi} \sin(120\pi t).$$

$$\frac{d}{dt} \bar{x} = A \bar{x} \quad \bar{x} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

$$\bar{x} = \left[ e^{At} \right] \bar{x}(0) \quad \bar{x}(0) = \begin{bmatrix} 2 \\ -5 \\ 4 \end{bmatrix}$$


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$$\mathcal{L} \left\{ \frac{d}{dt} \bar{x} \right\} = A \mathcal{L} \{ \bar{x} \}$$

$$s \mathcal{L} \{ \bar{x} \} - \bar{x}(0) = A \mathcal{L} \{ \bar{x} \}$$

$$s \mathcal{L} \{ \bar{x} \} - A \mathcal{L} \{ \bar{x} \} = \bar{x}(0)$$

$$(sI - A) \mathcal{L} \{ \bar{x} \} = \bar{x}(0)$$

$$\underbrace{(sI - A)^{-1} (sI - A)}_I \mathcal{L} \{ \bar{x} \} = (sI - A)^{-1} \bar{x}(0)$$

$$\mathcal{L} \{ \bar{x} \} = (sI - A)^{-1} \bar{x}(0)$$

$$\bar{x} = \mathcal{L}^{-1} \left\{ (sI - A)^{-1} \right\} \bar{x}(0)$$

$$e^{A(t)} = \mathcal{L}^{-1} \left\{ (sI - A)^{-1} \right\}$$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$sI - A = s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$(sI - A) = \begin{bmatrix} s-2 & -3 \\ -1 & s-4 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{\begin{bmatrix} s-4 & 3 \\ 1 & s-2 \end{bmatrix}}{(s-2)(s-4) - 3}$$

$$(sI - A)^{-1} = \frac{\begin{bmatrix} s-4 & 3 \\ 1 & s-2 \end{bmatrix}}{s^2 - 6s + 5} \Rightarrow \frac{\begin{bmatrix} s-4 & 3 \\ 1 & s-2 \end{bmatrix}}{(s-1)(s-5)}$$

$$(sI - A)^{-1} = \begin{bmatrix} \frac{s-4}{(s-1)(s-5)} & \frac{3}{(s-1)(s-5)} \\ \frac{1}{(s-1)(s-5)} & \frac{s-2}{(s-1)(s-5)} \end{bmatrix}$$

$$\frac{s-4}{(s-1)(s-5)} = \frac{A}{s-1} + \frac{B}{s-5}$$

$$s-4 = A(s-5) + B(s-1)$$

$$\text{Si } s=1$$

$$\text{Si } s=5$$

$$1-4 = -4A + (0)$$

$$5-4 = B(4)$$

$$\boxed{A = \frac{3}{4}}$$

$$\boxed{B = \frac{1}{4}}$$

$$s-2 = D(s-5) + E(s-1)$$

$$\text{Si } s=1$$

$$\text{Si } s=5$$

$$1-2 = -4D$$

$$5-2 = E(4)$$

$$D = \frac{1}{4}$$

$$E = \frac{3}{4}$$

$$3 = AA(s-5) + BB(s-1)$$

$$\text{Si } s=1$$

$$\text{Si } s=5$$

$$3 = AA(-4) \quad AA = -\frac{3}{4}$$

$$3 = BB(4) \quad BB = \frac{3}{4}$$

$$(sI - A)^{-1} = \begin{bmatrix} \frac{3}{4} \cdot \frac{1}{s-1} + \frac{1}{4} \cdot \frac{1}{s-5} & -\frac{3}{4} \frac{1}{s-1} + \frac{3}{4} \frac{1}{s-5} \\ -\frac{1}{4} \frac{1}{s-1} + \frac{1}{4} \frac{1}{s-5} & \frac{1}{4} \cdot \frac{1}{s-1} + \frac{3}{4} \cdot \frac{1}{s-5} \end{bmatrix}$$

$$e^{At} = \mathcal{L}^{-1} \left\{ (sI - A)^{-1} \right\} = \begin{bmatrix} \frac{3}{4} & -\frac{3}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{bmatrix} \mathcal{L}^{-1} \left\{ \frac{1}{s-1} \right\} +$$

$$+ \begin{bmatrix} \frac{1}{4} & \frac{3}{4} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix} \mathcal{L}^{-1} \left\{ \frac{1}{s-5} \right\}$$

$$e^{At} = \begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} \frac{e^t}{4} + \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} \frac{e^{5t}}{4}$$