PRIMER CAPÍTULO

- NOCIONES INICIALES
- LA ECUACIÓN DIFERENCIAL PRIMER ORDEN NO LINEAL
- LA ECUACION DIFERENCIAL PRIMER ORDEN LINEAL

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0 \quad \text{EDO } \underline{NL} (1)$$

+ MVS
+ MCH.

+ M EXACTA
+ M FACTOR INTEGRANTE.

EDO L (n)

$$a_0(x)\frac{d^ny}{dx^n} + a_1(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_{n-1}(x)\frac{dy}{dx} + a_n(x)y = Q(x).$$

orden $n=1$ EDO L (1) CV NH.

$$a_0(x)\frac{dy}{dx} + a_1(x)y = Q(x)$$ FORMA GENERAL

$$\frac{dy}{dx} + \frac{a_1(x)}{a_0(x)}y = \frac{Q(x)}{a_0(x)}$$

$$\boxed{\frac{dy}{dx} + p(x)y = q(x)}$$ FORMA SINTÉTICA EDO L (1) CV NH.

REGLA DOS

LA SOLUCIÓN GENERAL EDO LINEAL

$$y_{g/NH} = y_{g/H_A} + y_{p/q}.$$

HOMOGENEA ASOCIADA.

$$\frac{dy}{dx} + p(x)y = 0$$

$$\frac{dy}{dx} = -p(x)\,y$$

$$\frac{dy}{y} = -p(x)\,dx$$ VARIABLES SEPARABLES

$$\int \frac{dy}{y} = -\int p(x)\,dx$$

$$Ly + C_1 = -\int p(x)\,dx + C_2$$

$$Ly + (C_1 - C_2) = -\int p(x)\,dx$$

$$Ly - LC = -\int p(x)\,dx$$

$$L\left(\frac{y}{C}\right) = -\int p(x)\,dx$$

$$\frac{y}{C} = e^{-\int p(x)\,dx}$$

$$\boxed{y = Ce^{-\int p(x)\,dx}}$$

SOLUCIÓN GRAL EDO L (1) CV H ASOCIADA.

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} + p(x)\,y = 0$$

$$M(x,y) = p(x)\,y \qquad \frac{\partial M}{\partial y} = p(x)$$

$$N(x,y) = 1 \qquad \frac{\partial N}{\partial x} = 0$$

NO ES EXACTA.

$$\frac{d\mu(x)}{\mu(x)} = \left(\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}\right)dx$$

$$\frac{d\mu(x)}{\mu(x)} = \left(\frac{p(x) - (0)}{1}\right)dx$$

$$\frac{d\mu(x)}{\mu(x)} = p(x)\,dx$$

$$\int \frac{d\mu(x)}{\mu(x)} = \int p(x)\,dx$$

$$\ln \mu(x) = \int p(x)\,dx$$

$$\boxed{\mu(x) = e^{\int p(x)\,dx}}$$

FACTOR INTEGRANTE.

EDO L(1) CV HA.

$$\frac{dy}{dx} + p(x)y = 0$$

$$e^{\int p(x)dx}\frac{dy}{dx} + e^{\int p(x)dx}p(x)y = 0$$

$$MM(x,y) = e^{\int p(x)dx}p(x)y \qquad \frac{\partial MM}{\partial y} = e^{\int p(x)dx}p(x)$$

$$NN(x,y) = e^{\int p(x)dx} \qquad \frac{\partial NN}{\partial x} = e^{\int p(x)dx}p(x)$$

EXACTA.

$$\left(\int MM\,dx\right) \cup \left(\int NN\,dy\right) = C$$

SOLUCIÓN GENERAL

$$\int MM\,dx = y\int e^{\int p(x)dx}p(x)\,dx \Rightarrow ye^{\int p(x)dx}$$

$$\int NN\,dy = e^{\int p(x)dx}\int dy \Rightarrow ye^{\int p(x)dx}$$

$$ye^{\int p(x)dx} = C$$

$$\boxed{y = Ce^{-\int p(x)dx}}$$

SG.

$$\frac{dy}{dx} + p(x)y = q(x)$$

$$\underbrace{e^{\int p(x)dx}\frac{dy}{dx} + e^{\int p(x)dx}p(x)y}_{\text{EXACTA}} = e^{\int p(x)dx}q(x)$$

$$\frac{d}{dx}\left(ye^{\int p(x)dx}\right) = e^{\int p(x)dx}q(x)$$

$$d\left(ye^{\int p(x)dx}\right) = e^{\int p(x)dx}q(x)dx$$

$$\int d\left(ye^{\int p(x)dx}\right) = \int \cancel{e^{\int p(x)dx}q(x)dx}$$

$$ye^{\int p(x)dx} = \int e^{\int p(x)dx}q(x)dx + C$$

$$\boxed{y = Ce^{-\int p(x)dx} + e^{-\int p(x)dx}\int e^{\int p(x)dx}q(x)dx}$$

FÓRMULA GENERAL

SOLUCIÓN GENERAL EDO L (1) CU NH.

$$y_{g/NH} = y_{g/H_o} + y_{p/q(x)}$$

CAPÍTULO 2. LINEAL CC NH.

2° orden

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = Q(x).$$

$$y_{g/NH} = y_{g/H_A} + y_{p/Q}.$$

Homogenea asociada

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0$$

Hipótesis: supongamos $y_{p/\text{fundamental}} = e^{mx}$

$$y = c_1 y_1 + c_2 y_2$$

$y = e^{mx}$ $\quad \frac{dy}{dx} = me^{mx}$ $\quad \frac{d^2y}{dx^2} = m^2e^{mx}$

$(m^2e^{mx}) + a_1(me^{mx}) + a_2(e^{mx}) = 0$ trivial.

$(m^2 + a_1 m + a_2)e^{mx} = 0$ $\quad \boxed{e^{mx} = 0}$ ×

$m^2 + a_1 m + a_2 = 0$ ECUACIÓN (ALGEBRAICA) CARACTERÍSTICA.

$m_1 \quad m_2$

EJEMPLO

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

H: $y = e^{mx}$

$$(m^2 - 5m + 6)e^{mx} = 0$$

$$m^2 - 5m + 6 = 0 \quad \text{EC.}$$

$$(m-2)(m-3) = 0$$

$$m_1 = 2 \quad m_2 = 3.$$ VALORES CARACT.

$$y_1 = e^{2x} \quad y_2 = e^{3x}$$ SOLUCIONES PARTICULARES FUNDAMENTALES

$$y_1 = e^{2x} \quad \frac{dy_1}{dx} = 2e^{2x} \quad \frac{d^2y_1}{dx^2} = 4e^{2x}$$

$$(4e^{2x}) - 5(2e^{2x}) + 6(e^{2x}) = 0$$

$$(4 - 10 + 6)e^{2x} = 0$$

$$(0)e^{2x} = 0$$

$$0 \equiv 0.$$

$$\boxed{y_g = c_1 e^{2x} + c_2 e^{3x}}$$ SOLUCIÓN GENERAL.