

```
> restart
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3) DADO EL SISTEMA, Y CON LAS CONDICIONES INICIALES DADAS:

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$$\begin{aligned}\frac{d}{dt} x(t) &= x(t) - y(t) + z(t) \\ \frac{d}{dt} y(t) &= -x(t) + y(t) + z(t) + 2e^{-2t} \\ \frac{d}{dt} z(t) &= x(t) + y(t) - z(t) + e^{-3t}\end{aligned}\quad (1)$$

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$$\begin{aligned}x(0) &= 3 \\ y(0) &= -4 \\ z(0) &= 6\end{aligned}\quad (2)$$

RESOLUCION POR MATRIZ EXPONENCIAL

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```
> AA := array([ [1, -1, 1], [-1, 1, 1], [1, 1, -1] ])
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$$AA := \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix}\quad (3)$$

```
> BB := array([ 0, 2 e^{-2t}, e^{-3t} ])
```

$$BB := \begin{bmatrix} 0 & 2e^{-2t} & e^{-3t} \end{bmatrix}\quad (4)$$

```
> CondIni := array([ 3, -4, 6 ])
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$$CondIni := \begin{bmatrix} 3 & -4 & 6 \end{bmatrix}\quad (5)$$

```
> CondPara := array([ C[1], C[2], C[3] ])
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$$CondPara := \begin{bmatrix} C_1 & C_2 & C_3 \end{bmatrix}\quad (6)$$

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> with(linalg) :
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> det(AA)
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$$-4\quad (7)$$

```
> MatExp := exponential(AA, t)
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$$MatExp := \begin{bmatrix} \frac{1}{3}e^t + \frac{1}{2}e^{2t} + \frac{1}{6}e^{-2t} & -\frac{1}{2}e^{2t} + \frac{1}{3}e^t + \frac{1}{6}e^{-2t} & \frac{1}{3}e^t - \frac{1}{3}e^{-2t} \\ -\frac{1}{2}e^{2t} + \frac{1}{3}e^t + \frac{1}{6}e^{-2t} & \frac{1}{3}e^t + \frac{1}{2}e^{2t} + \frac{1}{6}e^{-2t} & \frac{1}{3}e^t - \frac{1}{3}e^{-2t} \\ \frac{1}{3}e^t - \frac{1}{3}e^{-2t} & \frac{1}{3}e^t - \frac{1}{3}e^{-2t} & \frac{2}{3}e^{-2t} + \frac{1}{3}e^t \end{bmatrix}\quad (8)$$

> *ComprobacionUno* := *map*(*rcurry*(*eval*, *t*='0'), *MatExp*)

$$\text{ComprobacionUno} := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (9)$$

> *SolGralHom* := *evalm*(*MatExp* & * *CondPara*) : *SGH*[1] := *simplify*(*x*[1])(*t*)
= *SolGralHom*[1]; *SGH*[2] := *simplify*(*x*[2])(*t*) = *SolGralHom*[2]; *SGH*[3]
:= *simplify*(*x*[3])(*t*) = *SolGralHom*[3];

$$\text{SGH}_1 := x_1(t) = \frac{1}{3} C_1 e^t + \frac{1}{2} C_1 e^{2t} + \frac{1}{6} C_1 e^{-2t} - \frac{1}{2} C_2 e^{2t} + \frac{1}{3} C_2 e^t + \frac{1}{6} C_2 e^{-2t} \\ + \frac{1}{3} C_3 e^t - \frac{1}{3} C_3 e^{-2t}$$

$$\text{SGH}_2 := x_2(t) = -\frac{1}{2} C_1 e^{2t} + \frac{1}{3} C_1 e^t + \frac{1}{6} C_1 e^{-2t} + \frac{1}{3} C_2 e^t + \frac{1}{2} C_2 e^{2t} + \frac{1}{6} C_2 e^{-2t} \\ + \frac{1}{3} C_3 e^t - \frac{1}{3} C_3 e^{-2t}$$

$$\text{SGH}_3 := x_3(t) = \frac{1}{3} C_1 e^t - \frac{1}{3} C_1 e^{-2t} + \frac{1}{3} C_2 e^t - \frac{1}{3} C_2 e^{-2t} + \frac{2}{3} C_3 e^{-2t} + \frac{1}{3} C_3 e^t \quad (10)$$

> *SolPartHom* := *evalm*(*MatExp* & * *CondIni*) : *SPH*[1] := *x*[1](*t*) = *SolPartHom*[1]; *SPH*[2]
:= *x*[2](*t*) = *SolPartHom*[2]; *SPH*[3] := *x*[3](*t*) = *SolPartHom*[3];

$$\text{SPH}_1 := x_1(t) = \frac{5}{3} e^t + \frac{7}{2} e^{2t} - \frac{13}{6} e^{-2t}$$

$$\text{SPH}_2 := x_2(t) = -\frac{7}{2} e^{2t} + \frac{5}{3} e^t - \frac{13}{6} e^{-2t}$$

$$\text{SPH}_3 := x_3(t) = \frac{5}{3} e^t + \frac{13}{3} e^{-2t} \quad (11)$$

> *CIH*[1] := *eval*(*subs*(*t*=0, *SPH*[1])); *CIH*[2] := *eval*(*subs*(*t*=0, *SPH*[2])); *CIH*[3]
:= *eval*(*subs*(*t*=0, *SPH*[3]))

$$\text{CIH}_1 := x_1(0) = 3$$

$$\text{CIH}_2 := x_2(0) = -4$$

$$\text{CIH}_3 := x_3(0) = 6 \quad (12)$$

> *CondIni*[1]; *CondIni*[2]; *CondIni*[3]

$$3$$

$$-4$$

$$6$$

(13)

> *BBtau* := *map*(*rcurry*(*eval*, *t*='tau'), *BB*)

$$\text{BBtau} := \begin{bmatrix} 0 & 2 e^{-2\tau} & e^{-3\tau} \end{bmatrix} \quad (14)$$

> *evalm*(*BB*)

$$\begin{bmatrix} 0 & 2 e^{-2t} & e^{-3t} \end{bmatrix} \quad (15)$$

> *MatExpTau* := *map*(*rcurry*(*eval*, *t*='t - tau'), *MatExp*) :

> *ProdTau* := *simplify*(*evalm*(*MatExpTau* & * *BBtau*)) : *ProdTau*[1]; *ProdTau*[2]; *ProdTau*[3]

$$\begin{aligned}
& -e^{-4\tau+2t} + \frac{2}{3} e^{-3\tau+t} + \frac{1}{3} e^{-2t} + \frac{1}{3} e^{-4\tau+t} - \frac{1}{3} e^{-\tau-2t} \\
& \frac{2}{3} e^{-3\tau+t} + e^{-4\tau+2t} + \frac{1}{3} e^{-2t} + \frac{1}{3} e^{-4\tau+t} - \frac{1}{3} e^{-\tau-2t} \\
& \frac{2}{3} e^{-3\tau+t} - \frac{2}{3} e^{-2t} + \frac{2}{3} e^{-\tau-2t} + \frac{1}{3} e^{-4\tau+t}
\end{aligned} \tag{16}$$

> *IntTau* := map(int, ProdTau, tau=0..t) : IntTau[1]; IntTau[2]; IntTau[3]

$$\begin{aligned}
& -\frac{1}{4} e^{2t} + \frac{11}{36} e^t - \frac{11}{36} e^{-2t} + \frac{1}{3} e^{-2t} t + \frac{1}{4} e^{-3t} \\
& \frac{11}{36} e^t + \frac{1}{4} e^{2t} - \frac{29}{36} e^{-2t} + \frac{1}{3} e^{-2t} t + \frac{1}{4} e^{-3t} \\
& \frac{11}{36} e^t + \frac{4}{9} e^{-2t} - \frac{2}{3} e^{-2t} t - \frac{3}{4} e^{-3t}
\end{aligned} \tag{17}$$

> *ComprobacionDos* := map(rcurry(eval, t=0'), IntTau)

$$ComprobacionDos := \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \tag{18}$$

> *SolGralComp* := evalm(*SolGralHom* + *IntTau*) : *SolGral*[1] := x[1](t) = *SolGralComp*[1];
SolGral[2] := x[2](t) = *SolGralComp*[2]; *SolGral*[3] := x[3](t) = *SolGralComp*[3];

$$\begin{aligned}
SolGral_1 := x_1(t) &= \left(\frac{1}{3} e^t + \frac{1}{2} e^{2t} + \frac{1}{6} e^{-2t} \right) C_1 + \left(-\frac{1}{2} e^{2t} + \frac{1}{3} e^t + \frac{1}{6} e^{-2t} \right) C_2 + \left(\frac{1}{3} e^t \right. \\
& \left. - \frac{1}{3} e^{-2t} \right) C_3 - \frac{1}{4} e^{2t} + \frac{11}{36} e^t - \frac{11}{36} e^{-2t} + \frac{1}{3} e^{-2t} t + \frac{1}{4} e^{-3t} \\
SolGral_2 := x_2(t) &= \left(-\frac{1}{2} e^{2t} + \frac{1}{3} e^t + \frac{1}{6} e^{-2t} \right) C_1 + \left(\frac{1}{3} e^t + \frac{1}{2} e^{2t} + \frac{1}{6} e^{-2t} \right) C_2 + \left(\frac{1}{3} e^t \right. \\
& \left. - \frac{1}{3} e^{-2t} \right) C_3 + \frac{11}{36} e^t + \frac{1}{4} e^{2t} - \frac{29}{36} e^{-2t} + \frac{1}{3} e^{-2t} t + \frac{1}{4} e^{-3t} \\
SolGral_3 := x_3(t) &= \left(\frac{1}{3} e^t - \frac{1}{3} e^{-2t} \right) C_1 + \left(\frac{1}{3} e^t - \frac{1}{3} e^{-2t} \right) C_2 + \left(\frac{2}{3} e^{-2t} + \frac{1}{3} e^t \right) C_3 \\
& + \frac{11}{36} e^t + \frac{4}{9} e^{-2t} - \frac{2}{3} e^{-2t} t - \frac{3}{4} e^{-3t}
\end{aligned} \tag{19}$$

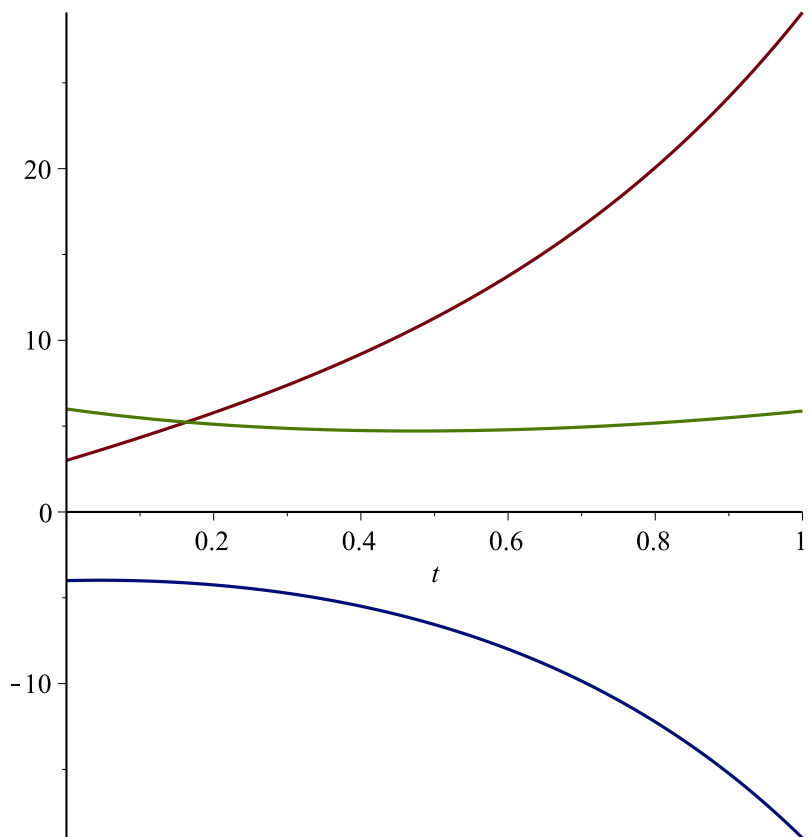
> *SolPartComp* := evalm(*SolPartHom* + *IntTau*) : *SolComp*[1] := x[1](t) = *SolPartComp*[1];
SolComp[2] := x[2](t) = *SolPartComp*[2]; *SolComp*[3] := x[3](t) = *SolPartComp*[3]

$$\begin{aligned}
SolComp_1 := x_1(t) &= \frac{71}{36} e^t + \frac{13}{4} e^{2t} - \frac{89}{36} e^{-2t} + \frac{1}{3} e^{-2t} t + \frac{1}{4} e^{-3t} \\
SolComp_2 := x_2(t) &= -\frac{13}{4} e^{2t} + \frac{71}{36} e^t - \frac{107}{36} e^{-2t} + \frac{1}{3} e^{-2t} t + \frac{1}{4} e^{-3t} \\
SolComp_3 := x_3(t) &= \frac{71}{36} e^t + \frac{43}{9} e^{-2t} - \frac{2}{3} e^{-2t} t - \frac{3}{4} e^{-3t}
\end{aligned} \tag{20}$$

> *ComprobacionTres* := map(rcurry(eval, t=0'), SolPartComp)

$$ComprobacionTres := \begin{bmatrix} 3 & -4 & 6 \end{bmatrix} \tag{21}$$

> + plot([*SolPartComp*[1], *SolPartComp*[2], *SolPartComp*[3]], t=0..1)



> *DerMatExp* := *map*(*diff*, *MatExp*, *t*)

$$DerMatExp := \begin{bmatrix} \frac{1}{3} e^t + e^{2t} - \frac{1}{3} e^{-2t} & -e^{2t} + \frac{1}{3} e^t - \frac{1}{3} e^{-2t} & \frac{2}{3} e^{-2t} + \frac{1}{3} e^t \\ -e^{2t} + \frac{1}{3} e^t - \frac{1}{3} e^{-2t} & \frac{1}{3} e^t + e^{2t} - \frac{1}{3} e^{-2t} & \frac{2}{3} e^{-2t} + \frac{1}{3} e^t \\ \frac{2}{3} e^{-2t} + \frac{1}{3} e^t & \frac{2}{3} e^{-2t} + \frac{1}{3} e^t & -\frac{4}{3} e^{-2t} + \frac{1}{3} e^t \end{bmatrix} \quad (22)$$

> *AAA* := *map*(*rcurry*(*eval*, *t*='0'), *DerMatExp*)

$$AAA := \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \quad (23)$$

> *InvMatExp* := *map*(*rcurry*(*eval*, *t*='-t'), *MatExp*)

$$InvMatExp := \begin{bmatrix} \frac{1}{3} e^{-t} + \frac{1}{2} e^{-2t} + \frac{1}{6} e^{2t} & -\frac{1}{2} e^{-2t} + \frac{1}{3} e^{-t} + \frac{1}{6} e^{2t} & \frac{1}{3} e^{-t} - \frac{1}{3} e^{2t} \\ -\frac{1}{2} e^{-2t} + \frac{1}{3} e^{-t} + \frac{1}{6} e^{2t} & \frac{1}{3} e^{-t} + \frac{1}{2} e^{-2t} + \frac{1}{6} e^{2t} & \frac{1}{3} e^{-t} - \frac{1}{3} e^{2t} \\ \frac{1}{3} e^{-t} - \frac{1}{3} e^{2t} & \frac{1}{3} e^{-t} - \frac{1}{3} e^{2t} & \frac{2}{3} e^{2t} + \frac{1}{3} e^{-t} \end{bmatrix} \quad (24)$$

> AAAA := simplify(evalm(DerMatExp &* InvMatExp))

$$AAAA := \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \quad (25)$$

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a) OBTENER LA SOLUCIÓN PARTICULAR UTILIZANDO dsolve

b) GRAFICAR LA SOLUCIÓN DEL SISTEMA OBTENIDA EN EL INCISO [a] (FUNCIONES JUNTAS EN UN SOLO GRÁFICO) CON UN INTERVALO $0 < t < 1$

> restart

ECUACIONES DIFERENCIALES EN DERIVADAS PARCIALES

> EDP := diff(z(x, y), x\$2) - 2·diff(z(x, y), x, y) + diff(z(x, y), y\$2) = 0

$$EDP := \frac{\partial^2}{\partial x^2} z(x, y) - 2 \left(\frac{\partial^2}{\partial y \partial x} z(x, y) \right) + \frac{\partial^2}{\partial y^2} z(x, y) = 0 \quad (26)$$

> SolGral := pdsolve(EDP)

$$SolGral := z(x, y) = _F1(y + x) + _F2(y + x) x \quad (27)$$

> SolGralDos := z(x, y) = _F1(y + x) + _F2(y + x) · y

$$SolGralDos := z(x, y) = _F1(y + x) + _F2(y + x) y \quad (28)$$

> ComprobacionUno := eval(subs(z(x, y) = rhs(SolGral), EDP))

$$ComprobacionUno := 0 = 0 \quad (29)$$

> ComprobacionDos := eval(subs(z(x, y) = rhs(SolGralDos), EDP))

$$ComprobacionDos := 0 = 0 \quad (30)$$

> SolPart := z(x, y) = cos(y + x) + x·exp(x + y)

$$SolPart := z(x, y) = \cos(y + x) + x e^{y+x} \quad (31)$$

```
> ComprobacionTres := eval(subs(z(x,y) = rhs(SolPart), EDP))
```

$$ComprobacionTres := 0 = 0 \quad (32)$$

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