

> restart

## PROBLEMA DE LA CUERDA DE GUITARRA

> EDP := diff(y(x, t), t\$2) - c·2·diff(y(x, t), x\$2) = 0

$$EDP := \frac{\partial^2}{\partial t^2} y(x, t) - c^2 \left( \frac{\partial^2}{\partial x^2} y(x, t) \right) = 0 \quad (1)$$

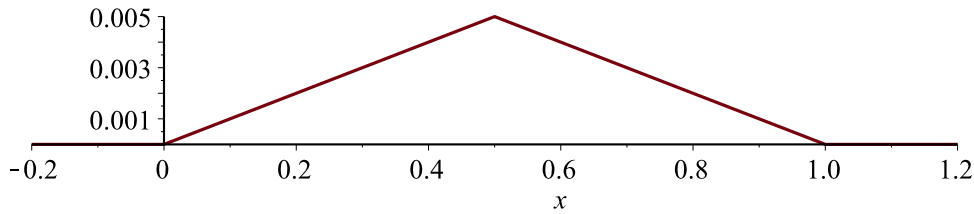
> CondFrot := y(0, t) = 0, y(1, t) = 0

$$CondFrot := y(0, t) = 0, y(1, t) = 0 \quad (2)$$

> CondIni := y(x, 0) =  $\frac{\left(\frac{5}{1000}\right)}{\left(\frac{5}{10}\right)} \cdot x \cdot \text{Heaviside}(x) - 2 \cdot \frac{\left(\frac{5}{1000}\right)}{\left(\frac{5}{10}\right)} \cdot \left(x - \frac{5}{10}\right) \cdot \text{Heaviside}\left(x - \frac{5}{10}\right) + \frac{\left(\frac{5}{1000}\right)}{\left(\frac{5}{10}\right)} \cdot (x - 1) \cdot \text{Heaviside}(x - 1)$

$$CondIni := y(x, 0) = \frac{1}{100} x \text{Heaviside}(x) - \frac{1}{50} \left(x - \frac{1}{2}\right) \text{Heaviside}\left(x - \frac{1}{2}\right) + \frac{1}{100} (x - 1) \text{Heaviside}(x - 1) \quad (3)$$

> plot(rhs(CondIni), x=-0.2..1.2)



> CondIniVel := DerYcero = 0

$$CondIniVel := DerYcero = 0 \quad (4)$$

> EcuaSep := eval(subs(y(x, t) = F(x)·G(t), EDP))

$$EcuaSep := F(x) \left( \frac{d^2}{dt^2} G(t) \right) - c^2 \left( \frac{d^2}{dx^2} F(x) \right) G(t) = 0 \quad (5)$$

> EcuaSeparada :=  $\frac{(lhs(EcuaSep) + c \cdot 2 \cdot diff(F(x), x\$2) \cdot G(t))}{F(x) \cdot G(t)}$   
=  $\frac{(rhs(EcuaSep) + c \cdot 2 \cdot diff(F(x), x\$2) \cdot G(t))}{F(x) \cdot G(t)}$

$$EcuaSeparada := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \frac{c^2 \left( \frac{d^2}{dx^2} F(x) \right)}{F(x)} \quad (6)$$

> EcuacionX := rhs(EcuaSeparada) = alpha; EcuacionT := lhs(EcuaSeparada) = alpha

$$EcuacionX := \frac{c^2 \left( \frac{d^2}{dx^2} F(x) \right)}{F(x)} = \alpha$$

$$EcuacionT := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \alpha \quad (7)$$

> EcuacionXneg := subs(alpha=-n·2·c·2·Pi·2, EcuacionX)

$$EcuacionXneg := \frac{c^2 \left( \frac{d^2}{dx^2} F(x) \right)}{F(x)} = -n^2 c^2 \pi^2 \quad (8)$$

> SolucionX := dsolve(EcuacionXneg)

$$SolucionX := F(x) = \_C1 \sin(\pi n x) + \_C2 \cos(\pi n x) \quad (9)$$

> EcuacionTneg := subs(alpha=-n·2·c·2·Pi·2, EcuacionT)

$$EcuacionTneg := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = -n^2 c^2 \pi^2 \quad (10)$$

> SolucionT := dsolve(EcuacionTneg)

$$SolucionT := G(t) = \_C1 \sin(\pi c n t) + \_C2 \cos(\pi c n t) \quad (11)$$

> SolucionPartX := subs(\\_C2=0, \\_C1=1, SolucionX)

$$SolucionPartX := F(x) = \sin(\pi n x) \quad (12)$$

> SolucionPart := y(x, t) = rhs(SolucionPartX) · rhs(SolucionT)

$$SolucionPart := y(x, t) = \sin(\pi n x) (\_C1 \sin(\pi c n t) + \_C2 \cos(\pi c n t)) \quad (13)$$

> SolucionGeneral := y(x, t) = Sum(sin(n·Pi·x) · (b[n]cos(Pi·c·n·t) + a[n]·sin(n·Pi·c·t)), n = 1 ..infinity)

$$SolucionGeneral := y(x, t) = \sum_{n=1}^{\infty} \sin(\pi n x) (b_n \cos(\pi c n t) + a_n \sin(\pi c n t)) \quad (14)$$

> SolucionInicial := eval(subs(t=0, SolucionGeneral))

$$SolucionInicial := y(x, 0) = \sum_{n=1}^{\infty} \sin(\pi n x) b_n \quad (15)$$

> b[n] :=  $\frac{1}{\left(\frac{5}{10}\right)} \cdot \text{int}\left(\text{rhs}(\text{CondIni}) \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{1}\right), x=0..1\right)$

$$b_n := \frac{1}{50} \frac{-\sin(\pi n) + 2 \sin\left(\frac{1}{2} \pi n\right)}{\pi^2 n^2} \quad (16)$$

> a[n] := 0

$$a_n := 0 \quad (17)$$

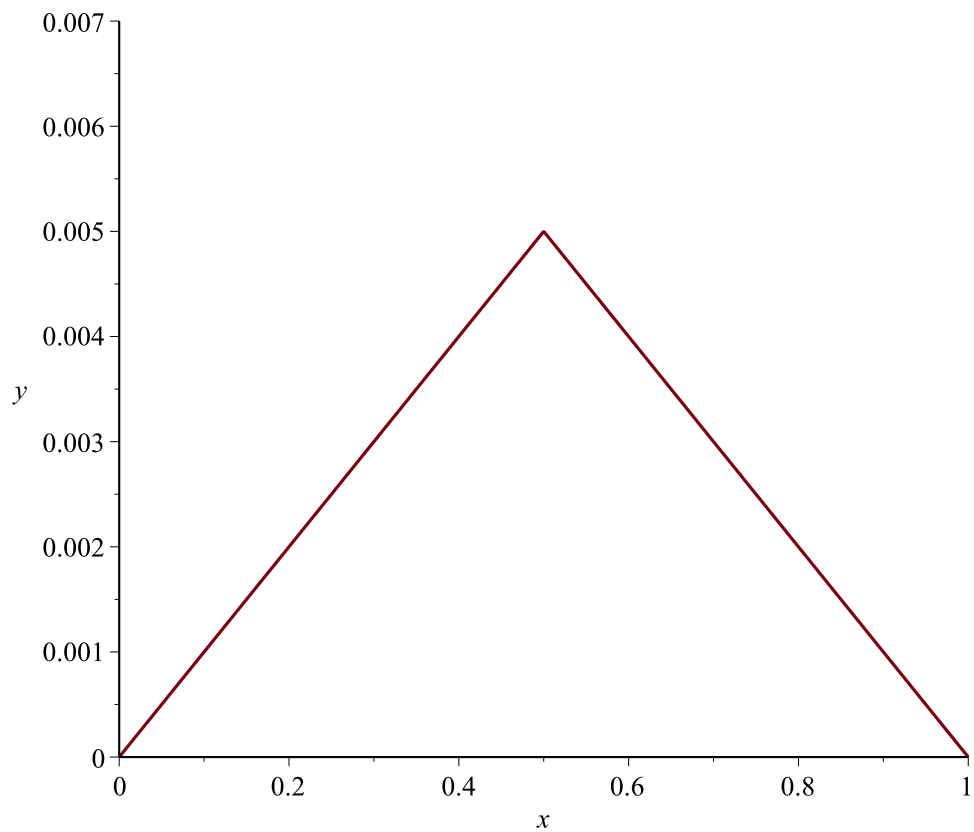
> SolucionGeneral

$$y(x, t) = \sum_{n=1}^{\infty} \frac{1}{50} \frac{\sin(\pi n x) \left( -\sin(\pi n) + 2 \sin\left(\frac{1}{2} \pi n\right) \right) \cos(\pi c n t)}{\pi^2 n^2} \quad (18)$$

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> SolucionParticula := y(x, t) = 
$$\sum_{n=1}^{500} \frac{1}{50} \frac{\sin(\pi n x) \left( -\sin(\pi n) + 2 \sin\left(\frac{1}{2} \pi n\right) \right) \cos(\pi c n t)}{n^2 \pi^2} :$$

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> plot(rhs(subs(c = 1, t = 0, SolucionParticula)), x = 0..1, y = 0..0.007)
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> with(plots) :
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> animate(rhs(subs(c = 1, SolucionParticula)), x = 0..1, t = 0..4, frames = 150, view = [0..1, -0.01..0.01])
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