Ejemplo EDO(2)LCCH.

$$M\frac{d^2s}{dt^2} = -Hs \qquad s(t)$$

MODELO
ARCO Y FLECHA.

$$\frac{d^2s}{dt^2} = -\frac{H}{M}s \Rightarrow \boxed{\frac{d^2s}{dt^2} + \frac{H}{M}s = 0}$$

EDO(2)LCCH -

$$m^2 + \frac{H}{M} = 0$$

$$m^2 = -\frac{H}{M}$$

$$m = \sqrt{-\frac{H}{M}} \qquad \boxed{m_{1,2} = \pm\sqrt{\frac{H}{M}}\,i}$$

CASO III

SG $\quad s(t) = C_1\cos\left(\sqrt{\frac{H}{M}}t\right) + C_2\,\text{Sen}\left(\sqrt{\frac{H}{M}}t\right).$

$$\left.\begin{aligned} &S_0(0) = 0{,}40 \\ \left.\frac{ds}{dt}\right|_{t=0} &\Rightarrow V_0(0) = 0 \end{aligned}\right\} \text{cond. iniciales}$$

$$S(0) \Rightarrow C_1\cancel{\cos}(0) + C_2\cancel{\text{sen}}(0) = 0.40$$

$$C_1(1) + C_2(0) = 0.40$$

$$C_1 = 0.40$$

$$s(t) = 0.40\cos\left(\sqrt{\frac{H}{M}}t\right) + C_2\,\text{sen}\left(\sqrt{\frac{H}{M}}t\right)$$

$$\frac{ds}{dt} = -0{,}40\sqrt{\frac{H}{M}}\,\text{sen}\left(\sqrt{\frac{H}{M}}t\right) + C_2\sqrt{\frac{H}{M}}\cos\left(\sqrt{\frac{H}{M}}t\right).$$

$$\left.\frac{ds}{dt}\right|_{t=0} \Rightarrow \cancel{-0.40\sqrt{\frac{H}{M}}(0)} + C_2\sqrt{\frac{H}{M}}\cancel{\cos(0)} = 0$$

$$C_2\sqrt{\frac{H}{M}}(1) = 0 \qquad \boxed{C_2 = 0}$$

SP $\quad \boxed{s(t) = 0{,}40\cos\left(\sqrt{\frac{H}{M}}t\right)} \qquad S(0) = 0{,}40 \quad \left.\frac{ds}{dt}\right|_{t=0} = 0.$

MÉTODO DEL OPERADOR DIFERENCIAL

$\frac{dy}{dt} \rightarrow$ Leibnitz. (Mat.)

$\dot{y} \rightarrow$ Newton (Físico)

$y' \Rightarrow$ Algebraica

Dy $D_t y \Rightarrow$ Operador Diferencial

$$\frac{d^2y}{dt^2} \to D^2 y \longrightarrow D(Dy)$$

$$D^{-1} y \longrightarrow \text{antiderivada}$$

$$D(D^{-1}y) = D^0 y \Rightarrow y$$

$$D^{-1}(Dy) = y$$

$$\int Dy = D^{-1}(Dy) + c \Rightarrow y + C$$

$$D(af + bg) = \qquad \left.\begin{matrix} a \\ b \end{matrix}\right\} \in \mathbb{R} \quad \left.\begin{matrix} f \\ g \end{matrix}\right\} \text{func.}$$

$$aDf + bDg = 0$$

$$D[f \cdot g] = f \cdot Dg + g \cdot Df.$$

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

$$D^2y - 5Dy + 6y = 0$$

$$(D^2 - 5D + 6)y = 0$$

$$(D-2)(D-3)y = 0$$

$$y_g = C_1 e^{2x} + C_2 e^{3x}$$

$$(D-2)(D-3)\left[C_1 e^{2x} + C_2 e^{3x}\right] = 0$$

$$(D-2)\left[2C_1 e^{2x} \cancel{+3C_2 e^{3x}} - 3C_1 e^{2x} \cancel{-3C_2 e^{3x}}\right] = 0$$

$$(D-2)\left[-C_1 e^{2x}\right] = 0$$

$$\left[\cancel{-2C_1 e^{2x}} \cancel{+2C_1 e^{2x}}\right] = 0$$

$$\underline{0 = 0}$$

$$(D^2 - 5D + 6)\left[C_1 e^{2x} + C_2 e^{3x}\right] = 0$$

$$\left[4C_1 e^{2x} + 9C_2 e^{3x} - 10C_1 e^{2x} - 15C_2 e^{3x} + 6C_1 e^{2x} + 6C_2 e^{3x}\right] = 0$$

$$(0)C_1 e^{2x} + (0)C_2 e^{3x} = 0$$

$$\underline{0 = 0}$$

EDO(2) L CC NH

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 8e^{x}$$

$$\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = Q(x)$$

$$y_{g/NH} = y_{g/H} + y_{p/Q}.$$

$$y_{g/NH} = \underbrace{C_1e^{m_1x} + C_2e^{m_2x}}\, + Ae^{m_3x}$$

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0 \longrightarrow y_{g/H_o} = C_1e^{2x} + C_2e^{3x}$$

EDO(2) LccH$_o$.

$$y_{p/Q} = Ae^{mx} \quad \boxed{m=1} \quad Q(x) = 8e^{x}$$

$$y_p = Ae^{x} \longrightarrow y' = Ae^{x} \quad y'' = Ae^{x}$$

$$(Ae^{x}) - 5(Ae^{x}) + 6(Ae^{x}) = 8e^{x}$$

$$2Ae^{x} = 8e^{x}$$

$$A = 4$$

$$\boxed{y_g = C_1e^{2x} + C_2e^{3x} + 4e^{x}}$$

$$y_{g/NH} = y_{g/H} + y_{p/Q}.$$

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 24e^{4x} + 78e^{5x}$$

$$(D^2 - 3D + 2)y = 24e^{4x} + 78e^{5x}$$

$$(D-1)(D-2)y = 24e^{4x} + 78e^{5x}$$

$$(D-1)(D-2)(D-4)_A(D-5)_A y = 0$$

$$\boxed{(D-4)_A\left[24e^{4x}\right] = 24\times 4e^{4x} - 4\times 24e^{4x} \Rightarrow 0}$$

$$(D-1)(D-2)(D-4)_A(D-5)_A y = 0$$

$$y_g = \underbrace{C_1e^{x} + C_2e^{2x}}_{y_{H/A}} + \underbrace{Ae^{4x} + Be^{5x}}_{y_{P/Q}}$$

$$(D^2 - 3D + 2)y = 24e^{4x} + 78e^{5x}$$

$y_p = Ae^{4x} + Be^{5x}$	$(16Ae^{4x} + 25Be^{5x}) - 3(4Ae^{4x} + 5Be^{5x}) + 2(Ae^{4x} + Be^{5x}) = 24e^{4x} + 78e^{5x}$
$\frac{dy}{dx} = 4Ae^{4x} + 5Be^{5x}$	$(16A - 12A + 2A)e^{4x} + (25B - 15B + 2B)e^{5x} = 24e^{4x} + 78e^{5x}$
$\frac{d^2y}{dx^2} = 16Ae^{4x} + 25Be^{5x}$	$6A = 24 \Rightarrow A = 4$ $12B = 78 \Rightarrow B = \frac{13}{2}$

$$y_g = C_1e^{x} + C_2e^{2x} + 4e^{4x} + \frac{13}{2}e^{5x}$$

$$(D^2 + aD + b)\,y = Q(x)$$

$P(D)$	$Q(x)$
$(D-a)$	Ce^{ax}
D	C
D^2	Cx
D^{n+1}	Cx^n
$(D-a)^2$	Cxe^{ax}
$(D-a)^3$	Cx^2e^{ax}
$(D-a)^{n+1}$	Cx^ne^{ax}
(D^2+b^2)	$C\cos(bx)$ $C\,\mathrm{sen}(bx)$
$(D^2+b^2)^2$	$Cx\cos(bx)$ $Cx\,\mathrm{sen}(bx)$
$(D^2-2aD)+(a^2+b^2)$	$Ce^{ax}\cos(bx)$ $Ce^{ax}\,\mathrm{sen}(bx)$

e^{ax}

x^n

$\begin{cases} \cos(bx) \\ \mathrm{sen}(bx) \end{cases}$

MÉTODO DE LOS COEFICIENTES INDETERMINADOS

O

DEL OPERADOR DIFERENCIAL

$$\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=4e^{-x}$$

$$(D^2+3D+2)y=4e^{-x}$$

$$(D+1)(D+2)y=4e^{-x}$$

$$(D+1)(D+2)(D+1)_A\, y=0$$ EDO(3) LccH.

$$(D+2)(D+1)^2y=0$$

$$y_g=c_1e^{-2x}+c_2e^{-x}+c_3xe^{-x}$$

$$y_g=c_1e^{-2x}+c_2e^{-x}+Axe^{-x}$$

$$y=Axe^{-x}$$

$$\frac{dy}{dx}=A\left(-xe^{-x}+e^{-x}\right)$$

$$\frac{d^2y}{dx^2}=A\left(-\left(-xe^{-x}+e^{-x}\right)-e^{-x}\right)\Rightarrow A\left(xe^{-x}-2e^{-x}\right)$$

$$\left(A\left(xe^{-x}-2e^{-x}\right)+3\left(A\left(-xe^{-x}+e^{-x}\right)\right)+2\left(Axe^{-x}\right)=4e^{-x}\right.$$

$$\left(\underbrace{Axe^{-x}-3Axe^{-x}+2Axe^{-x}}_{0}\right)+\left(-2Ae^{-x}+3Ae^{-x}\right)=4e^{-x}$$

$$Ae^{-x}=4e^{-x}$$

$$A=4$$

$$\boxed{y_g=c_1e^{-2x}+c_2e^{-x}+4xe^{-x}}$$