

EDO (1) NL.

- MÉTODOS $\left\{ \begin{array}{l} \text{VARIABLES SEPARABLES} \\ \text{EXACTA.} \\ \text{FACTOR INTEGRANTE.} \end{array} \right.$

MÉTODO DE COEFICIENTES HOMOGÉNEOS

$$M(x, y) + N(x, y) \frac{dy}{dx} = 0$$

$$F(x, y) \longrightarrow$$

si $F(\lambda x, \lambda y) = \lambda^n \cdot F(x, y) \quad \underline{n \in \mathbb{N}.}$

$$M(\lambda x, \lambda y) = \lambda^m M(x, y)$$

$$N(\lambda x, \lambda y) = \lambda^n N(x, y) \quad m = n$$

ENTONCES

$$M(x, y) + N(x, y) \frac{dy}{dx} = 0$$

ES UNA
EDO(1) NL
de C. H.

$$X \frac{dy}{dx} = \sqrt{x^2 - y^2} + y$$

$$\underbrace{(\sqrt{x^2 - y^2} + y)}_M - \underbrace{X \frac{dy}{dx}}_N = 0$$

$$\begin{aligned} M(\lambda x, \lambda y) &= \sqrt{(\lambda x)^2 - (\lambda y)^2} + (\lambda y) \\ &= \sqrt{\lambda^2 x^2 - \lambda^2 y^2} + \lambda y \\ &= \sqrt{\lambda^2 (x^2 - y^2)} + \lambda y \\ &= \sqrt{\lambda^2} \sqrt{x^2 - y^2} + \lambda y \\ &= \lambda \sqrt{x^2 - y^2} + \lambda y \\ &= \lambda (\sqrt{x^2 - y^2} + y) \quad m=1 \end{aligned}$$

$$\begin{aligned} N(\lambda x, \lambda y) &= -(\lambda x) \\ &= \lambda(-x) \quad n=1 \end{aligned}$$

$$\underbrace{\left((\sqrt{x^2 - y^2} + y) - x \frac{dy}{dx} \right)}_{\substack{\text{si } m=n \\ \text{EDO(1) NL} \\ \text{COEF. HOMOGÉNEOS}}} = 0$$

Para resolver EDO(1) NL (CH)

será necesario hacer un cambio de var.

$$y = vx \quad \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$(\sqrt{x^2 - (vx)^2} + vx) - x \left(v + x \frac{dv}{dx} \right) = 0$$

$$(\sqrt{x^2 - v^2 x^2} + vx) - vx - x^2 \frac{dv}{dx} = 0$$

$$\sqrt{x^2(1-v^2)} - x^2 \frac{dv}{dx} = 0 \quad \leftarrow$$

$$\sqrt{x^2} \sqrt{1-v^2} - x^2 \frac{dv}{dx} = 0$$

$$x \sqrt{1-v^2} - x^2 \frac{dv}{dx} = 0$$

$$\left. \begin{aligned} P(x) &= x \\ Q(v) &= \sqrt{1-v^2} \\ R(v) &= -v^2 \\ S(v) &= 1 \end{aligned} \right\} \text{EDO(1) NL (VS)}$$

$$\frac{x dx}{x^2} - \frac{dv}{\sqrt{1-v^2}} = 0 \quad \begin{array}{c} \text{1} \\ \text{v} \\ \text{1-v}^2 \end{array}$$

$$\textcircled{SG} \int \frac{dx}{x} - \int \frac{dv}{\sqrt{1-v^2}} = C \quad \frac{v}{1} = \sin(\theta) \quad \leftarrow$$

$$\ln x - \int \frac{\cos(\theta) d\theta}{\cos(\theta)} = C \quad \frac{\sqrt{1-v^2}}{1} = \cos(\theta)$$

$$\ln x - \int d\theta = C \quad d\theta = \cos(\theta) dv$$

$$\ln x - \theta = C \quad \theta = \arcsin(v)$$

$$\ln x - \theta = C \quad y = vx$$

$$\ln x - \arcsin\left(\frac{y}{x}\right) = C \quad v = \frac{y}{x}$$

$$\arcsin\left(\frac{y}{x}\right) = C_1 + \ln x$$

$$\frac{y}{x} = \sin(C_1 + \ln x)$$

$$\textcircled{SG} \boxed{y = x \sin(C_1 + \ln(x))}$$

EDO(1) L.C.V.N.H. FORMA GENERAL

$$a_0(x) \frac{dy}{dx} + a_1(x) y = Q(x).$$

FORMA SINTÉTICA.

$$\frac{dy}{dx} + \frac{a_1(x)}{a_0(x)} y = \frac{Q(x)}{a_0(x)}.$$

$$\left[\begin{array}{c} \frac{dy}{dx} + p(x) y = q(x) \\ \text{EDO(1) L.C.V.N.H.} \end{array} \right]$$

→ Homogenea asociada

$$\frac{dy}{dx} + p(x) y = 0$$

VARIABLES
SEPARABLES.

$$dy + p(x) y dx = 0$$

$$\frac{dy}{y} + p(x) dx = 0$$

(SOL.) → $\int \frac{dy}{y} = \int -p(x) dx + C$

$$\ln y = -\int p(x) dx + C$$

$$y = e^{-\int p(x) dx + C}$$

$$y = e^C e^{-\int p(x) dx}$$

$$y = C_1 e^{-\int p(x) dx}.$$

SOL GEN

EDO(1) L.C.V.N.H.

$$\frac{dy}{dx} + p(x)y = 0 \quad M(x,y) + N(x,y) \frac{dy}{dx} = 0$$

$$(p(x)y) + \frac{dy}{dx} = 0$$

$$\begin{aligned} M(x,y) &= p(x)y \\ N(x,y) &= 1. \end{aligned} \quad \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y} = p(x) \quad \frac{\partial N}{\partial x} = 0$$

NO ES EXACTA

BUSCAR FACTOR INTEGRANTE.

$$\mu(x) \Rightarrow \frac{d\mu(x)}{\mu(x)} = \left(\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} \right) dx$$

$$\frac{d\mu(x)}{\mu(x)} = \left(\frac{p(x) - 0}{1} \right) dx$$

$$\frac{d\mu(x)}{\mu(x)} = p(x) dx$$

$$\int \frac{d\mu}{\mu} = \int p(x) dx$$

$$\mu(x) = \int p(x) dx$$

FACTOR INTEGRANTE

$$\mu(x) = e^{\int p(x) dx}$$

$$e^{\int p(x) dx} (p(x)y) + e^{\int p(x) dx} \frac{dy}{dx} = 0$$

MM

NN

$$\frac{\partial MM}{\partial y} = p(x) e^{\int p(x) dx}$$

$$\frac{\partial MM}{\partial y} = \frac{\partial NN}{\partial x}$$

$$\frac{\partial NN}{\partial x} = e^{\int p(x) dx} p(x)$$

EXACTA.

$$\frac{dy}{dx} + p(x)y = 0 \rightarrow y = C e^{-\int p(x)dx}$$

$$p(x)y + \frac{dy}{dx} = 0 \quad \text{NO ES EXACTA.}$$

F.I.

$$\underbrace{e^{\int p(x)dx}}_{MM} p(x)y + \underbrace{e^{\int p(x)dx}}_{NN} \frac{dy}{dx} = 0 \quad \text{EXACTA.}$$

$$\text{SOL}_{\text{GZAL}} \Rightarrow \int MM dx + \int \left(NN - \frac{\partial}{\partial y} \int MM dx \right) dy = C$$

$$\int MM dx = y \int e^{\int p(x)dx} p(x) dx \Rightarrow y e^{\int p(x)dx}$$

$$\frac{\partial}{\partial y} \int MM dx = e^{\int p(x)dx}$$

$$NN = e^{\int p(x)dx}$$

$$\text{SOL}_{\text{GZAL}} \Rightarrow y e^{\int p(x)dx} = C$$

$$\boxed{y = C e^{-\int p(x)dx}}$$

