

EDOL(2) CCH.

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

$$H_0 = y = e^{mx}$$

$$\text{Ec. Carac. } m^2 + a_1 m + a_2 = 0 \quad \left\{ \begin{array}{l} m_1 \\ m_2 \end{array} \right.$$

Caso I- $m_1 \neq m_2 \in \mathbb{R}$

$$y_g = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$

$$\left| \begin{array}{l} y_g = C_1 y_1 + C_2 y_2 \end{array} \right. \quad y_1, y_2 \Rightarrow \begin{array}{l} \text{Sol} \\ \text{PART} \\ \text{FUND.} \end{array}$$

y_1 linealmente independiente de y_2

$$W(y_1, y_2) \neq 0$$

$$\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0$$

ECUACIÓN CARACTERÍSTICA DE LA EDO(2) LCC H.

$m_1, m_2 \}$ CASO I: raíces reales y distintas
 $m_1 \neq m_2 \in \mathbb{R}$

CASO II: - raíces iguales y reales
 $m_1 = m_2 \in \mathbb{R}$

CASO III: - raíces complejas (distintas)

$$\left. \begin{array}{l} m_1 = a + bi \\ m_2 = a - bi \end{array} \right\} \in \mathbb{C} \quad (m_1 \neq m_2)$$

CASO III:- raíces complejas

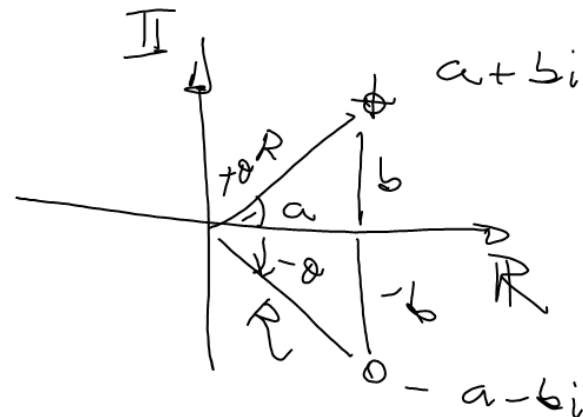
$$y = C_1 e^{(a+bi)x} + C_2 e^{(a-bi)x}$$

$$y_g = C_1 e^{ax} e^{(bx)i} + C_2 e^{ax} e^{(-bx)i}$$

$$y_g \in \mathbb{R} \quad x \in \mathbb{R} \quad \left. \begin{matrix} C_1 \\ C_2 \end{matrix} \right\} \in \mathbb{C}$$

$e^{\pi i} = -1$

T. Euler



$$re^{i\theta} = r\cos(\theta) + [r\sin(\theta)]i$$

$$re^{-i\theta} = r\cos(\theta) - [r\sin(\theta)]i$$

$$y_g = C_1 e^{ax} e^{bxi} + C_2 e^{ax} e^{-bxi}$$

$$y_g = C_1 e^{ax} \left(\cos(bx) + [\sin(bx)]i \right) + C_2 e^{ax} \left(\cos(bx) - [\sin(bx)]i \right)$$

$$y_g = (C_1 + C_2) e^{ax} \cos(bx) + (C_1 i + C_2 i) e^{ax} \sin(bx)$$

$$y_g = C_{10} e^{ax} \cos(bx) + C_{20} e^{ax} \sin(bx).$$

$$m_1, m_2 = a \pm bi \quad \begin{matrix} a \in \mathbb{R} \\ b \in \mathbb{R}^+ \end{matrix}$$

$$y = C_1 e^{5x} \cos(2x) + C_2 e^{5x} \sin(2x)$$

$$\text{EDO}(z) L \dots H.$$

$$\begin{cases} m_1 = 5 + 2i \\ m_2 = 5 - 2i \end{cases} \quad \mathbb{C}$$

ec. car. $(m - (5 + 2i)) \cdot (m - (5 - 2i)) = 0$

$$((m - 5) + 2i)((m - 5) - 2i) = 0$$

$$(m - 5)^2 - (2i)^2 = 0$$

$$m^2 - 10m + 25 + 4 = 0$$

$$\left| \frac{d^2 y}{dx^2} - 10 \frac{dy}{dx} + 29 y = 0 \right.$$

$$m = 4i \quad m = -4i$$

$$y_g = C_1 \cos(4x) + C_2 \sin(4x)$$

$$(m - 4i)(m + 4i) = 0$$

$$\frac{d^2 y}{dx^2} + 16y = 0$$

$$m^2 - (4i)^2 = 0$$

$$m^2 + 16 = 0$$

$$\frac{d^2 y}{dx^2} - 16y = 0$$

$$m^2 - 16 = 0$$

$$(m - 4)(m + 4) = 0$$

$$m_1 = 4$$

$$m_2 = -4$$

$$y = C_1 e^{4x} + C_2 e^{-4x}$$

$$m^2 + a_1 m + a_2 = 0$$

$$m_{1,2} = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_2}}{2}$$

CASO IV. - $m_1 = m_2 \in \mathbb{R}$

$$y_g = C_1 e^{m_1 x} + C_2 (x e^{m_1 x})$$

$$m^2 + a_1 m + a_2 = 0$$

$$\frac{d}{dm} [(m - m_1)(m - m_2)] = 0 \quad m_1 \neq m_2$$

$$(m - m_1) + (m - m_2) = 0$$

$$\frac{d}{dm} (m - m_1)^2 = 0 \quad m_1 = m_2$$

$$2(m - m_1) = 0$$

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

$$m^2 + a_1 m + a_2 = 0$$

$$\left. \begin{array}{l} m^2 + a_1 m + a_2 = 0 \\ m_1 = m_2 \end{array} \right\}$$

$$\frac{dy}{dx} \begin{cases} y = e^{mx} \xrightarrow{m=m_1} e^{m_1 x} \quad y_{p.F.} \\ y = x e^{mx} \xrightarrow{m=m_1} x e^{m_1 x} \end{cases}$$

$$\frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 9y = 0$$

$$m^2 - 6m + 9 = 0$$

$$(m-3)^2 = 0 \quad m_1 = m_2 = 3$$

$$y_1 = e^{3x} \longrightarrow y_2 = x e^{3x}$$

$$\frac{d^2 y}{dx^2} = 0 \quad m_1 = m_2 = 0$$

$$y_g = c_1 e^{m_1 x} + c_2 x e^{m_1 x}$$

$$y_g = c_1 + c_2 x$$

$$\frac{dy}{dx} = c_2 \rightarrow \frac{d^2 y}{dx^2} = 0$$

$$\frac{d}{dx} \left(\frac{dy}{dx} \right) = 0$$

$$\frac{dy}{dx} = c_2$$

$$dy = c_2 dx$$

$$\int dy = c_2 \int dx + c_3$$

$$y_g = c_2 x + (c_4 + c_3)$$

$$y_g = c_1 + c_2 x$$

$$\frac{d^3 y}{dt^3} = 0$$

$$m^3 = 0 \quad m_1 = m_2 = m_3 = 0$$

$$\frac{d}{dm} \left(\begin{array}{l} y = e^{m,t} \quad m = m_1 \quad \rightarrow \quad 1 \\ y = t e^{m,t} \quad m = m_1 \quad \rightarrow \quad t \end{array} \right.$$

$$\frac{d}{dm} \left(y = t^2 e^{m,t} \quad m = m_1 \quad \rightarrow \quad t^2 \right.)$$

$$y = C_1 + C_2 t + C_3 t^2$$

EDOL(4)ccH.

$$m^4 + a_1 m^3 + a_2 m^2 + a_3 m + a_4 = 0$$

$$\left. \begin{array}{l} m_{1,2} = a \pm bi \\ m_{3,4} = a \pm bi \end{array} \right\} \begin{array}{l} m_1 = m_3 \\ m_2 = m_4 \end{array}$$

$$y_g = C_1 e^{ax} \cos(bx) + C_2 e^{ax} \sin(bx) + C_3 x e^{ax} \cos(bx) + C_4 x e^{ax} \sin(bx).$$

$$y_g = e^{2x} \left(C_1 + C_2 x + C_3 \cos(5x) + C_4 \sin(5x) \right)$$

$$(m-2)^2 \cdot (m - (-2+5i)) \cdot (m - (-2-5i)) = 0$$

$$(m^2 - 4m + 4) \left((m+2)^2 - (5i)^2 \right) = 0$$

$$(m^2 - 4m + 4) (m^2 + 2m + 4 + 25) = 0$$

$$m^4 - 2m^3 + (4 - 8 + 1)m^2 + \dots$$

$$\frac{d^4 y}{dx^4} - 2 \frac{d^3 y}{dx^3} + 25 \frac{d^2 y}{dx^2} - 108 \frac{dy}{dx} + 116 y = 0$$