```
> restart;
> Ecua:=diff(y(x),x$3)+y(x)=x;
```

$$Ecua := \frac{d^3}{dx^3} y(x) + y(x) = x \tag{1}$$

```
> EcuaHom:=lhs(Ecua)=0;
```

$$EcuaHom := \frac{d^3}{dx^3} y(x) + y(x) = 0 \tag{2}$$

```
> Q:=rhs(Ecua);
```

$$Q := x \tag{3}$$

```
> EcuaCarac:=m**3+1=0;
```

$$EcuaCarac := m^3 + 1 = 0 \tag{4}$$

```
> Raiz:=solve(EcuaCarac);
```

$$Raiz := -1, \; \frac{1}{2} - \frac{1}{2} I\sqrt{3}, \; \frac{1}{2} + \frac{1}{2} I\sqrt{3} \tag{5}$$

```
> yUno:=exp(Raiz[1]*x);
```

$$yUno := e^{-x} \tag{6}$$

```
> yDos:=exp(Re(Raiz[3])*x)*cos(Im(Raiz[3])*x);
```

$$yDos := e^{\frac{1}{2} x} \cos\left(\frac{1}{2} \sqrt{3} \, x \right) \tag{7}$$

```
> yTres:=exp(Re(Raiz[3])*x)*sin(Im(Raiz[3])*x);
```

$$yTres := e^{\frac{1}{2} x} \sin\left(\frac{1}{2} \sqrt{3} \, x \right) \tag{8}$$

```
> with(linalg):
> WW:=wronskian([yUno,yDos,yTres],x):
> BB:=array([0,0,Q]);
```

$$BB := \begin{bmatrix} 0 & 0 & x \end{bmatrix} \tag{9}$$

```
> Para:=simplify(linsolve(WW,BB));
```

$$Para := \left[\frac{1}{3} x e^x, \; -\frac{1}{9} e^{-\frac{1}{2} x} \sqrt{3} \, x \left(\sqrt{3} \cos\left(\frac{1}{2} \sqrt{3} \, x \right) + 3 \sin\left(\frac{1}{2} \sqrt{3} \, x \right) \right), \tag{10}$$

$$-\frac{1}{9} e^{-\frac{1}{2} x} \sqrt{3} \, x \left(\sqrt{3} \sin\left(\frac{1}{2} \sqrt{3} \, x \right) - 3 \cos\left(\frac{1}{2} \sqrt{3} \, x \right) \right) \right]$$

```
> Aprima:=Para[1];
```

$$Aprima := \frac{1}{3} x e^x \tag{11}$$

```
> Bprima:=Para[2];
```

$$Bprima := -\frac{1}{9}\, e^{-\frac{1}{2}x}\, \sqrt{3}\, x \left(\sqrt{3}\, \cos\!\left(\frac{1}{2}\, \sqrt{3}\, x \right) + 3\, \sin\!\left(\frac{1}{2}\, \sqrt{3}\, x \right) \right) \tag{12}$$

```
> Dprima:=Para[3];
```

$$Dprima := -\frac{1}{9}\, e^{-\frac{1}{2}x}\, \sqrt{3}\, x \left(\sqrt{3}\, \sin\!\left(\frac{1}{2}\, \sqrt{3}\, x \right) - 3\, \cos\!\left(\frac{1}{2}\, \sqrt{3}\, x \right) \right) \tag{13}$$

```
> A:=int(Aprima,x)+C1;
```

$$A := \frac{1}{3}\, (x-1)\, e^{x} + C1 \tag{14}$$

```
> B:=int(Bprima,x)+C2;
```

$$B := -\frac{1}{3} \left(-\frac{1}{2}\, x + \frac{1}{2} \right) e^{-\frac{1}{2}x} \cos\!\left(\frac{1}{2}\, \sqrt{3}\, x \right) + \frac{1}{3} \left(-\frac{1}{2}\, \sqrt{3}\, x \right.$$
$$\left. -\frac{1}{2}\, \sqrt{3} \right) e^{-\frac{1}{2}x} \sin\!\left(\frac{1}{2}\, \sqrt{3}\, x \right) - \frac{1}{3}\, \sqrt{3} \left(\left(-\frac{1}{2}\, \sqrt{3}\, x \right. \right.$$
$$\left. -\frac{1}{2}\, \sqrt{3} \right) e^{-\frac{1}{2}x} \cos\!\left(\frac{1}{2}\, \sqrt{3}\, x \right) + \left(-\frac{1}{2}\, x + \frac{1}{2} \right) e^{-\frac{1}{2}x} \left. \sin\!\left(\frac{1}{2}\, \sqrt{3}\, x \right) \right) + C2 \tag{15}$$

```
> DD:=int(Dprima,x)+C3;
```

$$DD := -\frac{1}{3} \left(-\frac{1}{2}\, \sqrt{3}\, x - \frac{1}{2}\, \sqrt{3} \right) e^{-\frac{1}{2}x} \cos\!\left(\frac{1}{2}\, \sqrt{3}\, x \right) - \frac{1}{3} \left(-\frac{1}{2}\, x \right.$$
$$\left. + \frac{1}{2} \right) e^{-\frac{1}{2}x} \sin\!\left(\frac{1}{2}\, \sqrt{3}\, x \right) + \frac{1}{3}\, \sqrt{3} \left(\left(-\frac{1}{2}\, x + \frac{1}{2} \right) e^{-\frac{1}{2}x} \cos\!\left(\frac{1}{2}\, \sqrt{3}\, x \right) - \left(\right. \right.$$
$$\left. \left. -\frac{1}{2}\, \sqrt{3}\, x - \frac{1}{2}\, \sqrt{3} \right) e^{-\frac{1}{2}x} \sin\!\left(\frac{1}{2}\, \sqrt{3}\, x \right) \right) + C3 \tag{16}$$

```
> SolGral:=y(x)=simplify(A*yUno+B*yDos+DD*yTres);
```

$$SolGral := y(x) = x + e^{\frac{1}{2}x} \cos\!\left(\frac{1}{2}\, \sqrt{3}\, x \right) C2 + e^{\frac{1}{2}x} \sin\!\left(\frac{1}{2}\, \sqrt{3}\, x \right) C3 + e^{-x} C1 \tag{17}$$

```
> Comprobacion:=simplify(eval(subs(y(x)=rhs(SolGral),Ecua)));
```

$$Comprobacion := x = x \tag{18}$$