$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+9)^2}\right\} =$$

$$\mathcal{L}^{-1}\left\{F(s)\cdot G(s)\right\} = f(t) * g(t)$$

$$f(t) * g(t) = \int_0^t f(z)\cdot g(t-z)\,dz.$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+9)^2}\right\} = \mathcal{L}^{-1}\left\{\left(\frac{s}{s^2+9}\cdot\frac{1}{s^2+9}\right)\right\}$$

$$= \frac{1}{3}\mathcal{L}^{-1}\left\{\left(\frac{s}{s^2+9}\right)\cdot\left(\frac{3}{s^2+9}\right)\right\}$$

$$= \frac{1}{3}\left(\cos(3t) * \operatorname{sen}(3t)\right).$$

$$\cos(3t) * \operatorname{sen}(3t) = \int_0^t \cos(3z)\cdot\operatorname{sen}\left(3(t-z)\right)dz$$

$$= \left[\int \cos(3z)\cdot\left(\operatorname{sen}(3t)\cos(3z) - \cos(3t)\cdot\operatorname{sen}(3z)\right)dz\right]_0^t$$

$$= \left[\int\left(\operatorname{sen}(3t)\cos^2(3z) - \cos(3z)\cos(3t)\cdot\operatorname{sen}(3z)\right)dz.\right]_0^t$$

$$= \left[\operatorname{sen}(3t)\int\cos^2(3z)\,dz - \cos(3t)\int\cos(3z)\operatorname{sen}(3z)\,dz\right]_0^t$$

$$= \left[\operatorname{sen}(3t)\int\left(\frac{1}{2}+\frac{1}{2}\cos(6z)\right)dz - \frac{1}{3}\cos(3t)\left(\int\operatorname{sen}(3z)\cdot 3\cos(3z)\,dz\right)\right]_0^t$$

$$= \left[\frac{\operatorname{sen}(3t)}{2}\left(\int dz + \frac{1}{12}\int 6\cos(6z)\,dz\right) - \frac{1}{3}\cos(3t)\int\operatorname{sen}(3z)\cdot\left(3\cos(3z)\,dz\right)\right]_0^t$$

$$= \left[\frac{\operatorname{sen}(3t)}{2}\left(z + \frac{1}{12}\operatorname{sen}(6z)\right) - \frac{1}{3}\cos(3t)\left(\frac{\operatorname{sen}^2(3z)}{2}\right)\right]_0^t$$

$$= \left[\frac{\operatorname{sen}(3t)}{2}\left((t-0) + \frac{1}{12}\left(\operatorname{sen}(6t) - 0\right)\right) - \frac{1}{3}\cos(3t)\cdot\left(\frac{\operatorname{sen}^2(3t)}{2} - 0\right)\right.$$

$$= \frac{t\operatorname{sen}(3t)}{2} + \frac{1}{12}\left(2\left(\operatorname{sen}^2(3t)\cdot\cos(3t)\right)\right) - \frac{1}{6}\cos(3t)\cdot\operatorname{sen}^2(3t)$$

$$= \frac{t\operatorname{sen}(3t)}{2} + \frac{1}{6}\cancel{\operatorname{sen}^2(3t)\cdot\cos(3t)} - \frac{1}{6}\cancel{\cos(3t)\cdot\operatorname{sen}^2(3t)}$$

$$= \frac{t\operatorname{sen}(3t)}{2}$$

$$\cos(3t) * \operatorname{sen}(3t) = \frac{1}{3}\left(\frac{t\operatorname{sen}(3t)}{2}\right) \Rightarrow \frac{1}{6}t\operatorname{sen}(3t).$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+9)^2}\right\} = \frac{1}{6}t\operatorname{sen}(3t).$$

$$\mathcal{L}^{-1}\left\{e^{-as}F(s)\right\} = f(t-a)\cdot u(t-a)$$

$$\mathcal{L}^{-1}\left\{e^{-2s}\frac{s}{s^2+25}\right\} = \cos\left(5(t-2)\right)\cdot u(t-2)$$

$$\mathcal{L}\left\{e^{at}f(t)\right\} = F(s-a)$$

$$\mathcal{L}\left\{t^2e^{3t}\right\} = \frac{2}{(s-3)^3}$$

$$\mathcal{L}\left\{t^2\right\} = \frac{2!}{s^3}$$

$$\mathcal{L}\left\{t^n\right\} = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}\{y\} = \frac{e^{-4s} \cdot 5}{(s^2+25)^2}$$

$$y = \mathcal{L}^{-1}\left\{ \frac{e^{-4s} \cdot 5}{(s^2+25)^2} \right\}$$

$$y = \frac{1}{5}\mathcal{L}^{-1}\left\{ e^{-4s}\left(\frac{5}{s^2+25}\right) \cdot \left(\frac{5}{s^2+25}\right) \right\}$$

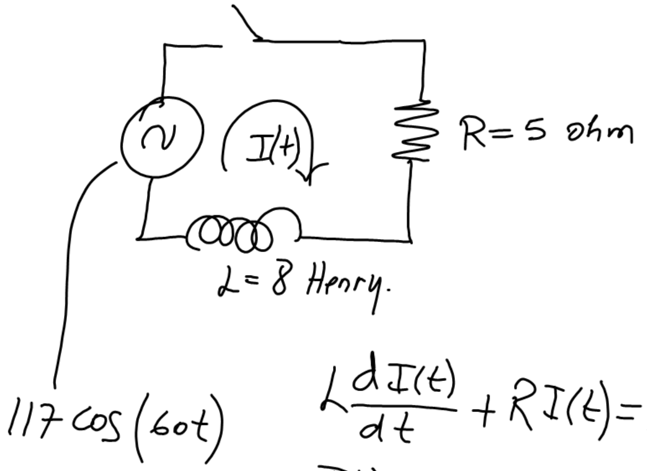
$$\mathcal{L}^{-1}\left\{ \frac{5}{s^2+25} \cdot \frac{5}{s^2+25} \right\} = \operatorname{sen}(5t) * \operatorname{sen}(5t)$$

$$\frac{1}{5}\mathcal{L}^{-1}\left\{ e^{-4s}\left(\frac{5}{s^2+25} \cdot \frac{5}{s^2+25}\right) \right\} = \left(\operatorname{sen}(5(t-4)) * \operatorname{sen}(5(t-4)) \right) u(t-4)$$

$$\mathcal{L}^{-1}\left\{ F(s) \cdot G(s) \right\} = f(t) * g(t)$$

$$\boxed{f(t) * g(t) = \int_0^t f(\tau) \cdot g(t-\tau)\, d\tau}$$

convolución



$$L\frac{dI(t)}{dt} + RI(t) = \mu(t-4)\cdot 117\cos(60t).$$

$$I(0) = 0$$

$$8\frac{dI}{dt} + 5I = 117\,\mu(t-4)\cos(60(t-4)). \quad I(0)=0$$

$$\mathcal{L}\left\{8\frac{dI}{dt} + 5I\right\} = 117\,\mathcal{L}\left\{\mu(t-4)\cos(60(t-4))\right\}$$

$$8\left(s\mathcal{L}\{I\} - I(0)\right) + 5\mathcal{L}\{I\} = 117\left(e^{-4s}\frac{s}{s^2+3600}\right)$$

$$s\mathcal{L}\{I\} + \frac{5}{8}\mathcal{L}\{I\} = \frac{117}{8}\,e^{-4s}\frac{s}{s^2+3600}$$

$$\mathcal{L}\{I\} = \frac{117}{8}\left(\frac{e^{-4s}\cdot s}{(s^2+3600)(s+\frac{5}{8})}\right)$$

EDO(2)LCCNH. C.i.

$$x'' + 4x' + 4x = 8e^{-2t} \quad x(0)=1 \quad x'(0)=1$$

$$\mathcal{L}\{x'' + 4x' + 4x\} = 8\mathcal{L}\{e^{-2t}\}$$

$$\mathcal{L}\{x''\} + 4\mathcal{L}\{x'\} + 4\mathcal{L}\{x\} = 8\left(\frac{1}{s+2}\right)$$

$$\left(s^2\mathcal{L}\{x\} - s(1) - (1)\right) + 4\left(s\mathcal{L}\{x\} - (1)\right) + 4\mathcal{L}\{x\} = \frac{8}{s+2}$$

$$(s^2+4s+4)\mathcal{L}\{x\} - s - 5 = \frac{8}{s+2}$$

$$(s^2+4s+4)\mathcal{L}\{x\} = \frac{8}{s+2} + (s+5)$$

$$= \frac{8 + (s+5)(s+2)}{(s+2)}$$

$$= \frac{s^2+7s+10+8}{(s+2)}$$

$$(s^2+4s+4)\mathcal{L}\{x\} = \frac{s^2+7s+18}{(s+2)}$$

$$\mathcal{L}\{x\} = \frac{s^2+7s+18}{(s+2)(s^2+4s+4)} \Rightarrow \frac{s^2+7s+18}{(s+2)^3}$$

$$\frac{s^2+7s+18}{(s+2)^3} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{(s+2)^3}$$

$$s^2+7s+18 = A(s+2)^2 + B(s+2) + C$$

$$= As^2 + 4sA + 4A + Bs + 2B + C$$

$$s^2+7s+18 = As^2 + (4A+B)s + (4A+2B+C)$$

$$\boxed{A=1}$$

$$4A+B=7 \qquad \boxed{B = 7-4 \Rightarrow 3}$$

$$4A+2B+C=18$$

$$\boxed{C = 18-4-6 \Rightarrow 8}$$

$$\mathcal{L}\{x\} = \frac{1}{s+2} + \frac{3}{(s+2)^2} + \frac{8}{(s+2)^3}$$

$$x(t) = \mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} + 3\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2}\right\} + \frac{8}{2}\mathcal{L}^{-1}\left\{\frac{2}{(s+2)^3}\right\}$$

$$\boxed{x(t) = e^{-2t} + 3te^{-2t} + 4t^2e^{-2t}}$$

SOLUCIÓN PARTICULAR

SISTEMAS DE E.D.O. (1) L.

$$\frac{dx}{dt} = 2x + 3y \quad -(1) \qquad x(t) \qquad x(0) = 5$$

$$\frac{dy}{dt} = x - y \quad -(2) \qquad y(t) \qquad y(0) = -4$$

$$x = \frac{dy}{dt} + y$$

$$\frac{dx}{dt} = \frac{d^2y}{dt^2} + \frac{dy}{dt}$$

$$\left(\frac{d^2y}{dt^2} + \frac{dy}{dt}\right) = 2\left(\frac{dy}{dt} + y\right) + 3y$$

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 5y = 0$$

$$m^2 - m - 5 = 0$$

$$m = \frac{1 \pm \sqrt{1+20}}{2}$$

$$\left(m - \frac{1+\sqrt{21}}{2}\right)\left(m - \frac{1-\sqrt{21}}{2}\right) = 0$$

$$m = \frac{1}{2} \pm \frac{\sqrt{21}}{2}$$

$$\boxed{y(t) = C_1 e^{\frac{1+\sqrt{21}}{2}t} + C_2 e^{\frac{1-\sqrt{21}}{2}t}}$$

$$m_1 = \frac{1+\sqrt{21}}{2} \qquad m_2 = \frac{1-\sqrt{21}}{2}$$

$$\frac{dy}{dt} = \left(\frac{1+\sqrt{21}}{2}\right) C_1 e^{\frac{1+\sqrt{21}}{2}t} + \left(\frac{1-\sqrt{21}}{2}\right) C_2 e^{\frac{1-\sqrt{21}}{2}t}$$

$$\boxed{x(t) = \left(1 + \frac{1+\sqrt{21}}{2}\right) C_1 e^{\frac{1+\sqrt{21}}{2}t} + \left(1 + \frac{1-\sqrt{21}}{2}\right) C_2 e^{\frac{1-\sqrt{21}}{2}t}}$$

$$x(0) \Rightarrow 5 = \left(1 + \frac{1+\sqrt{21}}{2}\right) C_1(1) + \left(1 + \frac{1-\sqrt{21}}{2}\right) C_2(1)$$

$$y(0) \Rightarrow -4 = C_1(1) + C_2(1)$$

$$C_1 + C_2 = -4$$

$$\left(1 + \frac{1+\sqrt{21}}{2}\right) C_1 + \left(1 + \frac{1-\sqrt{21}}{2}\right) C_2 = 5$$

$$\left(1 + \frac{1+\sqrt{21}}{2}\right) C_1 + \left(1 + \frac{1-\sqrt{21}}{2}\right)(-4 - C_1) = 5$$

$$\left(1 + \frac{1+\sqrt{21}}{2} - 1 - \frac{1-\sqrt{21}}{2}\right) C_1 = 5 + 4\left(1 + \frac{1-\sqrt{21}}{2}\right)$$

$$\left\{\begin{array}{l} C_1 = \dfrac{5 + 4\left(1 + \frac{1-\sqrt{21}}{2}\right)}{\sqrt{21}} \\[2ex] C_2 = -4 - \dfrac{5 + 4\left(1 + \frac{1-\sqrt{21}}{2}\right)}{\sqrt{21}} \end{array}\right.$$