

$$\frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + b_1(t) \quad x_1(t) \quad x_1(0)$$

$$\frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + b_2(t) \quad x_2(t) \quad x_2(0)$$

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$$\bar{X} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad \frac{d}{dt} \bar{X} = \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} \quad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \bar{b} = \begin{bmatrix} b_1(t) \\ b_2(t) \end{bmatrix} \quad \bar{X}(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix}$$

$$\begin{bmatrix} e^{At} \end{bmatrix} \rightarrow \frac{d}{dt} \begin{bmatrix} e^{At} \end{bmatrix} = A \cdot \begin{bmatrix} e^{At} \end{bmatrix} \quad \begin{bmatrix} e^{At} \end{bmatrix}_{t=0} = I$$

$$\begin{bmatrix} e^{At} \end{bmatrix}^{-1} = \begin{bmatrix} e^{A(-t)} \end{bmatrix}$$

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} b_1(t) \\ b_2(t) \end{bmatrix}$$

$A_{n \times n}$

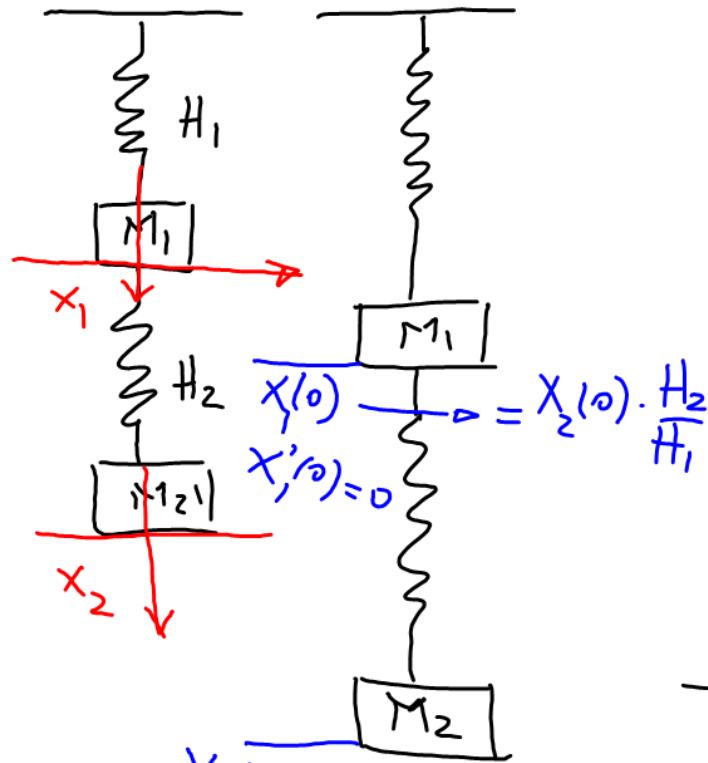
$$\frac{d}{dt} \bar{X} = A \bar{X} + \bar{b}(t)$$

$$\bar{X} = \underbrace{\begin{bmatrix} e^{At} \end{bmatrix} \bar{X}(0)}_{\text{Sol Hom}} + \underbrace{\int_0^t e^{A(t-\tau)} \bar{b}(\tau) d\tau}_{\text{Sol No Hom}}.$$

$$\begin{array}{l|l} x_1(0)=4 & \frac{dx_1}{dt} = x_1 + 2x_2 + 4e^{2t} + \cos(3t) \\ x_2(0)=-2 & \frac{dx_2}{dt} = -3x_1 + 4x_2 + 2e^{2t} + 8\sin(3t). \end{array}$$

$$A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} \quad \bar{X}(0) = \begin{bmatrix} 4 \\ -2 \end{bmatrix} \quad \bar{b}(t) = \begin{bmatrix} 4e^{2t} + \cos(3t) \\ 2e^{2t} + 8\sin(3t) \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} e^{\frac{5}{2}t} \cos\left(\frac{1}{2}t\sqrt{15}\right) - \frac{1}{5}\sqrt{15} e^{\frac{5}{2}t} \sin\left(\frac{1}{2}t\sqrt{15}\right) & \frac{4}{15}\sqrt{15} e^{\frac{5}{2}t} \sin\left(\frac{1}{2}t\sqrt{15}\right) \\ -\frac{2}{5}\sqrt{15} e^{\frac{5}{2}t} \sin\left(\frac{1}{2}t\sqrt{15}\right) & e^{\frac{5}{2}t} \cos\left(\frac{1}{2}t\sqrt{15}\right) + \frac{1}{5}\sqrt{15} e^{\frac{5}{2}t} \sin\left(\frac{1}{2}t\sqrt{15}\right) \end{bmatrix}$$



$$M_1 \frac{dx_3}{dt} = -H_1 x_1 + H_2 (x_2 - x_1)$$

$$M_2 \frac{dx_4}{dt} = -H_2 (x_2 - x_1)$$

$$\frac{dx_1}{dt} = x_3$$

$$\frac{dx_2}{dt} = x_4$$

$$x_2(0) = 5 \text{ [cm]}$$

$$x_2'(0) = 0$$

$$x_1(0) = \frac{H_2}{H_1} \cdot 5$$

$$x_2(0) = 5$$

$$x_3(0) = 0$$

$$x_4(0) = 0$$

$$\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{H_1+H_2}{M_1} & \frac{H_2}{M_1} & 0 & 0 \\ \frac{H_2}{M_2} & -\frac{H_2}{M_2} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

A.

