$$\frac{dx_1}{dt} = x_1 + 2x_2 \qquad x_1(0) = 2$$

$$\frac{dx_2}{dt} = 4x_1 + 3x_2 \qquad x_2(0) = -2$$

$$L\left\{\frac{dx_1}{dt}\right\} = L\{x_1\} + 2L\{x_2\}$$

$$L\left\{\frac{dx_2}{dt}\right\} = 4L\{x_1\} + 3L\{x_2\}$$

$$sL\{x_1\} - x_1(0) = L\{x_1\} + 2L\{x_2\}$$

$$sL\{x_2\} - x_2(0) = 4L\{x_1\} + 3L\{x_2\}$$

$$sL\{x_1\} - L\{x_1\} = 2L\{x_2\} + 2$$

$$sL\{x_2\} - 3\{x_2\} = 4L\{x\} - 2$$

$$(s-1)L\{x_1\} = 2L\{x_2\} + 2$$

$$(s-3)L\{x_1\} = 4L\{x_2\} - 2$$

$$(s-1)L\{x_1\} - 2L\{x_2\} = 2$$

$$-4L\{x_1\} + (s-3)L\{x_2\} = -2$$

$$\begin{bmatrix} (s-1) & -2 \\ -4 & (s-3) \end{bmatrix} \begin{bmatrix} L\{x_1\} \\ L\{x_2\} \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\left[s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \right] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \left[s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \right]^{-1} \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$\rightarrow \begin{cases} \frac{d}{dt}\bar{x} = A\bar{x} \\ \bar{x} = e^{At}\bar{x}(0) \end{cases}$$

$$\Rightarrow e^{At} = \mathcal{L}^{-1}\left\{ \underline{(sI-A)^{-1}} \right\}$$

$$e^{A(0)} = I \qquad \frac{d}{dt}e^{At} = Ae^{At}$$

$$\rightarrow e^{A(t)} \times e^{A(-t)} = I$$

$$e^{At} = B_0(t)\,I + B_1(t)\,A + B_2(t)\,A^2 + \dots + B_{n-1}(t)\,A^{n-1}$$

$A_{n\times n}$

$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}_{2\times 2}$

$e^{At} = B_0(t)\,I + B_1(t)\,A.$

λ_1, λ_2

$$\begin{cases} e^{\lambda_1 t} = B_0(t) + \lambda_1 B_1(t) \\ e^{\lambda_2 t} = B_0(t) + \lambda_2 B_1(t) \end{cases}$$

$\det(A - \lambda I) = 0$

$$\begin{vmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{vmatrix} = 0$$

$(1-\lambda)(3-\lambda) - (4)(2) = 0$

$\lambda^2 - 4\lambda + 3 - 8 = 0$

$\lambda^2 - 4\lambda - 5 = 0$

$(\lambda+1)(\lambda-5) = 0$

$\lambda_1 = -1$

$\lambda_2 = 5$

$e^{-t} = B_0 - B_1$

$e^{5t} = B_0 + 5B_1$

-1

$-e^{-t} = -B_0 + B_1$

$e^{5t} - e^{-t} = 6B_1$

$B_0 = e^{-t} + B_1$

$B_0 = e^{-t} + \frac{1}{6}(e^{5t} - e^{-t}).$

$$\begin{cases} B_1 = \frac{1}{6}(e^{5t} - e^{-t}) \\ B_0 = \frac{1}{6}(e^{5t} + 5e^{-t}) \end{cases}$$

$e^{At} = B_0 I + B_1 A.$

$$e^{At} = \frac{1}{6}\underbrace{\left(e^{5t} + 5e^{-t}\right)}_{B_0}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{1}{6}\underbrace{\left(e^{5t} - e^{-t}\right)}_{B_1}\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

$$e^{At} = \frac{1}{6}\begin{bmatrix} 1+1 & 2 \\ 4 & 1+3 \end{bmatrix} e^{5t} + \frac{1}{6}\begin{bmatrix} 5-1 & -2 \\ -4 & 5-3 \end{bmatrix} e^{-t}$$

$$e^{At} = \frac{1}{6}\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix} e^{5t} + \frac{1}{6}\begin{bmatrix} 4 & -2 \\ -4 & 2 \end{bmatrix} e^{-t}$$

$$e^{At} = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} e^{5t} + \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix} e^{-t}$$

$$e^{At} = \begin{bmatrix} \frac{e^{5t}+2e^{-t}}{3} & \frac{e^{5t}-e^{-t}}{3} \\ \frac{2e^{5t}-2e^{-t}}{3} & \frac{2e^{5t}+e^{-t}}{3} \end{bmatrix} \qquad e^{A(0)} = I.$$

$$e^{A(0)} = \begin{bmatrix} \frac{3}{3} & 0 \\ 0 & \frac{3}{3} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$e^{At}$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ \\ x_n \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \\ \\ x_n \end{bmatrix} + \bar{b}(t)$$

$$\bar{x} = e^{At}\bar{x}(0) + \int_0^t e^{A(t-\tau)}\bar{b}(\tau)\,d\tau$$

$$\left[e^{At}\bar{x}(0)\right]_{t=0} = \bar{x}(0)$$

$$\left[\int_0^t e^{A(t-\tau)}\bar{b}(\tau)\,d\tau\right]_{t=0} = \begin{bmatrix} 0 \\ 0 \\ \\ 0 \end{bmatrix}$$

NP	19	19
CERO	6	
5	4	10
6	6	
7	9	
8	2	
9	2	
10	6	25
		54