```
> restart
> Sist := diff(x[1](t), t) = x[1](t) + 2·x[2](t) + 5·exp(3·t), diff(x[2](t), t) = 4·x[1](t) + 3
      ·x[2](t) − 7·exp(2·t) : Sist[1]; Sist[2]
```

$$\frac{d}{dt} x_1(t) = x_1(t) + 2\,x_2(t) + 5\,e^{3t}$$

$$\frac{d}{dt} x_2(t) = 4\,x_1(t) + 3\,x_2(t) - 7\,e^{2t} \quad (1)$$

```
> Cond := x[1](0) = 2, x[2](0) = -2 : Cond[1]; Cond[2]
```

$$x_1(0) = 2$$

$$x_2(0) = -2 \quad (2)$$

```
> BB := 5·exp(3·t), -7·exp(2·t) : BB[1]; BB[2]
```

$$5\,e^{3t}$$

$$-7\,e^{2t} \quad (3)$$

```
> with(LinearAlgebra) :
> with(inttrans) :
> with(linalg) :
> AA := Matrix([[1, 2], [4, 3]])
```

$$AA := \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \quad (4)$$

```
> II := Matrix([[1, 0], [0, 1]])
```

$$II := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (5)$$

```
> MatUno := evalm(s·II − AA)
```

$$MatUno := \begin{bmatrix} s-1 & -2 \\ -4 & s-3 \end{bmatrix} \quad (6)$$

```
> InvMatUno := inverse(MatUno)
```

$$InvMatUno := \begin{bmatrix} \frac{s-3}{s^2-4s-5} & \frac{2}{s^2-4s-5} \\ \frac{4}{s^2-4s-5} & \frac{s-1}{s^2-4s-5} \end{bmatrix} \quad (7)$$

```
> MatExp := map(expand, (map(convert, map(invlaplace, InvMatUno, s, t), exp)))
```

$$MatExp := \begin{bmatrix} \frac{1}{3}\,(e^t)^5 + \frac{2}{3\,e^t} & \frac{1}{3}\,(e^t)^5 - \frac{1}{3\,e^t} \\ \frac{2}{3}\,(e^t)^5 - \frac{2}{3\,e^t} & \frac{2}{3}\,(e^t)^5 + \frac{1}{3\,e^t} \end{bmatrix} \quad (8)$$

```
> MatExpDos := exponential(AA, t)
```

(9)

$$MatExpDos := \begin{bmatrix} \frac{1}{3}e^{5t} + \frac{2}{3}e^{-t} & \frac{1}{3}e^{5t} - \frac{1}{3}e^{-t} \\ \frac{2}{3}e^{5t} - \frac{2}{3}e^{-t} & \frac{2}{3}e^{5t} + \frac{1}{3}e^{-t} \end{bmatrix} \quad (9)$$

> $MatExpTres := MatrixExponential(AA, t)$

$$MatExpTres := \begin{bmatrix} \frac{1}{3}e^{5t} + \frac{2}{3}e^{-t} & \frac{1}{3}e^{5t} - \frac{1}{3}e^{-t} \\ \frac{2}{3}e^{5t} - \frac{2}{3}e^{-t} & \frac{2}{3}e^{5t} + \frac{1}{3}e^{-t} \end{bmatrix} \quad (10)$$

> $Identidad := map(rcurry(eval, t='0'), MatExpDos)$

$$Identidad := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (11)$$

> $DerMatExp := map(diff, MatExpDos, t)$

$$DerMatExp := \begin{bmatrix} \frac{5}{3}e^{5t} - \frac{2}{3}e^{-t} & \frac{5}{3}e^{5t} + \frac{1}{3}e^{-t} \\ \frac{10}{3}e^{5t} + \frac{2}{3}e^{-t} & \frac{10}{3}e^{5t} - \frac{1}{3}e^{-t} \end{bmatrix} \quad (12)$$

> $Comprobacion := evalm(DerMatExp - evalm(AA \&* MatExpDos))$

$$Comprobacion := \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (13)$$

> $IdentidadDos := simplify(evalm(map(rcurry(eval, t='-t'), MatExpDos) \&* MatExpDos))$

$$IdentidadDos := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (14)$$

> $evalm(MatExpDos)$

$$\begin{bmatrix} \frac{1}{3}e^{5t} + \frac{2}{3}e^{-t} & \frac{1}{3}e^{5t} - \frac{1}{3}e^{-t} \\ \frac{2}{3}e^{5t} - \frac{2}{3}e^{-t} & \frac{2}{3}e^{5t} + \frac{1}{3}e^{-t} \end{bmatrix} \quad (15)$$

> $Xcero := array([[2], [-2]])$

$$Xcero := \begin{bmatrix} 2 \\ -2 \end{bmatrix} \quad (16)$$

> $MatExpHom := evalm(MatExpDos \&* Xcero)$

$$MatExpHom := \begin{bmatrix} 2e^{-t} \\ -2e^{-t} \end{bmatrix} \quad (17)$$

> $BBtau := map(rcurry(eval, t='tau'), array([[BB[1]], [BB[2]]]))$

(18)

$$BBtau := \begin{bmatrix} 5\,e^{3\tau} \\ -7\,e^{2\tau} \end{bmatrix} \tag{18}$$

> $MatExpTau := map(rcurry(eval, t='t-tau'), MatExpDos)$

$$MatExpTau := \begin{bmatrix} \frac{1}{3}\,e^{5t-5\tau} + \frac{2}{3}\,e^{-t+\tau} & \frac{1}{3}\,e^{5t-5\tau} - \frac{1}{3}\,e^{-t+\tau} \\ \frac{2}{3}\,e^{5t-5\tau} - \frac{2}{3}\,e^{-t+\tau} & \frac{2}{3}\,e^{5t-5\tau} + \frac{1}{3}\,e^{-t+\tau} \end{bmatrix} \tag{19}$$

> $ProdTau := evalm(\ MatExpTau\ \&*\ BBtau) : ProdTau[1,1]; ProdTau[2,1]$

$$5\left(\frac{1}{3}\,e^{5t-5\tau} + \frac{2}{3}\,e^{-t+\tau}\right)e^{3\tau} - 7\left(\frac{1}{3}\,e^{5t-5\tau} - \frac{1}{3}\,e^{-t+\tau}\right)e^{2\tau}$$

$$5\left(\frac{2}{3}\,e^{5t-5\tau} - \frac{2}{3}\,e^{-t+\tau}\right)e^{3\tau} - 7\left(\frac{2}{3}\,e^{5t-5\tau} + \frac{1}{3}\,e^{-t+\tau}\right)e^{2\tau} \tag{20}$$

> $MatExpNoHom := map(int, ProdTau, tau = 0..t)$

$$MatExpNoHom := \begin{bmatrix} \frac{1}{18}\left(e^{6t} + 28\,e^{3t} - 29\right)e^{-t} \\ -\frac{1}{18}\left(-2\,e^{6t} + 45\,e^{4t} - 14\,e^{3t} - 29\right)e^{-t} \end{bmatrix} \tag{21}$$

> $Compr := map(rcurry(eval, t='0'), MatExpNoHom)$

$$Compr := \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{22}$$

> $Sol[1] := x[1](t) = simplify(MatExpHom[1,1] + MatExpNoHom[1,1])$

$$Sol_1 := x_1(t) = \frac{7}{18}\,e^{-t} + \frac{1}{18}\,e^{5t} + \frac{14}{9}\,e^{2t} \tag{23}$$

> $Sol[2] := x[2](t) = simplify(MatExpHom[2,1] + MatExpNoHom[2,1])$

$$Sol_2 := x_2(t) = -\frac{7}{18}\,e^{-t} + \frac{1}{9}\,e^{5t} - \frac{5}{2}\,e^{3t} + \frac{7}{9}\,e^{2t} \tag{24}$$

> $Sist[1]$

$$\frac{d}{dt}x_1(t) = x_1(t) + 2\,x_2(t) + 5\,e^{3t} \tag{25}$$

> $Sist[2]$

$$\frac{d}{dt}x_2(t) = 4\,x_1(t) + 3\,x_2(t) - 7\,e^{2t} \tag{26}$$

> $CompUno := simplify(eval(subs(x[1](t) = rhs(Sol[1]), x[2](t) = rhs(Sol[2]), lhs(Sist[1]) - rhs(Sist[1]) = 0)))$

$$CompUno := 0 = 0 \tag{27}$$

> $CompDos := simplify(eval(subs(x[1](t) = rhs(Sol[1]), x[2](t) = rhs(Sol[2]), lhs(Sist[2]) - rhs(Sist[2]) = 0)))$

$$CompDos := 0 = 0 \tag{28}$$

> $CompTres := simplify(subs(t = 0, Sol[1]))$

$$CompTres := x_1(0) = 2 \tag{29}$$

```
> CompCuatro := simplify(subs(t = 0, Sol[2]))
```

$$CompCuatro := x_2(0) = -2 \quad \textbf{(30)}$$

```
>
```