Capítulo 3. Transformada de Laplace y sistemas de EDO's.

$y(t) \Rightarrow y_1(t)$

$y'(t) \Rightarrow \frac{dy_1}{dt}$

$y''(t) \Rightarrow \frac{dy_2}{dt}$

$y'''(t) \Rightarrow \frac{dy_3}{dt}$

$$\frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} + \frac{dy}{dt} + y = 4e^{2t}$$

$y(0) = 1$

$y'(0) = 2$

$y''(0) = 3.$

$$\boxed{\begin{aligned} \frac{dy_1}{dt} &= y_2 \\ \frac{dy_2}{dt} &= y_3 \\ \frac{dy_3}{dt} &= -y_1 - y_2 - y_3 + 4e^{2t} \end{aligned}}$$

$y_1(0) = 1$

$y_2(0) = 2$

$y_3(0) = 3.$

$$\frac{d}{dt}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 4e^{2t} \end{bmatrix} \qquad \bar{y}(0) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\frac{d}{dt}\bar{y} = A\bar{y} + \bar{b}(t)$$

$$\bar{y} = \underbrace{e^{At}\bar{y}(0)}_{\text{Sol Hom}} + \underbrace{\int_0^t e^{A(t-z)}\, b(z)\,dz}_{\text{Sol No Hom}}$$

$$\frac{dx_1}{dt} = 2x_1 + 3x_2 + 8e^{t}$$

$$\frac{dx_2}{dt} = 5x_1 + 4x_2 + 2e^{2t}$$

$$3x_2 = \frac{dx_1}{dt} - 2x_1 - 8e^{t}$$

$$x_2 = \frac{1}{3}\left(\frac{dx_1}{dt} - 2x_1 - 8e^{t}\right)$$

$$\frac{dx_2}{dt} = \frac{1}{3}\left(\frac{d^2x_1}{dt^2} - 2\frac{dx_1}{dt} - 8e^{t}\right)$$

$$\frac{1}{3}\left(\frac{d^2x_1}{dt^2} - 2\frac{dx_1}{dt} - 8e^{t}\right) = 5x_1 + \frac{4}{3}\left(\frac{dx_1}{dt} - 2x_1 - 8e^{t}\right) + 2e^{2t}$$

$$\frac{d^2x_1}{dt^2} - 2\frac{dx_1}{dt} - 8e^{t} = 15x_1 + 4\left(\frac{dx_1}{dt} - 2x_1 - 8e^{t}\right) + 6e^{2t}$$

$$\frac{d^2x_1}{dt^2} - 2\frac{dx_1}{dt} - 15x_1 - 4\frac{dx_1}{dt} + 8x_1 = -32e^{t} + 8e^{t} + 6e^{2t}$$

$$\boxed{\frac{d^2x_1}{dt^2} - 6\frac{dx_1}{dt} - 7x_1 = -24e^{t} + 6e^{2t}}$$

$$y''' + y'' + y' + y = 4e^{2t}$$

$$y''' = -y'' - y' - y + 4e^{2t}$$