```
> restart
> Ecua := diff(y(t), t$3) + diff(y(t), t$2) + diff(y(t), t) + y(t) = 4·exp(2·t)
```

$$Ecua := \frac{d^3}{dt^3} y(t) + \frac{d^2}{dt^2} y(t) + \frac{d}{dt} y(t) + y(t) = 4\, e^{2t} \tag{1}$$

```
> Cond := y(0) = 1, D(y)(0) = 2, D(D(y))(0) = 3
```

$$Cond := y(0) = 1, D(y)(0) = 2, D^{(2)}(y)(0) = 3 \tag{2}$$

```
> with(inttrans) :
> EcuaTrans := subs(Cond, laplace(Ecua, t, s))
```

$$EcuaTrans := s^3\, laplace(y(t), t, s) - 6 - 3\, s - s^2 + s^2\, laplace(y(t), t, s) + s\, laplace(y(t), t, s) + laplace(y(t), t, s) = \frac{4}{s-2} \tag{3}$$

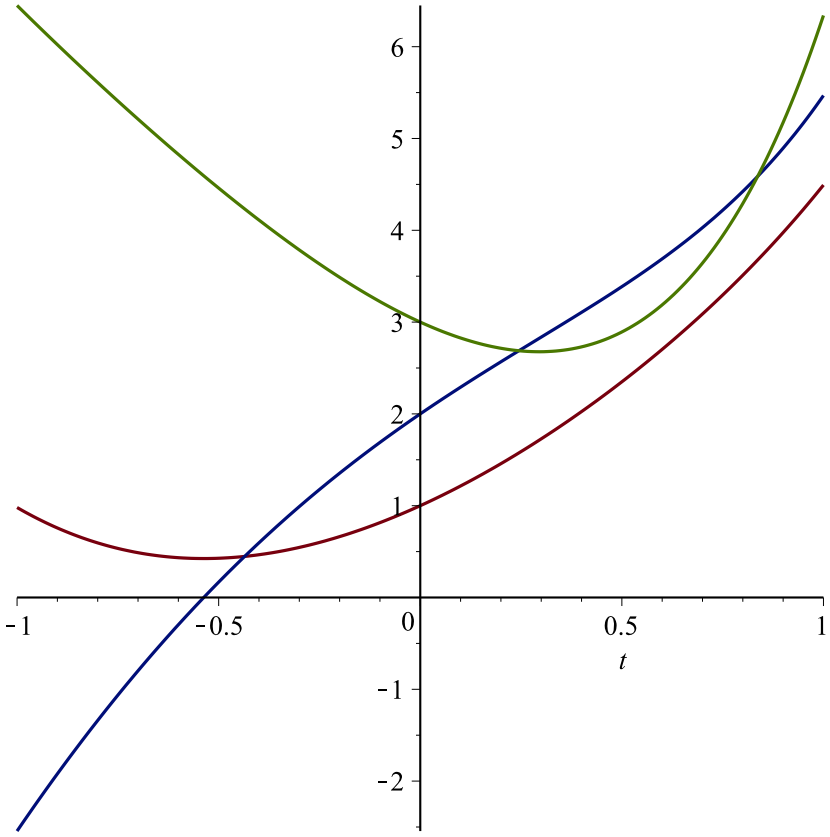
```
> SolTrans := isolate(EcuaTrans, laplace(y(t), t, s))
```

$$SolTrans := laplace(y(t), t, s) = \frac{\frac{4}{s-2} + s^2 + 3\, s + 6}{s^3 + s^2 + s + 1} \tag{4}$$

```
> SolPart := invlaplace(SolTrans, s, t)
```

$$SolPart := y(t) = \frac{4}{15}\, e^{2t} - \frac{3}{5}\cos(t) + \frac{14}{5}\sin(t) + \frac{4}{3}\, e^{-t} \tag{5}$$

```
> plot([rhs(SolPart), rhs(diff(SolPart, t)), rhs(diff(SolPart, t$2))], t = -1..1)
```




```
> restart
> Sist := diff(y[1](t), t) = y[2](t), diff(y[2](t), t) = y[3](t), diff(y[3](t), t) = -y[1](t)
      - y[2](t) - y[3](t) + 4·exp(2·t) : Sist[1]; Sist[2]; Sist[3]
```

$$\frac{d}{dt} y_1(t) = y_2(t)$$

$$\frac{d}{dt} y_2(t) = y_3(t)$$

$$\frac{d}{dt} y_3(t) = -y_1(t) - y_2(t) - y_3(t) + 4\, e^{2t} \quad \textbf{(6)}$$

```
> Cond := y[1](0) = 1, y[2](0) = 2, y[3](0) = 3
```

$$Cond := y_1(0) = 1, y_2(0) = 2, y_3(0) = 3 \quad \textbf{(7)}$$

```
> AA := array([[0, 1, 0], [0, 0, 1], [-1, -1, -1]])
```

$$AA := \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix} \quad \textbf{(8)}$$

```
> BB := array([0, 0, 4·exp(2·t)])
```

(9)

$$BB := \begin{bmatrix} 0 & 0 & 4\,e^{2t} \end{bmatrix} \quad (9)$$

> $Cond := array([1, 2, 3])$

$$Cond := \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \quad (10)$$

> $with(linalg)$:

> $MatExp := exponential(AA, t)$

$$MatExp := \begin{bmatrix} \frac{1}{2}e^{-t} + \frac{1}{2}\cos(t) + \frac{1}{2}\sin(t) & \sin(t) & \frac{1}{2}\sin(t) - \frac{1}{2}\cos(t) + \frac{1}{2}e^{-t} \\ -\frac{1}{2}\sin(t) + \frac{1}{2}\cos(t) - \frac{1}{2}e^{-t} & \cos(t) & \frac{1}{2}\cos(t) - \frac{1}{2}e^{-t} + \frac{1}{2}\sin(t) \\ -\frac{1}{2}\cos(t) + \frac{1}{2}e^{-t} - \frac{1}{2}\sin(t) & -\sin(t) & \frac{1}{2}e^{-t} - \frac{1}{2}\sin(t) + \frac{1}{2}\cos(t) \end{bmatrix} \quad (11)$$

> $CompUno := map(rcurry(eval, t = '0'), MatExp)$

$$CompUno := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (12)$$

> $DerMatExp := map(diff, MatExp, t)$

$$DerMatExp := \begin{bmatrix} -\frac{1}{2}\sin(t) + \frac{1}{2}\cos(t) - \frac{1}{2}e^{-t} & \cos(t) & \frac{1}{2}\cos(t) - \frac{1}{2}e^{-t} + \frac{1}{2}\sin(t) \\ -\frac{1}{2}\cos(t) + \frac{1}{2}e^{-t} - \frac{1}{2}\sin(t) & -\sin(t) & \frac{1}{2}e^{-t} - \frac{1}{2}\sin(t) + \frac{1}{2}\cos(t) \\ \frac{1}{2}\sin(t) - \frac{1}{2}e^{-t} - \frac{1}{2}\cos(t) & -\cos(t) & -\frac{1}{2}e^{-t} - \frac{1}{2}\cos(t) - \frac{1}{2}\sin(t) \end{bmatrix} \quad (13)$$

> $CompDos := evalm(DerMatExp - evalm(AA \&* MatExp))$

$$CompDos := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (14)$$

> $SolHom := evalm(MatExp \&* Cond) : SolHom[1]; SolHom[2]; SolHom[3]$

$$2\,e^{-t} - \cos(t) + 4\sin(t)$$
$$\sin(t) + 4\cos(t) - 2\,e^{-t}$$
$$\cos(t) + 2\,e^{-t} - 4\sin(t) \quad (15)$$

> $MatExpTau := map(rcurry(eval, t = 't - tau'), MatExp) : MatExpTau[2, 2]$

$$\cos(t - \tau) \quad (16)$$

> $BBtau := map(rcurry(eval, t = 'tau'), BB)$

$$BBtau := \begin{bmatrix} 0 & 0 & 4\,e^{2\tau} \end{bmatrix} \quad (17)$$

> $ProdTau := evalm(MatExpTau \&* BBtau) : ProdTau[1]; ProdTau[2]; ProdTau[3]$

$$4\left(\frac{1}{2}\sin(t-\tau)-\frac{1}{2}\cos(t-\tau)+\frac{1}{2}e^{-t+\tau}\right)e^{2\tau}$$

$$4\left(\frac{1}{2}\cos(t-\tau)-\frac{1}{2}e^{-t+\tau}+\frac{1}{2}\sin(t-\tau)\right)e^{2\tau}$$

$$4\left(\frac{1}{2}e^{-t+\tau}-\frac{1}{2}\sin(t-\tau)+\frac{1}{2}\cos(t-\tau)\right)e^{2\tau} \quad \textbf{(18)}$$

> `SolNoHom := simplify(map(int, ProdTau, tau = 0..t)) : SolNoHom[1]; SolNoHom[2]; SolNoHom[3]`

$$\frac{2}{15}e^{-t}\left(-9\sin(t)e^{t}+3\cos(t)e^{t}+2e^{3t}-5\right)$$

$$\frac{2}{15}e^{-t}\left(4e^{3t}-3\sin(t)e^{t}-9\cos(t)e^{t}+5\right)$$

$$\frac{2}{15}e^{-t}\left(8e^{3t}+9\sin(t)e^{t}-3\cos(t)e^{t}-5\right) \quad \textbf{(19)}$$

> `CompTres := map(rcurry(eval, t='0'), SolNoHom)`

$$CompTres := \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \quad \textbf{(20)}$$

> `SolPart[1] := y[1](t) = simplify(SolHom[1] + SolNoHom[1])`

$$SolPart_1 := y_1(t) = \frac{14}{5}\sin(t)-\frac{3}{5}\cos(t)+\frac{4}{15}e^{2t}+\frac{4}{3}e^{-t} \quad \textbf{(21)}$$

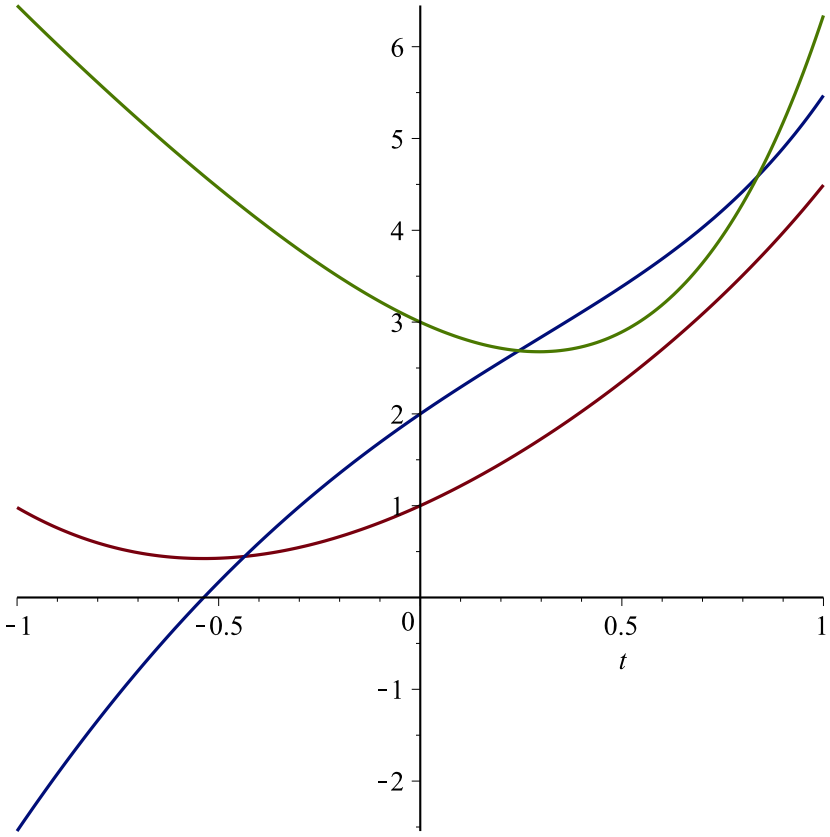
> `SolPart[2] := y[2](t) = simplify(SolHom[2] + SolNoHom[2])`

$$SolPart_2 := y_2(t) = \frac{3}{5}\sin(t)+\frac{14}{5}\cos(t)+\frac{8}{15}e^{2t}-\frac{4}{3}e^{-t} \quad \textbf{(22)}$$

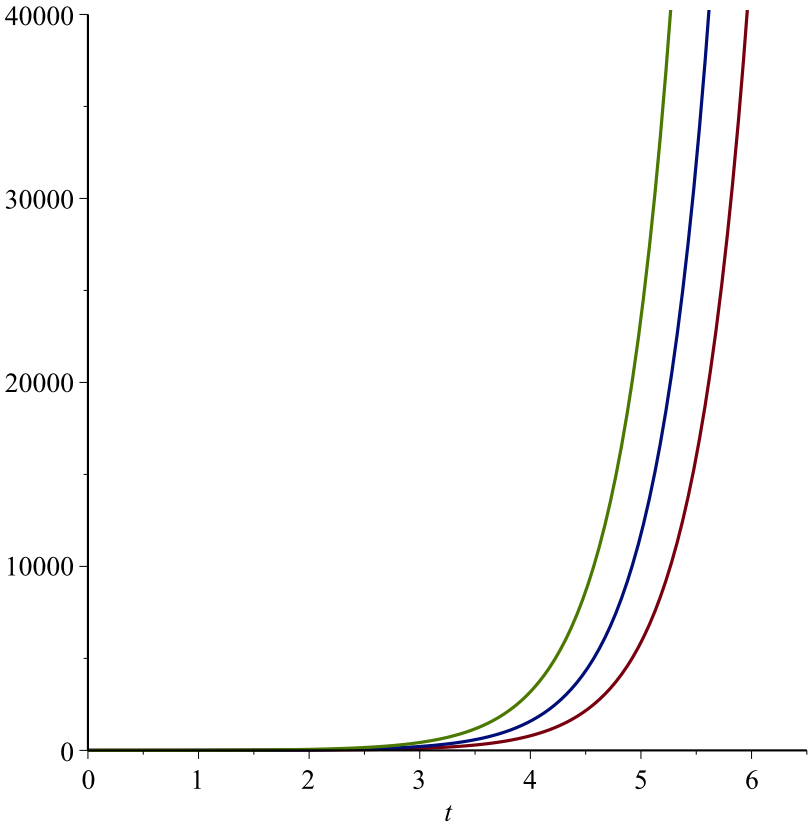
> `SolPart[3] := y[3](t) = simplify(SolHom[3] + SolNoHom[3])`

$$SolPart_3 := y_3(t) = -\frac{14}{5}\sin(t)+\frac{3}{5}\cos(t)+\frac{16}{15}e^{2t}+\frac{4}{3}e^{-t} \quad \textbf{(23)}$$

> `plot([rhs(SolPart[1]), rhs(SolPart[2]), rhs(SolPart[3])], t = -1..1)`



> $plot([rhs(SolPart[1]), rhs(SolPart[2]), rhs(SolPart[3])], t = 0..10)$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