TRANSFORMADA DE LAPLACE.

ESTÁ DISEÑADA PARA PROBLEMAS CON CONDICIONES INICIALES.

$$\frac{d^2y}{dt^2}+\frac{dy}{dt}+y=3e^t \qquad y(0)=5 \qquad y(2)=10$$

$$y'(0)=a$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2}\right\}+\mathcal{L}\left\{\frac{dy}{dt}\right\}+\mathcal{L}\{y\}=\frac{3}{s-1}$$

$$\left[s^2\mathcal{L}\{y\}-s(5)-a\right]+\left[s\mathcal{L}\{y\}-(5)\right]+\mathcal{L}\{y\}=\frac{3}{s-1}$$

$$(s^2+s+1)\mathcal{L}\{y\}-5s-(a+5)=\frac{3}{s-1}$$

$$(s^2+s+1)\mathcal{L}\{y\}=\frac{3}{s-1}+5s+a+5$$

$$=\frac{3+(5s+a+5)(s-1)}{(s-1)}$$

$$=\frac{5s^2+(\cancel{-5}+a+\cancel{5})s+3-a-5}{(s-1)}$$

$$(s^2+s+1)\mathcal{L}\{y\}=\frac{5s^2+as-(a+2)}{(s-1)}$$

$$\mathcal{L}\{y\}=\frac{5s^2+as-(a+2)}{(s-1)(s^2+s+1)}$$

$$=\frac{A}{s-1}+\frac{Bs+C}{s^2+s+1}$$

$$=A(s^2+s+1)+(Bs+C)(s-1)\Rightarrow 5s^2+as-(a+2)$$

$$=As^2+As+A+Bs^2+Cs-Bs-C=5s^2+as-(a+2)$$

$$=(A+B)s^2+(A-B+C)s+(A-C)=5s^2+as-(a+2)$$

$$A+B=5 \longrightarrow -(a+2)+C+B=5$$

$$A-B+C=a \longrightarrow -(a+2)+C-B+C=a$$

$$A-C=-(a+2)$$

$$\boxed{A=-(a+2)+C}$$

$$C+B=5+a+2$$

$$2C-B=a+a+2$$

$$3C=9+3a$$

$$\boxed{C=3+a}$$

$$A=-(\cancel{a}+2)+3+\cancel{a}$$

$$\boxed{A=1}$$

$$B=7+\cancel{a}-3-\cancel{a}$$

$$\boxed{B=4}$$

$$\mathcal{L}\{y\}=\frac{1}{s-1}+\frac{4s+(3+a)}{s^2+s+1}$$

$$\mathcal{L}\{\cos(bt)\}=\frac{s}{s^2+b^2}$$

$$\mathcal{L}\{e^{at}\cos(bt)\}=\frac{(s-a)}{(s-a)^2+b^2}$$

$$\mathcal{L}\{\operatorname{sen}(bt)\}=\frac{b}{s^2+b^2}$$

$$\mathcal{L}\{e^{at}\operatorname{sen}(bt)\}=\frac{b}{(s-a)^2+b^2}$$

$$y_p=\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}+4\mathcal{L}^{-1}\left\{\frac{s+\left(\frac{3+a}{4}\right)}{s^2+s+1}\right\}$$

$$y=e^t+4\mathcal{L}^{-1}\left\{\frac{s+\left(\frac{3+a}{4}\right)}{(s^2+s+\frac14)+(1-\frac14)}\right\}$$

$$y=e^t+4\mathcal{L}^{-1}\left\{\frac{s+1/2}{(s+\frac12)^2+\left(\frac{\sqrt3}{2}\right)^2}+\frac{\left(\frac{3+a}{4}-\frac12\right)}{(s+\frac12)^2+\left(\frac{\sqrt3}{2}\right)^2}\right\}$$

$$y=e^t+4e^{-\frac12 t}\cos\left(\frac{\sqrt3}{2}t\right)+\cancel{4}\left(\frac{\frac14+\frac{a}{4}}{\cancel{4}}\right)\frac{1}{\frac{\sqrt3}{2}}\mathcal{L}^{-1}\left\{\frac{\frac{\sqrt3}{2}}{(s+\frac12)^2+\left(\frac{\sqrt3}{2}\right)^2}\right\}$$

$$y=e^t+4e^{-\frac12 t}\cos\left(\frac{\sqrt3}{2}t\right)+\frac{\frac12+\frac{a}{2}}{\sqrt3}e^{-\frac12 t}\operatorname{sen}\left(\frac{\sqrt3}{2}t\right)$$

$$y(2)\Rightarrow 10=e^2+4e^{-1}\cos(\sqrt3)+\frac{1+a}{2\sqrt3}e^{-1}\operatorname{sen}(\sqrt3)$$

$$\frac{1+a}{2\sqrt3}=\frac{10-e^2-4e^{-1}\cos(\sqrt3)}{e^{-1}\operatorname{sen}(\sqrt3)}$$

$$a=2\sqrt3\left(\frac{10-e^2-4e^{-1}\cos(\sqrt3)}{e^{-1}\operatorname{sen}(\sqrt3)}\right)-1$$