CAPÍTULO 4

ECUACIONES EN DERIVADAS PARCIALES

EDO $F(x, y, y', y'' \ldots y^{(n)}) = 0$

$y(x)$

VARIABLE INDEPENDIENTE

FUNCIÓN INCÓGNITA.

SOLUCIÓN GENERAL ES SIEMPRE ÚNICA.

$$F\left(x, y, z(x,y), \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \ldots\right) = 0$$

$$z(x,y)$$

INCÓGNITA

VAR. INDEP.

EL ORDEN EDenDP SERÁ LA DERIVADA PARCIAL DE MAYOR ORDEN

$$\frac{\partial^3 z}{\partial x^2 \partial y} + \frac{\partial^2 z}{\partial y^2} = 0 \quad \text{EDenDP}(3).$$

EDO — orden 3

$$y_g = c_1 y_1 + c_2 y_2 + c_3 y_3$$

$$z_g(x,y) = F_1(x,y) + F_2(x,y) + F(x,y)$$

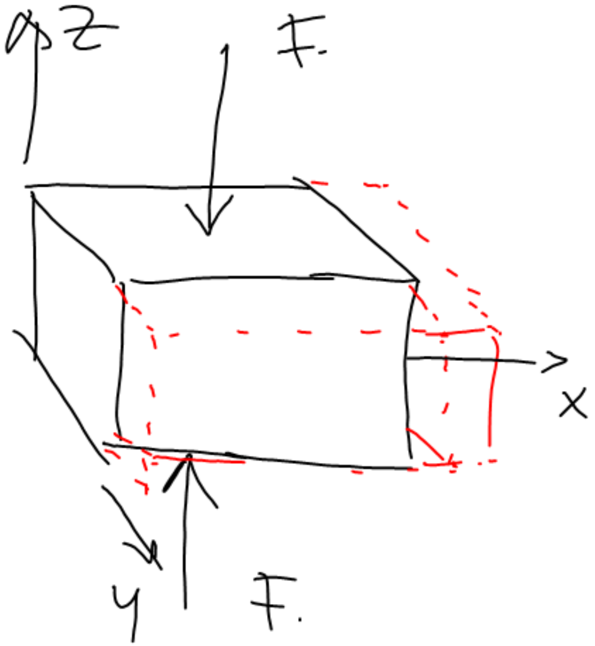
$$\text{EDO}\begin{cases}\text{LINEAL}\\ \\ \text{NO-LINEAL}\end{cases}$$

$$\text{EDenDP}\begin{cases}\text{LINEAL}\\ \text{CUASI LINEAL}\\ \text{NO-LINEAL}\end{cases}$$

$$\frac{\partial F}{\partial x}+\frac{\partial F}{\partial y}=F \qquad \text{LINEAL}$$

$$\left(\frac{\partial F}{\partial x}\right)^2+\left(\frac{\partial F}{\partial y}\right)^2=F \qquad \text{NO LINEAL}$$

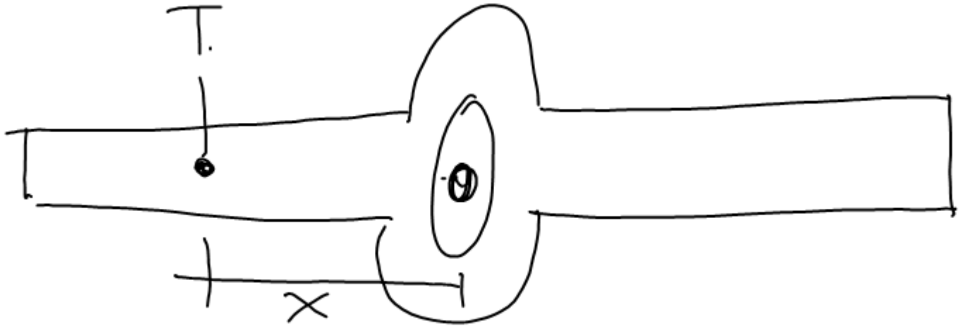
$$\frac{\partial F}{\partial x}+\frac{\partial F}{\partial y}=F^2 \qquad \text{CUASI LINEAL}$$



$Z(x, y)$

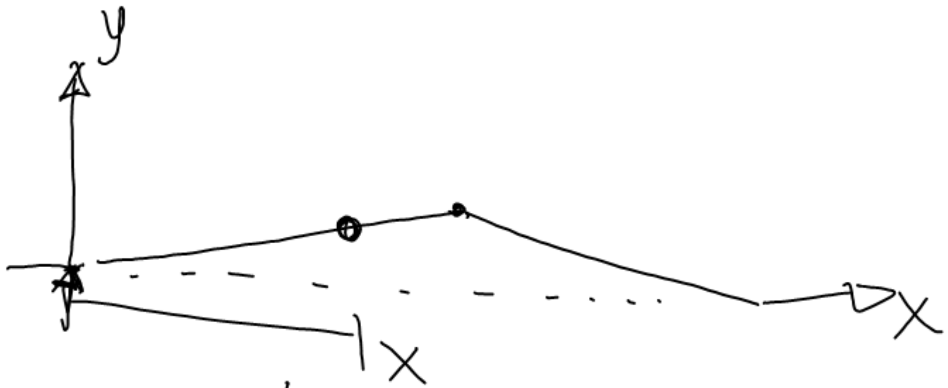
$$\frac{\partial^2 z}{\partial x^2} + a_1 \frac{\partial^2 z}{\partial y^2} = 0$$

1. MECÁNICA DEL MEDIO CONTINUO.



$T(x,t)$

$$\frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial t^2} = 0$$



$$y(x,t)$$

	C.	REALIDAD
EDO	80%	20%
EenDP	20%	80%

$$\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 6\frac{\partial^2 z}{\partial y^2} = 0$$

$f(y+ax) \Rightarrow f(u) \quad u = y + ax$

$\frac{\partial f}{\partial x} = f'(a) \quad \frac{\partial f}{\partial y} = f'(1)$

$\frac{\partial^2 f}{\partial x^2} = a^2 f'' \quad \frac{\partial^2 f}{\partial x \partial y} = af'' \quad \frac{\partial^2 f}{\partial y^2} = f''$

$a^2 f'' + 5af'' + 6f'' = 0$

$(a^2 + 5a + 6)f'' = 0 \quad f'' = 0$

$a^2 + 5a + 6 = 0$

$(a+2)(a+3) = 0$

$a_1 = -2$

$a_2 = -3$

$f' = c_1$

$f = c_1(y+ax) + c_2$

Solución trivial

$f = c_1 y + c_1 ax + c_2$

$f(x,y) = F_1(y-2x) + F_2(y-3x)$ SOLUCIÓN GENERAL

SOLUCIÓN GENERAL

$\frac{\partial f}{\partial x} = -2F_1' - 3F_2'$ $\quad \frac{\partial^2 f}{\partial x^2} = 4F_1'' + 9F_2''$

$\frac{\partial f}{\partial y} = F_1' + F_2'$ $\quad \frac{\partial^2 f}{\partial x \partial y} = -2F_1'' - 3F_2''$ $\quad \frac{\partial^2 f}{\partial y^2} = F_1'' + F_2''$

$(4F_1'' + 9F_2'') + 5(-2F_1'' - 3F_2'') + 6(F_1'' + F_2'') = 0$

$(4 - 10 + 6)F_1'' + (9 - 15 + 6)F_2'' = 0$

$(0)F_1'' + (0)F_2'' = 0$

$0 = 0$

$$f(x,y) = F_1(y-3x) + F_2(y-2x)$$

SERIE TRIGONOMÉTRICA DE FOURIER

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{L}\right)x + b_n \operatorname{sen}\left(\frac{n\pi}{L}\right)y\right).$$