ECUACIONES EN DERIVADAS PARCIALES

MÉTODO DE SEPARACIÓN DE VARIABLES

1.- TIPO PRUEBA Y ERROR

2.- SÓLO SE PUEDE APLICAR PARA $f(x,y)$ DE DOS VARIABLES INDEPENDIENTES.

$$\frac{\partial^2 z(x,y)}{\partial x^2} - 8\frac{\partial z(x,y)}{\partial y} = 0$$

$$\boxed{\text{EenDP }(2)\ L.}$$

$$z(x,y) = F_1(x,y) + F_2(x,y)$$

$$H_0:\ z(x,y) = F(x)\cdot G(y)$$

$$\frac{\partial z}{\partial x} = F'(x)\cdot G(y) \qquad \frac{\partial^2 z}{\partial x^2} = F''(x)\cdot G(y)$$

$$\frac{\partial z}{\partial y} = F(x)\cdot G'(y)$$

$$F''(x)\cdot G(y) - 8\,F(x)\cdot G'(y) = 0$$

$$F''(x)\cdot G(y) = 8\,F(x)\cdot G'(y)$$

EenDP

↓

S.EDO

$$\boxed{\frac{F''(x)}{8\,F(x)} = \frac{G'(y)}{G(y)}}$$

$$\frac{F''(x)}{8F(x)} = \frac{G'(y)}{G(y)}$$

$$\text{SEDO} \left\{ \begin{array}{l} \dfrac{F''(x)}{8F(x)} = \alpha \\ \dfrac{G'(y)}{G(y)} = \alpha \end{array} \right\} \quad \begin{array}{l} \alpha = 0 \\ \alpha > 0 \\ \alpha < 0 \end{array}$$

para $\alpha = 0$

$$\frac{F''(x)}{8F(x)} = 0 \qquad \frac{G'(y)}{G(y)} = 0$$

$$F(x) \neq 0 \quad F''(x) = 0 \qquad G(y) \neq 0 \quad G'(y) = 0$$

$$F'(x) = C_1$$

$$\boxed{\underset{\alpha=0}{F(x)} = C_1 x + C_2} \qquad \boxed{\underset{\alpha=0}{G(y)} = k_1}$$

$$\underset{\alpha=0}{Z(x,y)} = \left(C_1 x + C_2\right) \cdot \left(k_1\right)$$

$$\boxed{\underset{\alpha=0}{Z(x,y)} = C_{10} X + C_{20}.}$$

$$\frac{\partial z}{\partial x} = C_{10} \qquad \frac{\partial^2 z}{\partial x^2} = 0 \qquad \frac{\partial z}{\partial y} = 0$$

$$(0) - 8(0) = 0$$

$$\boxed{0 = 0}$$

para $\alpha > 0$ $\alpha = \beta^2$ $\forall \beta \neq 0$

$$\frac{F''(x)}{8F(x)} = \beta^2 \qquad \frac{G'(y)}{G(y)} = \beta^2$$

$$F''(x) = \beta^2 8F(x) \qquad G'(y) = \beta^2 G(y)$$

$$F''(x) - 8\beta^2 F(x) = 0 \qquad G'(y) - \beta^2 G(y) = 0$$

EDOL(2) CCH EDOL(1) CCH.

$$(D^2 - 8\beta^2)F(x) = 0 \qquad (D - \beta^2)G(y) = 0$$

$$\boxed{\underset{\alpha>0}{G(y)} = k_1 e^{\beta^2 y}}$$

$$m^2 - 8\beta^2 = 0$$

$$(m - \sqrt{8}\beta)(m + \sqrt{8}\beta) = 0$$

$$m_1 = \sqrt{8}\beta \quad m_2 = -\sqrt{8}\beta.$$

$$\boxed{\underset{\alpha>0}{F(x)} = C_1 e^{\sqrt{8}\beta x} + C_2 e^{-\sqrt{8}\beta x}}$$

$$\underset{\alpha>0}{z(x,y)} = \left(C_1 e^{\sqrt{8}\beta x} + C_2 e^{-\sqrt{8}\beta x}\right) k_1 e^{\beta^2 y}$$

$$\boxed{\underset{\alpha>0}{z(x,y)} = C_{10} e^{\sqrt{8}\beta x} e^{\beta^2 y} + C_{20} e^{-\sqrt{8}\beta x} e^{\beta^2 y}}$$

$$\frac{\partial z}{\partial x} = C_{10}\sqrt{8}\beta e^{\sqrt{8}\beta x} e^{\beta^2 y} - C_{20}\sqrt{8}\beta e^{-\sqrt{8}\beta x} e^{\beta^2 y}$$

$$\frac{\partial^2 z}{\partial x^2} = C_{10} 8\beta^2 e^{\sqrt{8}\beta x} e^{\beta^2 y} + C_{20} 8\beta^2 e^{-\sqrt{8}\beta x} e^{\beta^2 y}$$

$$\frac{\partial z}{\partial y} = C_{10}\beta^2 e^{\sqrt{8}\beta x} e^{\beta^2 y} + C_{20}\beta^2 e^{-\sqrt{8}\beta x} e^{\beta^2 y}$$

$$C_{10} 8\beta^2 e^{\sqrt{8}\beta x} e^{\beta^2 y} + C_{20} 8\beta^2 e^{-\sqrt{8}\beta x} e^{\beta^2 y} - 8\left(C_{10}\beta^2 e^{\sqrt{8}\beta x} e^{\beta^2 y} + C_{20}\beta^2 e^{-\sqrt{8}\beta x} e^{\beta^2 y}\right) = 0$$

$$(\cancel{C_{10} 8\beta^2} - \cancel{C_{10} 8\beta^2}) e^{\sqrt{8}\beta x} e^{\beta^2 y} + (\cancel{C_{20} 8\beta^2} - \cancel{C_{20} 8\beta^2}) e^{-\sqrt{8}\beta x} e^{\beta^2 y} = 0$$

$$(0) e^{\sqrt{8}\beta x} e^{\beta^2 y} + (0) e^{-\sqrt{8}\beta x} e^{\beta^2 y} = 0$$

$$0 \equiv 0$$

para $\alpha < 0$ $\quad \alpha = -\beta^2 \quad \forall \beta \neq 0$

$$\frac{F''(x)}{8F(x)} = -\beta^2 \qquad \frac{G'(y)}{G(y)} = -\beta^2$$

$$F''(x) = -\beta^2 8F(x) \qquad G'(y) = -\beta^2 G(y)$$

$$F''(x) + 8\beta^2 F(x) = 0 \qquad G'(y) + \beta^2 G(y) = 0$$

$$(D^2 + 8\beta^2)F(x) = 0 \qquad (D + \beta^2)G(y) = 0$$

$$m^2 + 8\beta^2 = 0 \qquad m_1 = \sqrt{8}\beta i \qquad m_2 = -\sqrt{8}\beta i$$

$$\boxed{G_{\alpha<0}(y) = k_1 e^{-\beta^2 y}}$$

$$\boxed{F_{\alpha<0}(x) = C_1 \cos(\sqrt{8}\beta x) + C_2 \operatorname{sen}(\sqrt{8}\beta x)}$$

$$Z_{\alpha<0}(x,y) = k_1 e^{-\beta^2 y}\left(C_1 \cos(\sqrt{8}\beta x) + C_2 \operatorname{sen}(\sqrt{8}\beta x)\right)$$

$$\boxed{Z_{\alpha<0}(x,y) = C_{10} e^{-\beta^2 y} \cos(\sqrt{8}\beta x) + C_{20} e^{-\beta^2 y} \operatorname{sen}(\sqrt{8}\beta x)}$$

$$SolDerParc := z(x, y) = e^{\sqrt{\_c_1}\, x}\ \_C3\ e^{\frac{1}{8}\,\_c_1\, y}\ \_C1 + \frac{\_C3\ e^{\frac{1}{8}\,\_c_1\, y}\ \_C2}{e^{\sqrt{\_c_1}\, x}}$$

$$z(x, y) = C_{10}\, e^{\sqrt{8\beta}x}\, e^{\beta^2 y} + C_{20}\, e^{-\sqrt{8\beta}x}\, e^{\beta^2 y}$$

$\alpha > 0$