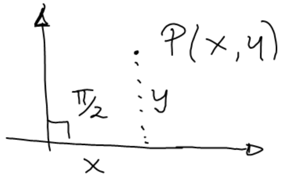
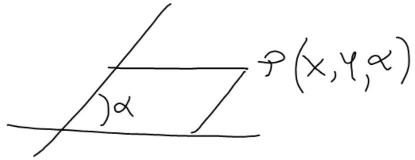
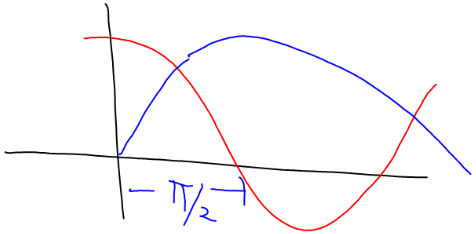
SERIE TRIGONOMÉTRICA DE FOURIER





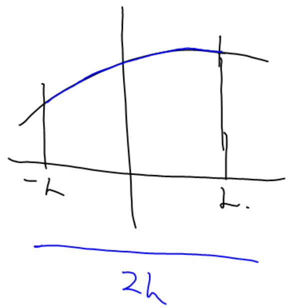


$$f(t) = C + \sum_{n=1}^{\infty}\left(a_n \cos \frac{n\pi}{L}t + b_n \operatorname{sen} \frac{n\pi}{L}t\right)$$

$$C = \frac{a_0}{2} \qquad a_0 = \frac{1}{L}\int_{-L}^{L} f(t)\,dt$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos\left(\frac{n\pi}{L}t\right)dt$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(t)\operatorname{sen}\left(\frac{n\pi}{L}t\right)dt.$$



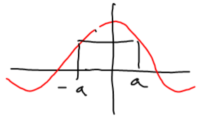
L semidistancia del periodo o intervalo

una función $f(t)$ es par

$-L \le t \le L$

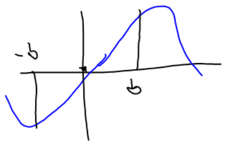
cos(t)

$f(-t) = f(t)$



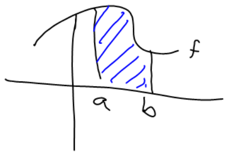
una función $f(t)$ es impar

$f(-t) = -f(t)$ sen(t)



$$\int_{-L}^{L} f\,dt = 2\int_{0}^{L} f(t)\,dt \quad \text{PAR}$$

$$\int_{-L}^{L} f\,dt = 0 \quad \text{IMPAR}$$



$\int_{a}^{b} f\,dt$ $\int_{a}^{a} f\,dt = 0.$

[par]·[par] = par [impar]·[impar] = par

[impar][par] = impar

$$f = C + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi t}{L}\right) + b_n \operatorname{sen}\left(\frac{n\pi t}{L}\right) \right).$$

$$C = \frac{a_0}{2} \qquad a_0 = \frac{1}{L} \int_{-L}^{L} f \, dt$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f \cdot \cos\left(\frac{n\pi t}{L}\right) dt$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f \cdot \operatorname{sen}\left(\frac{n\pi t}{L}\right) dt.$$

cos → par
sen → impar

f → par

$$STF = C + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi t}{L}$$

SERIE COS.

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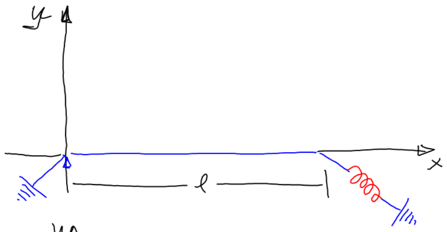
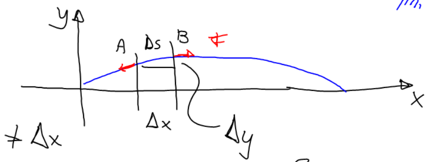
f → impar

$$STF \sum_{n=1}^{\infty} b_n \operatorname{sen}\left(\frac{n\pi t}{L}\right).$$

SERIE SENO

Ejercicio de la cuerda de guitarra





$\Delta s \neq \Delta x$

$$F_r = m a \qquad a = \frac{\partial^2 y(x,t)}{\partial t^2}$$

$$m = \rho \Delta s$$

$$F_r = \rho \Delta s \cdot \frac{\partial^2 y}{\partial t^2}$$

$$F_r = T_B - T_A \qquad \alpha < 4^\circ$$

$$\operatorname{sen}\alpha \doteq \tan\alpha \doteq \frac{\Delta y}{\Delta x}$$

$$T_A = T \cdot \frac{\Delta y}{\Delta x} \qquad \Delta x \to 0$$

$$T_A = T \frac{\partial y}{\partial x}$$

$$T_B = T \frac{\partial y}{\partial x} + \frac{\partial}{\partial x} T \left(\frac{\partial y}{\partial x}\right) \Delta x$$

$$= T \frac{\partial y}{\partial x} + T \frac{\partial^2 y}{\partial x^2} \cdot \Delta x$$

$$F = T \frac{\partial^2 y}{\partial x^2} \cdot \Delta x$$

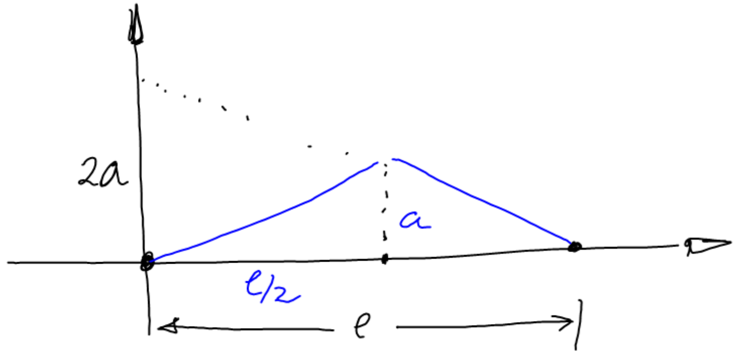
$$T \frac{\partial^2 y}{\partial x^2} \Delta x = \rho \Delta s \frac{\partial^2 y}{\partial t^2}$$

$$\Delta x \to 0 \qquad \Delta x \doteq \Delta s$$

$$T \frac{\partial^2 y}{\partial x^2} = \rho \frac{\partial^2 y}{\partial t^2}$$

$$\frac{T}{\rho} = c^2$$

$$\boxed{\frac{\partial^2 y}{\partial t^2} - c^2 \frac{\partial^2 y}{\partial x^2} = 0}$$



Condición inicial $y(x,t)\Big|_{t=0} = \begin{cases} \frac{2a}{l}x & : 0 \le x \le l/2 \\ 2a - \frac{2a}{l}x & ; l/2 < x \le l. \end{cases}$

$$\left.\frac{\partial y}{\partial x}\right|_{t=0} = 0$$

$\forall t \quad y(0,t) = 0$ Condición frontera.

$y(l,t) = 0$

$$\frac{\partial^2 y}{\partial t^2} - c^2 \frac{\partial^2 y}{\partial x^2} = 0$$