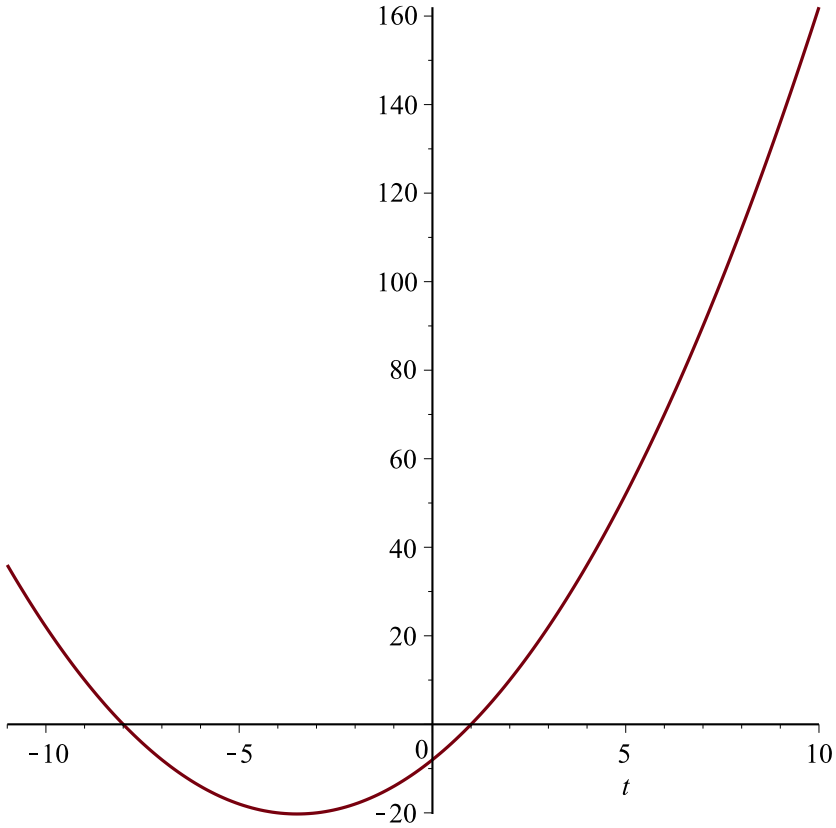
> *restart*

> $f := t\cdot\cdot 2 + 7\cdot t - 8$

$$f := t^2 + 7\,t - 8 \tag{1}$$

> $plot(f, t = -11..10)$



> $L := 10; a[0] := \frac{1}{L}\cdot int(f, t = -L..L); C := \frac{a[0]}{2}$

$$L := 10$$

$$a_0 := \frac{152}{3}$$

$$C := \frac{76}{3} \tag{2}$$

> $a[n] := \frac{1}{L}\cdot int\left(f\cdot\cos\left(\frac{n\cdot Pi\cdot t}{L}\right), t = -L..L\right)$

$$a_n := \frac{8\left(23\,\pi^2\,n^2\sin(n\,\pi) + 50\,n\,\pi\cos(n\,\pi) - 50\sin(n\,\pi)\right)}{n^3\,\pi^3} \tag{3}$$

> $b[n] := \frac{1}{L}\cdot int\left(f\cdot\sin\left(\frac{n\cdot Pi\cdot t}{L}\right), t = -L..L\right)$

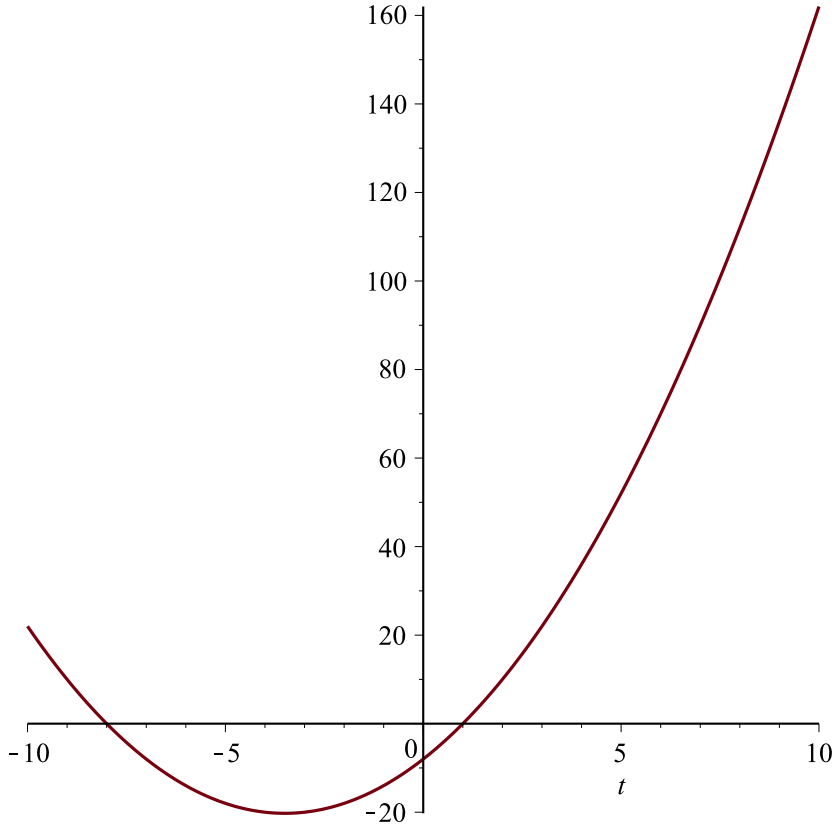
$$b_n := -\frac{140\,(n\,\pi\cos(n\,\pi) - \sin(n\,\pi))}{n^2\,\pi^2} \tag{4}$$

> $g := C + Sum\left(a[n]\cdot\cos\left(\frac{n\cdot \mathrm{Pi}\cdot t}{L}\right) + b[n]\cdot\sin\left(\frac{n\cdot \mathrm{Pi}\cdot t}{L}\right), n = 1\,..infinity\right)$

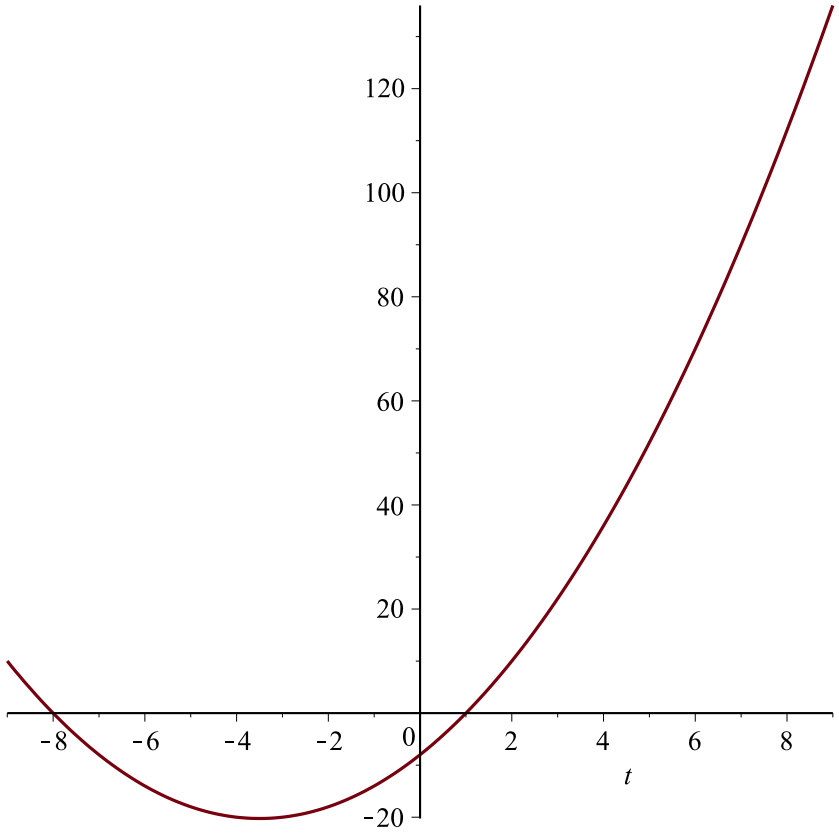
$$g := \frac{76}{3} + \sum_{n=1}^{\infty}\left(\frac{8\,(23\,\pi^2\,n^2\sin(n\,\pi) + 50\,n\,\pi\cos(n\,\pi) - 50\sin(n\,\pi))\cos\left(\frac{1}{10}\,n\,\pi\,t\right)}{n^3\,\pi^3} - \frac{140\,(n\,\pi\cos(n\,\pi) - \sin(n\,\pi))\sin\left(\frac{1}{10}\,n\,\pi\,t\right)}{n^2\,\pi^2}\right) \tag{5}$$

> $STF10000 := C + sum\left(a[n]\cdot\cos\left(\frac{n\cdot \mathrm{Pi}\cdot t}{L}\right) + b[n]\cdot\sin\left(\frac{n\cdot \mathrm{Pi}\cdot t}{L}\right), n = 1\,..10000\right)$:

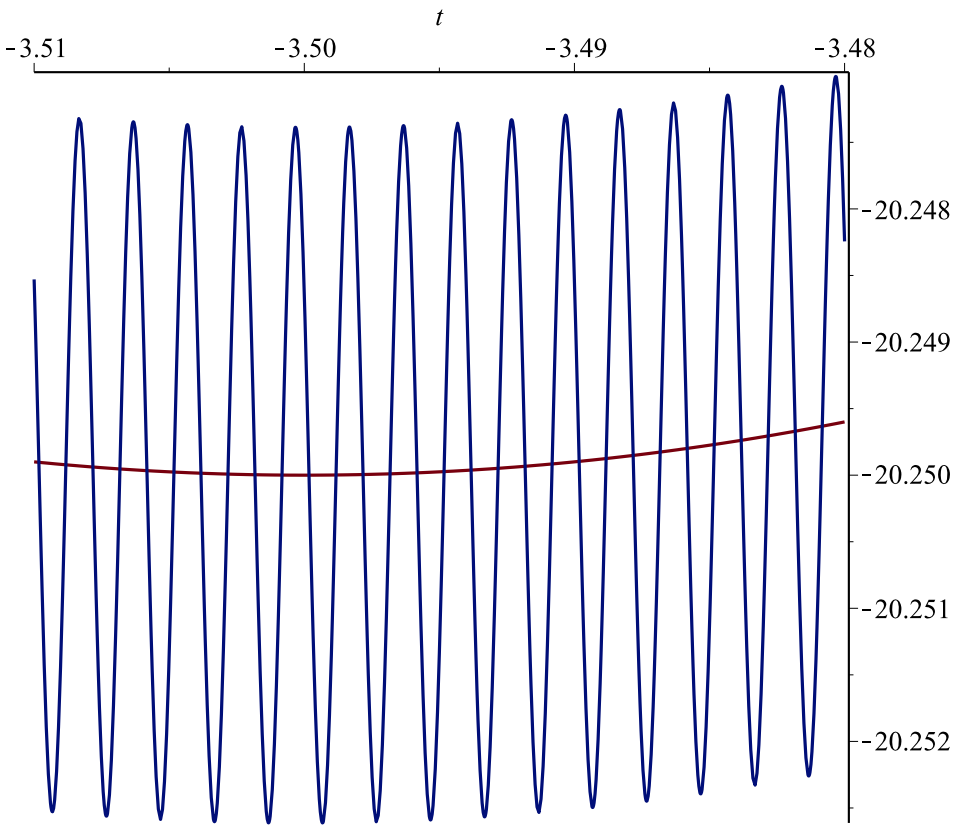
> $plot(f, t = -10\,..10)$



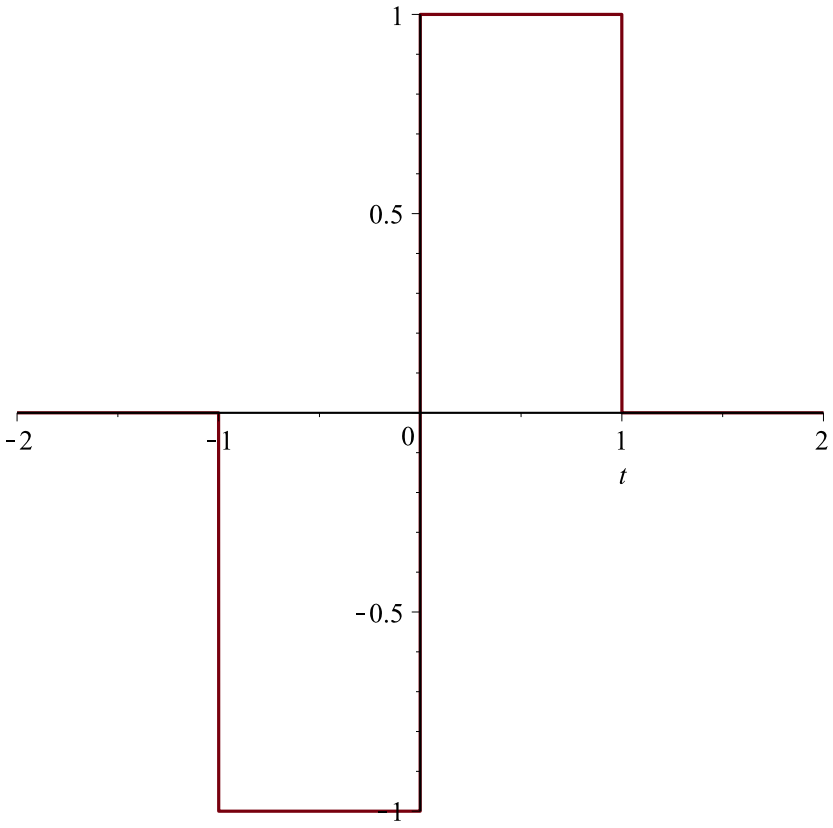
> $plot(STF10000, t = -9\,..9)$



> $plot([f, STF10000], t = -3.51 .. -3.48)$




```
> restart
> f := -Heaviside(t + 1) + 2·Heaviside(t) - Heaviside(t - 1) : plot(f, t = -2..2)
```



> $L := 2$

$$L := 2 \tag{6}$$

> $a[0] := \frac{1}{L} \cdot int(f, t = -L..L)$

$$a_0 := 0 \tag{7}$$

> $C := \frac{a[0]}{2}$

$$C := 0 \tag{8}$$

> $a[n] := \frac{1}{L} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), t = -L..L\right)$

$$a_n := 0 \tag{9}$$

> $b[n] := \frac{1}{L} int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), t = -L..L\right)$

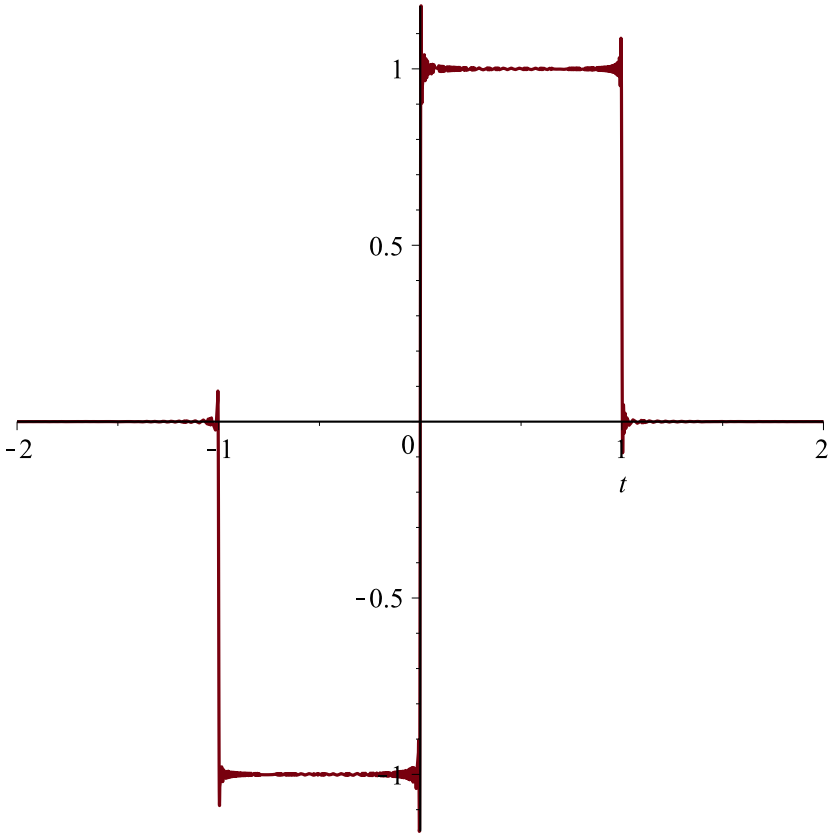
$$b_n := -\frac{2\cos\left(\frac{1}{2} n \pi\right)}{n \pi} + \frac{2}{n \pi} \tag{10}$$

> $STF := Sum\left(b[n]\cdot\sin\left(\frac{n\cdot\mathrm{Pi}\cdot t}{L}\right), n=1..infinity\right)$

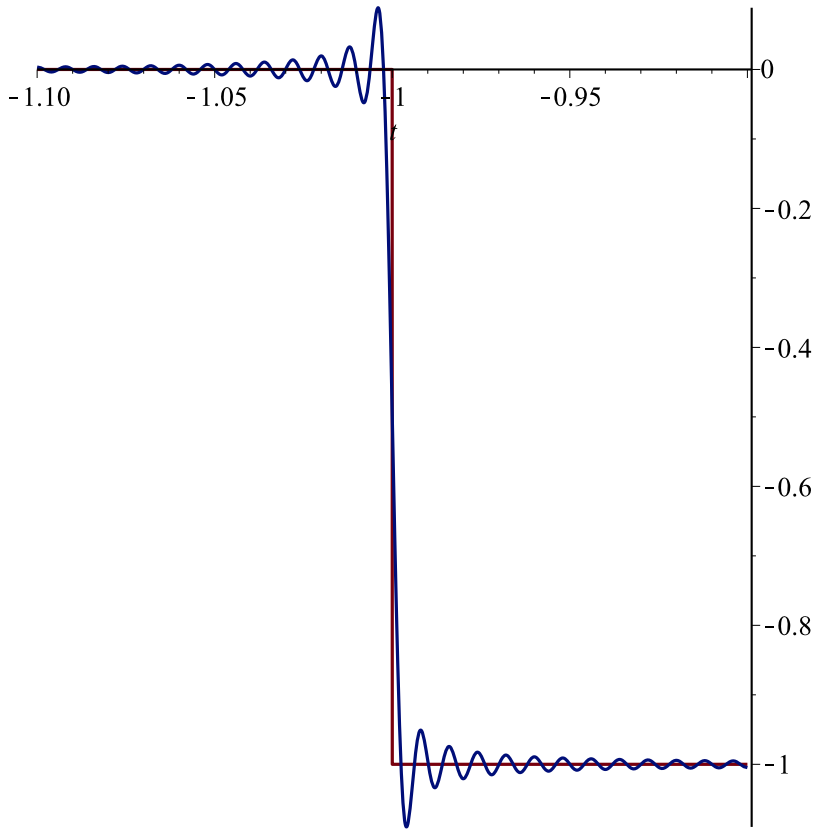
$$STF := \sum_{n=1}^{\infty}\left(-\frac{2\cos\left(\frac{1}{2}n\pi\right)}{n\pi}+\frac{2}{n\pi}\right)\sin\left(\frac{1}{2}n\pi t\right) \quad (11)$$

> $\mathrm{STF500} := C + sum\left(a[n]\cdot\cos\left(\frac{n\cdot\mathrm{Pi}\cdot t}{L}\right)+b[n]\cdot\sin\left(\frac{n\cdot\mathrm{Pi}\cdot t}{L}\right), n=1..500\right):$

> $plot(STF500, t=-2..2)$

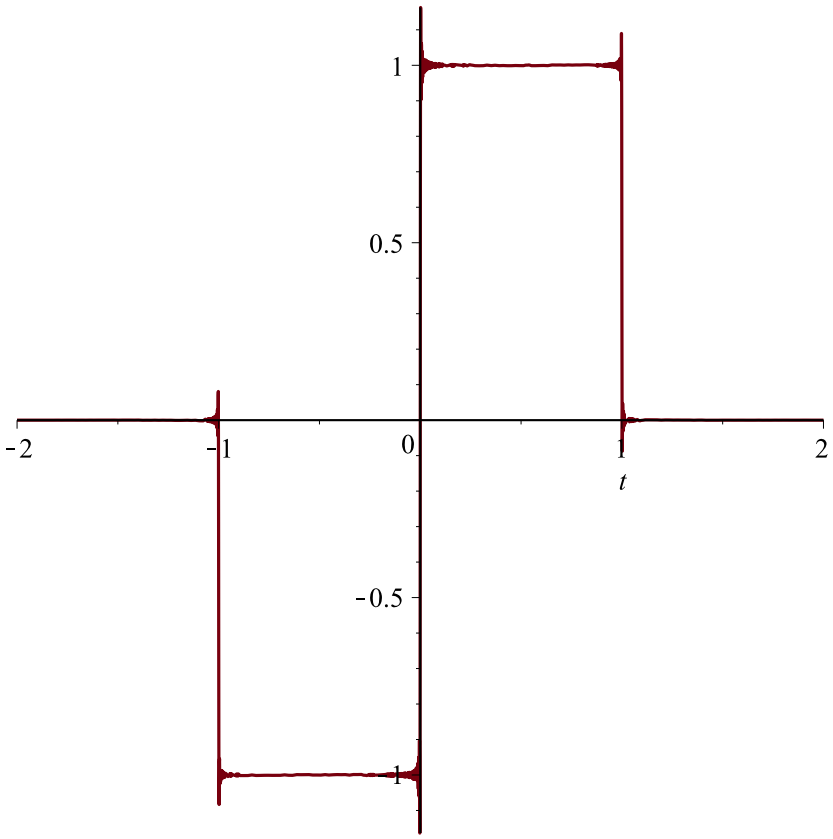


> $plot([f, STF500], t=-1.1..-0.9)$

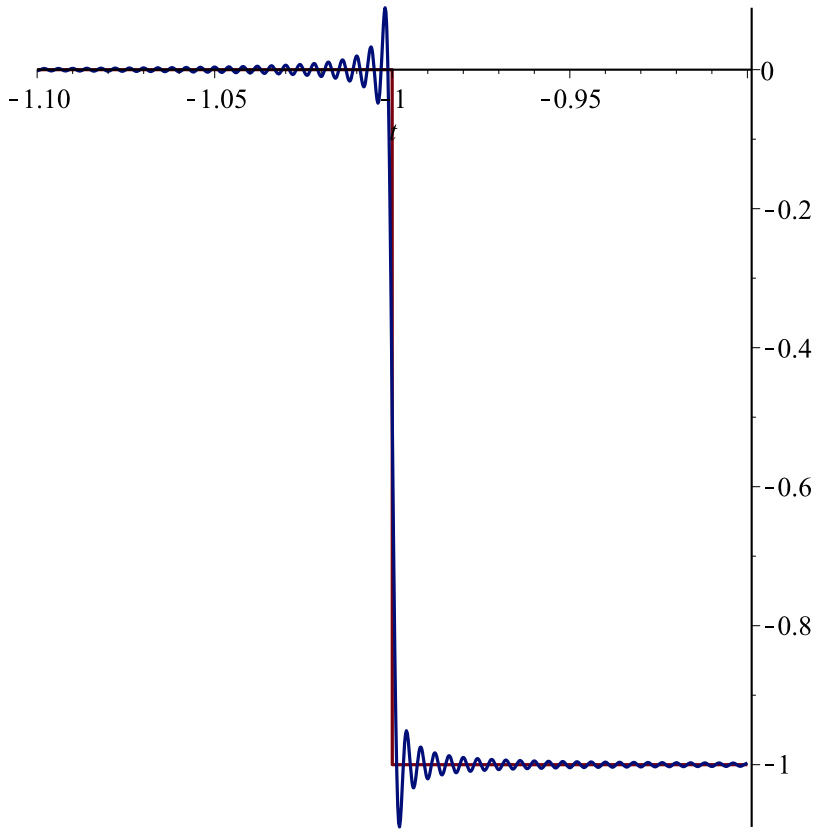


> $\mathrm{STF10000} := C + sum\left(a[n]\cdot\cos\left(\frac{n\cdot\mathrm{Pi}\cdot t}{L}\right) + b[n]\cdot\sin\left(\frac{n\cdot\mathrm{Pi}\cdot t}{L}\right), n = 1..1000\right):$

> $plot(STF10000, t = -2..2)$



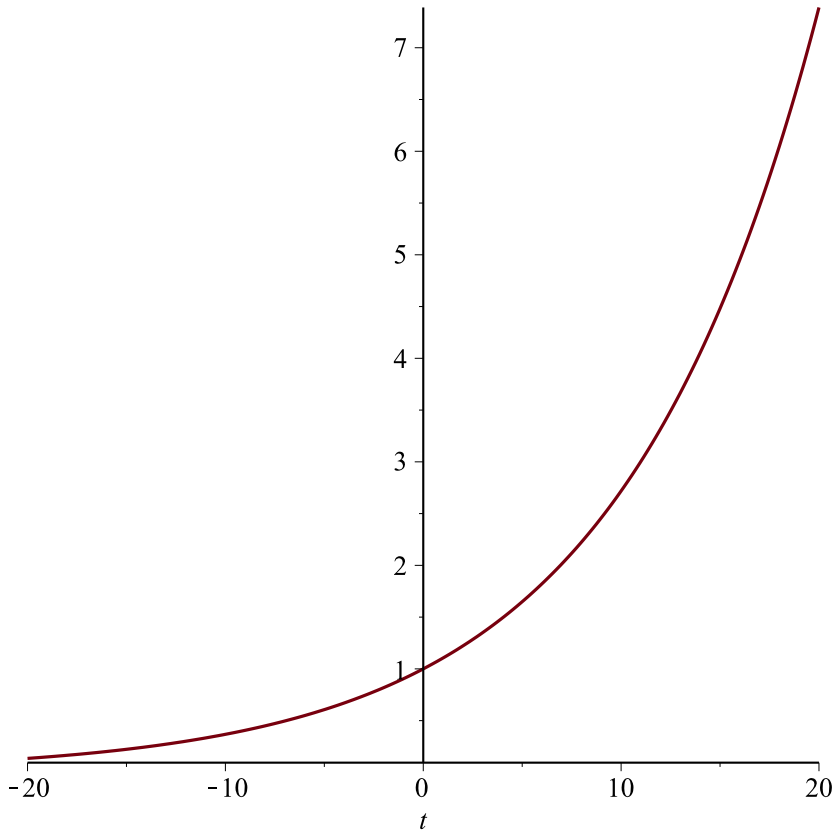
> $plot([f, STF10000], t = -1.1 .. -0.9)$




```
> restart
> f := exp(t/10)
```

$$f := e^{\frac{1}{10} t} \tag{12}$$

```
> plot(f, t = -20..20)
```



> $L := 25$

$$L := 25 \tag{13}$$

> $a[0] := \frac{1}{L} \cdot int(f, t = -L..L); evalf(\%, 3)$

$$a_0 := -\frac{2}{5} e^{-\frac{5}{2}} + \frac{2}{5} e^{\frac{5}{2}}$$

$$4.85 \tag{14}$$

> $C := \frac{a[0]}{2}; evalf(\%, 3)$

$$C := -\frac{1}{5} e^{-\frac{5}{2}} + \frac{1}{5} e^{\frac{5}{2}}$$

$$2.42 \tag{15}$$

> $a[n] := \frac{1}{L} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), t = -L..L\right):$

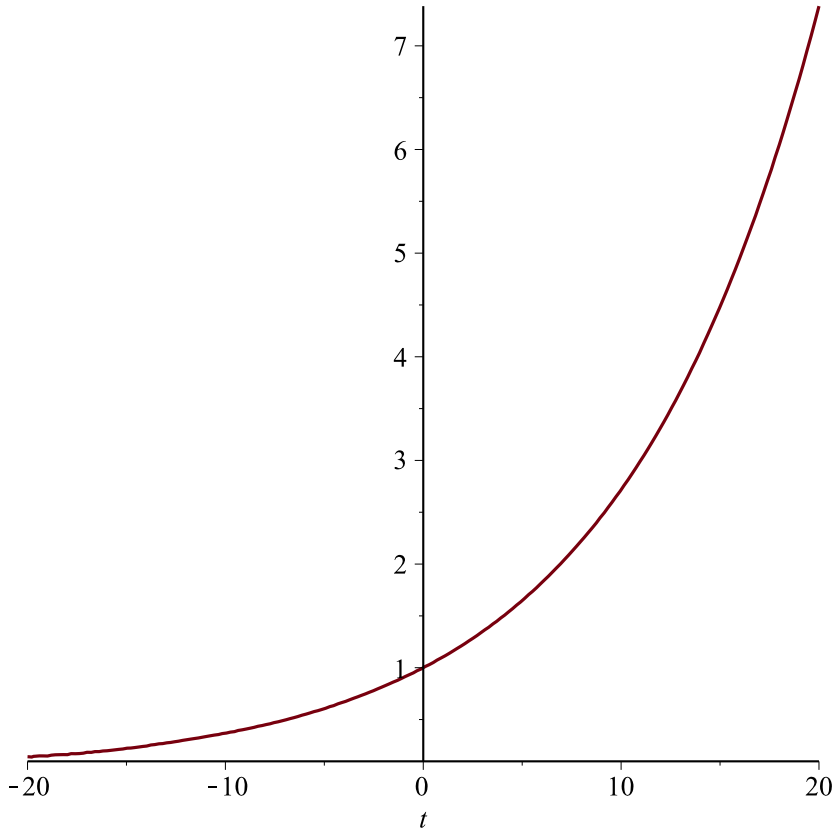
> $b[n] := \frac{1}{L} \cdot int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), t = -L..L\right):$

> $STF := C + Sum\left(a[n]\cdot\cos\left(\frac{n\cdot \mathrm{Pi}\cdot t}{L}\right) + b[n]\cdot\sin\left(\frac{n\cdot \mathrm{Pi}\cdot t}{L}\right), n = 1..infinity\right)$

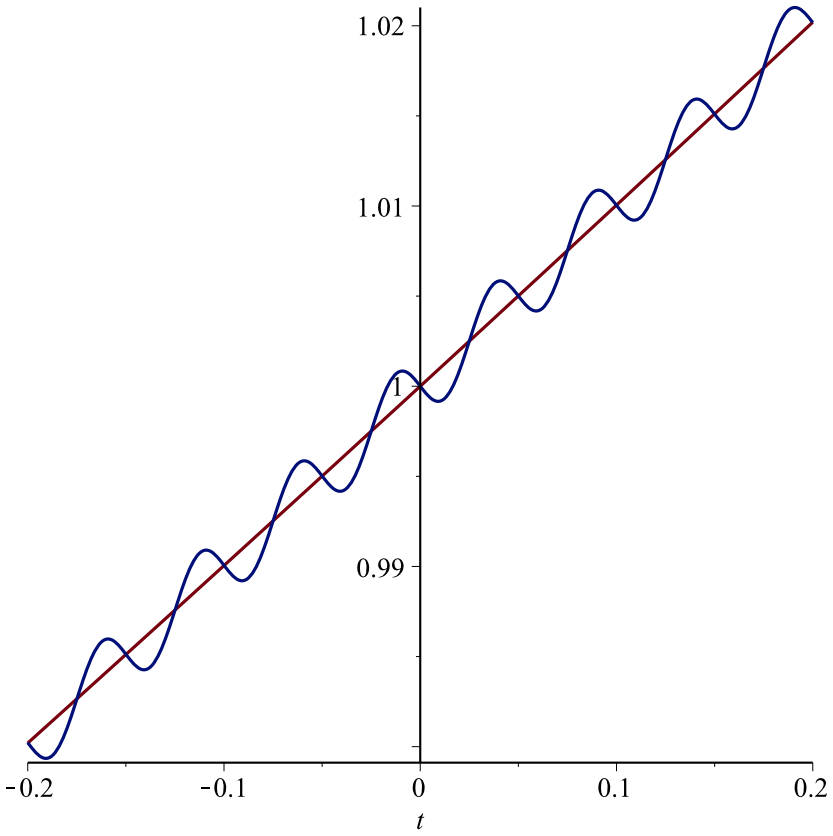
$$STF := -\frac{1}{5}\,e^{-\frac{5}{2}} + \frac{1}{5}\,e^{\frac{5}{2}} + \sum_{n=1}^{\infty}\left(\frac{1}{4\pi^2 n^2 + 25}\left(2\left(2\,e^{\frac{5}{2}}\sin(n\pi)\,\pi n + 2\,e^{-\frac{5}{2}}\sin(n\pi)\,\pi n + 5\,e^{\frac{5}{2}}\cos(n\pi) - 5\,e^{-\frac{5}{2}}\cos(n\pi)\right)\cos\left(\frac{1}{25}\,n\pi t\right)\right) + \frac{1}{4\pi^2 n^2 + 25}\left(2\left(-2\,e^{\frac{5}{2}}\cos(n\pi)\,\pi n + 2\,e^{-\frac{5}{2}}\cos(n\pi)\,\pi n + 5\,e^{\frac{5}{2}}\sin(n\pi) + 5\,e^{-\frac{5}{2}}\sin(n\pi)\right)\sin\left(\frac{1}{25}\,n\pi t\right)\right)\right) \quad \textbf{(16)}$$

> $STF1000 := C + sum\left(a[n]\cdot\cos\left(\frac{n\cdot \mathrm{Pi}\cdot t}{L}\right) + b[n]\cdot\sin\left(\frac{n\cdot \mathrm{Pi}\cdot t}{L}\right), n = 1..1000\right):$

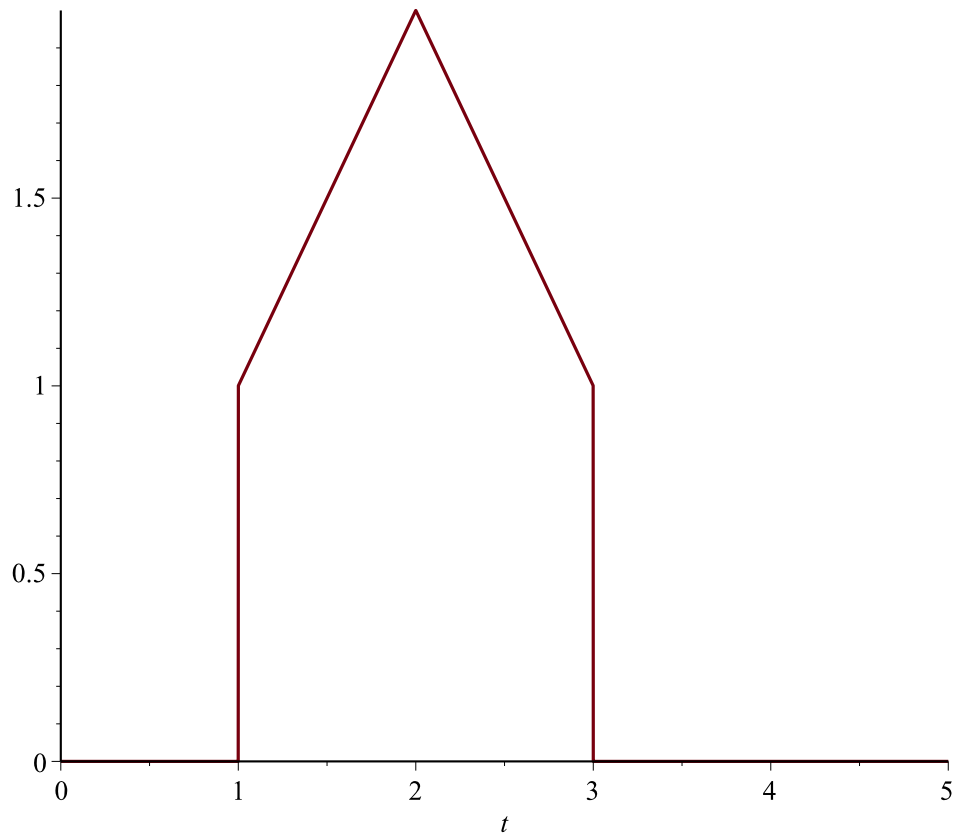
> $plot(STF1000, t = -20..20)$



> $plot([f, STF1000], t = -0.2..0.2)$




```
> restart
> f := Heaviside(t - 1) + (t - 1)·Heaviside(t - 1) - 2·(t - 2)·Heaviside(t - 2) + (t - 3)
      · Heaviside(t - 3) - Heaviside(t - 3) : plot(f, t = 0 ..5)
```



> $L := 2; a[0] := \frac{1}{L} \cdot int(f, t = 0..4); C := \frac{a[0]}{2}$

$$L := 2$$

$$a_0 := \frac{3}{2}$$

$$C := \frac{3}{4} \quad \textbf{(17)}$$

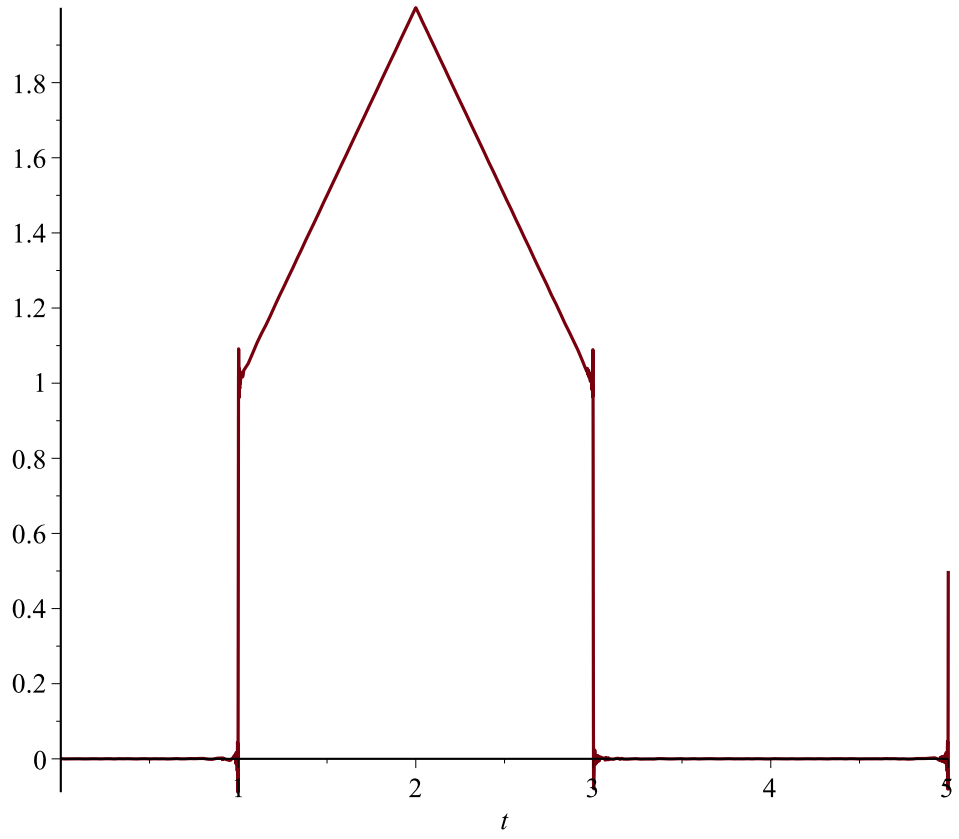
> $a[n] := \frac{1}{L} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), t = 0..4\right)$

$$a_n := -\frac{2\left(\cos\left(\frac{1}{2} n \pi\right) + \frac{1}{2} n \pi \sin\left(\frac{1}{2} n \pi\right)\right)}{n^2 \pi^2} - \frac{2\left(\cos\left(\frac{3}{2} n \pi\right) + \frac{3}{2} n \pi \sin\left(\frac{3}{2} n \pi\right)\right)}{n^2 \pi^2} + \frac{4\left(\cos(n \pi) + n \pi \sin(n \pi)\right)}{n^2 \pi^2} + \frac{4 \sin\left(\frac{3}{2} n \pi\right)}{n \pi} - \frac{4 \sin(n \pi)}{n \pi} \quad \textbf{(18)}$$

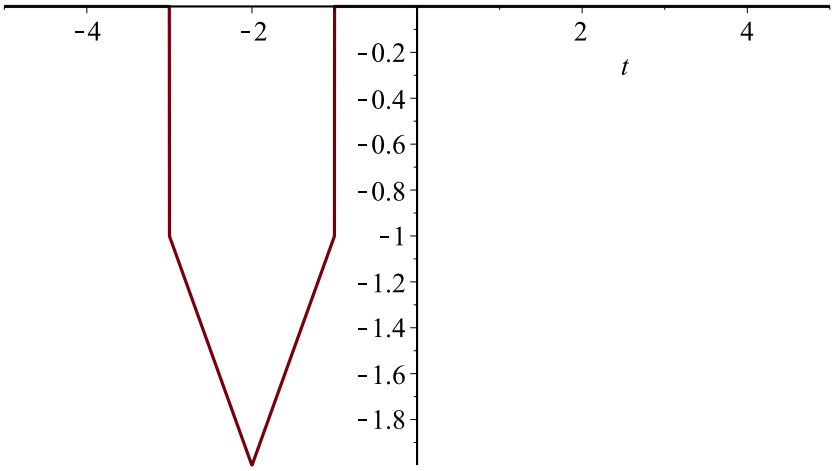
> $b[n] := \frac{1}{L} \cdot int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), t = 0..4\right)$

$$b_n := -\frac{2\left(\sin\left(\frac{1}{2}n\pi\right)-\frac{1}{2}\cos\left(\frac{1}{2}n\pi\right)n\pi\right)}{n^2\pi^2}-\frac{2\left(\sin\left(\frac{3}{2}n\pi\right)-\frac{3}{2}\cos\left(\frac{3}{2}n\pi\right)n\pi\right)}{n^2\pi^2}+\frac{4\left(\sin(n\pi)-\cos(n\pi)n\pi\right)}{n^2\pi^2}-\frac{4\cos\left(\frac{3}{2}n\pi\right)}{n\pi}+\frac{4\cos(n\pi)}{n\pi} \quad \textbf{(19)}$$

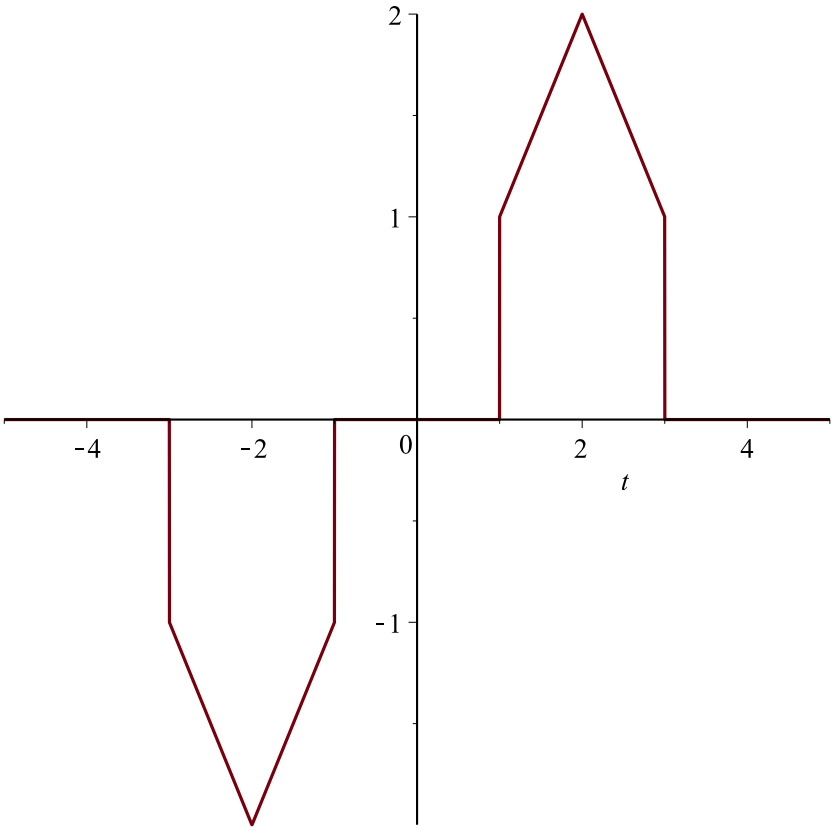
```
> STF10000 := C + sum(a[n]·cos(n·Pi·t/L) + b[n]·sin(n·Pi·t/L), n = 1..1000) :
> plot(STF10000, t = 0..5)
```




```
> g := -Heaviside(t + 3) - (t + 3)·Heaviside(t + 3) + 2·(t + 2)·Heaviside(t + 2) - (t + 1)
      ·Heaviside(t + 1) + Heaviside(t + 1) : plot(g, t = -5..5)
```



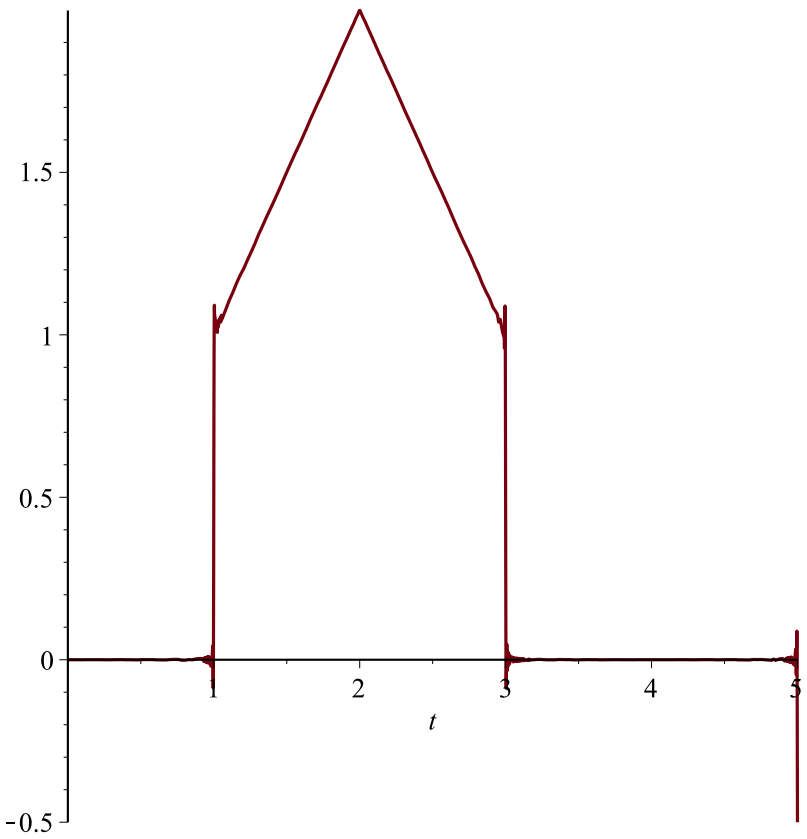
> $h := f + g : plot(h, t = -5..5)$



> $b[n] := \frac{1}{4} \cdot int\left(h \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot t}{4}\right), t = -4..4\right)$

$$b_n := -\frac{4\left(\sin\left(\frac{1}{4}n\pi\right)-\frac{1}{4}\cos\left(\frac{1}{4}n\pi\right)n\pi\right)}{n^2\pi^2}+\frac{8\left(\sin\left(\frac{1}{2}n\pi\right)-\frac{1}{2}\cos\left(\frac{1}{2}n\pi\right)n\pi\right)}{n^2\pi^2}$$
$$-\frac{4\left(\sin\left(\frac{3}{4}n\pi\right)-\frac{3}{4}\cos\left(\frac{3}{4}n\pi\right)n\pi\right)}{n^2\pi^2}$$
$$+\frac{4\left(-\sin\left(\frac{3}{4}n\pi\right)+\frac{3}{4}\cos\left(\frac{3}{4}n\pi\right)n\pi\right)}{n^2\pi^2}$$
$$-\frac{8\left(-\sin\left(\frac{1}{2}n\pi\right)+\frac{1}{2}\cos\left(\frac{1}{2}n\pi\right)n\pi\right)}{n^2\pi^2}$$
$$+\frac{4\left(-\sin\left(\frac{1}{4}n\pi\right)+\frac{1}{4}\cos\left(\frac{1}{4}n\pi\right)n\pi\right)}{n^2\pi^2}+\frac{8\cos\left(\frac{1}{2}n\pi\right)}{n\pi}-\frac{8\cos\left(\frac{3}{4}n\pi\right)}{n\pi} \quad (20)$$

```
> STFseno1000 := sum(b[n]·sin((n·Pi·t)/4), n = 1..1000):
> plot(STFseno1000, t = 0..5)
```


1.5
1
0.5
0
-0.5
1
2
3
4
5
t