> *restart*

Problema de la cuerda de guitarra de 1 mt. largo y rasgando 1 mm

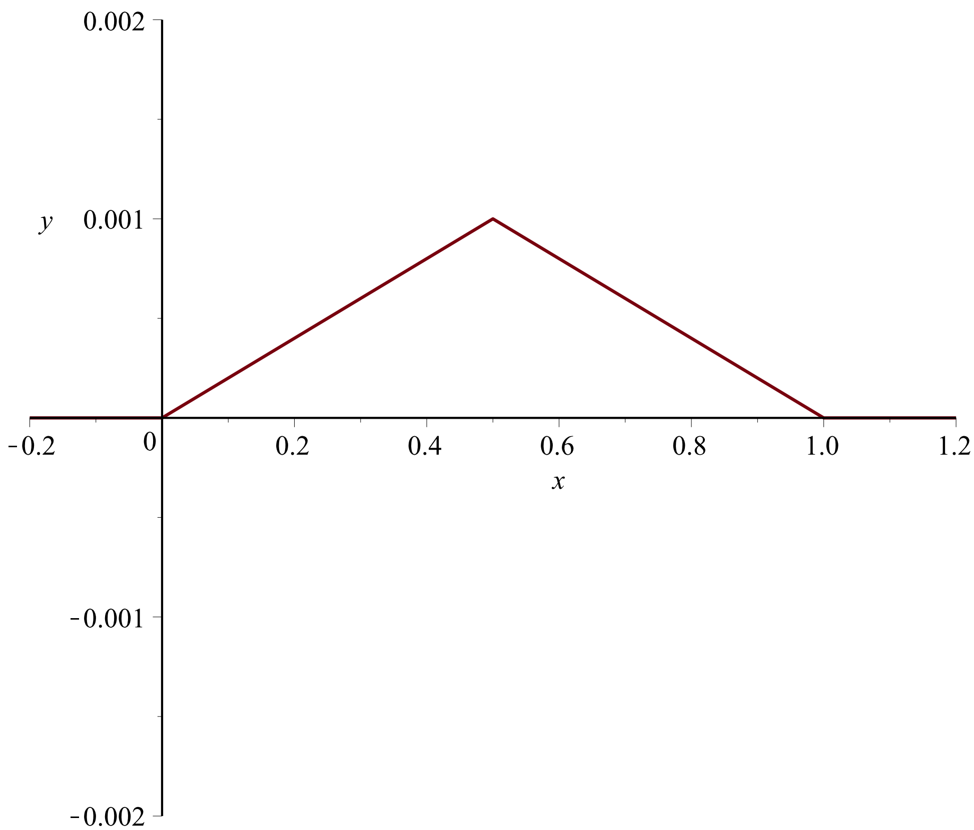
> $Ecuacion := diff(y(x,t), t\$2) = c\cdot\cdot 2\cdot diff(y(x,t), x\$2)$

$$Ecuacion := \frac{\partial^2}{\partial t^2}\, y(x,t) = c^2 \left(\frac{\partial^2}{\partial x^2}\, y(x,t)\right) \tag{1}$$

> $CondIniTray := f = \frac{\left(\frac{1}{1000}\right)}{\left(\frac{5}{10}\right)}\cdot x\cdot \text{Heaviside}(x) - 2\cdot\left(\frac{\left(\frac{1}{1000}\right)}{\left(\frac{5}{10}\right)}\cdot\left(x-\frac{5}{10}\right)\cdot\text{Heaviside}\left(x-\frac{5}{10}\right)\right) + \frac{\left(\frac{1}{1000}\right)}{\left(\frac{5}{10}\right)}\cdot(x-1)\cdot\text{Heaviside}(x-1)$

$$CondIniTray := f = \frac{1}{500}\, x\, \text{Heaviside}(x) - \frac{1}{250}\left(x-\frac{1}{2}\right)\text{Heaviside}\left(x-\frac{1}{2}\right) + \frac{1}{500}\,(x-1)\,\text{Heaviside}(x-1) \tag{2}$$

> $plot(rhs(CondIniTray), x = -0.2..1.2, y = -0.002..0.002)$



> $CondIniVel := 0$

(3)

$$CondIniVel := 0 \quad \textbf{(3)}$$

> $CondFrontera := F(0) = 0, F(1) = 0$

$$CondFrontera := F(0) = 0, F(1) = 0 \quad \textbf{(4)}$$

> $Ecuacion$

$$\frac{\partial^2}{\partial t^2} y(x,t) = c^2 \left(\frac{\partial^2}{\partial x^2} y(x,t) \right) \quad \textbf{(5)}$$

> $Hipotesis := y(x,t) = F(x) \cdot G(t)$

$$Hipotesis := y(x,t) = F(x)\, G(t) \quad \textbf{(6)}$$

> $EcuaSeparable := eval(subs(y(x,t) = rhs(Hipotesis), c \cdot \cdot 2 = 1, Ecuacion))$

$$EcuaSeparable := F(x) \left(\frac{d^2}{dt^2} G(t) \right) = \left(\frac{d^2}{dx^2} F(x) \right) G(t) \quad \textbf{(7)}$$

> $EcuacionSeparada := simplify\left(\frac{lhs(EcuaSeparable)}{F(x) \cdot G(t)} \right) = simplify\left(\frac{rhs(EcuaSeparable)}{F(x) \cdot G(t)} \right)$

$$EcuacionSeparada := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \frac{\frac{d^2}{dx^2} F(x)}{F(x)} \quad \textbf{(8)}$$

> $EcuacionX := rhs(EcuacionSeparada) = \mathrm{alpha}$

$$EcuacionX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \alpha \quad \textbf{(9)}$$

> $EcuacionT := lhs(EcuacionSeparada) = \mathrm{alpha}$

$$EcuacionT := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \alpha \quad \textbf{(10)}$$

> $EcuacionXcero := subs(\mathrm{alpha} = 0, EcuacionX)$

$$EcuacionXcero := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = 0 \quad \textbf{(11)}$$

> $SolucionXcero := dsolve(EcuacionXcero)$

$$SolucionXcero := F(x) = \_C1\, x + \_C2 \quad \textbf{(12)}$$

> $SolucionPartXcero := dsolve(\{EcuacionXcero, CondFrontera\})$

$$SolucionPartXcero := F(x) = 0 \quad \textbf{(13)}$$

SE DESCARTA ESTA SOLUCIÓN GENERAL PUES SI F(x)=0 PARA TODA "x" ENTONCES NO HAY SOLUCIÓN PARTICULAR POSIBLE.

> $EcuacionXpos := subs(\mathrm{alpha} = \mathrm{beta} \cdot \cdot 2, EcuacionX)$

$$EcuacionXpos := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \beta^2 \quad \textbf{(14)}$$

> $SolucionPartXpos := dsolve(\{EcuacionXpos, CondFrontera\})$

$$SolucionPartXpos := F(x) = 0 \quad \textbf{(15)}$$

SE DESCARTA ESTA SOLUCIÓN GENERAL PUES SI F(x)=0 PARA TODA "x" ENTONCES NO HAY SOLUCIÓN PARTICULAR POSIBLE.

> $EcuacionXneg := subs(\text{alpha} = -\text{beta}\cdot\cdot 2, EcuacionX)$

$$EcuacionXneg := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = -\beta^2 \quad \textbf{(16)}$$

> $SolucionXneg := dsolve(EcuacionXneg)$

$$SolucionXneg := F(x) = \_C1 \sin(\beta x) + \_C2 \cos(\beta x) \quad \textbf{(17)}$$

> $ParametroDos := simplify(subs(x = 0, rhs(SolucionXneg) = 0))$

$$ParametroDos := \_C2 = 0 \quad \textbf{(18)}$$

> $SolucionXnegBis := subs(\_C2 = rhs(ParametroDos), SolucionXneg)$

$$SolucionXnegBis := F(x) = \_C1 \sin(\beta x) \quad \textbf{(19)}$$

> $\text{beta} := n\cdot\text{Pi}$

$$\beta := n\pi \quad \textbf{(20)}$$

> $SolucionXnegPart := SolucionXnegBis$

$$SolucionXnegPart := F(x) = \_C1 \sin(n\pi x) \quad \textbf{(21)}$$

> $EcuacionTneg := subs(\text{alpha} = -\text{beta}\cdot\cdot 2, EcuacionT)$

$$EcuacionTneg := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = -n^2\pi^2 \quad \textbf{(22)}$$

> $SolucionTneg := dsolve(EcuacionTneg)$

$$SolucionTneg := G(t) = \_C1 \sin(n\pi t) + \_C2 \cos(n\pi t) \quad \textbf{(23)}$$

> $SolucionUno := y(x,t) = subs(\_C1 = 1, rhs(SolucionXnegPart))\cdot rhs(SolucionTneg)$

$$SolucionUno := y(x,t) = \sin(n\pi x)\,(\_C1 \sin(n\pi t) + \_C2 \cos(n\pi t)) \quad \textbf{(24)}$$

EXISTE UNA POSIBLE SOLUCIÓN GENERAL QUE CONTENGA LA SOLUCIÓN PARTICULAR REQUERIDA.

> $SolucionGeneral := y(x,t) = Sum(subs(\_C2 = b[n], \_C1 = a[n], rhs(SolucionUno)), n = 1..infinity)$

$$SolucionGeneral := y(x,t) = \sum_{n=1}^{\infty} \sin(n\pi x)\,(a_n \sin(n\pi t) + b_n \cos(n\pi t)) \quad \textbf{(25)}$$

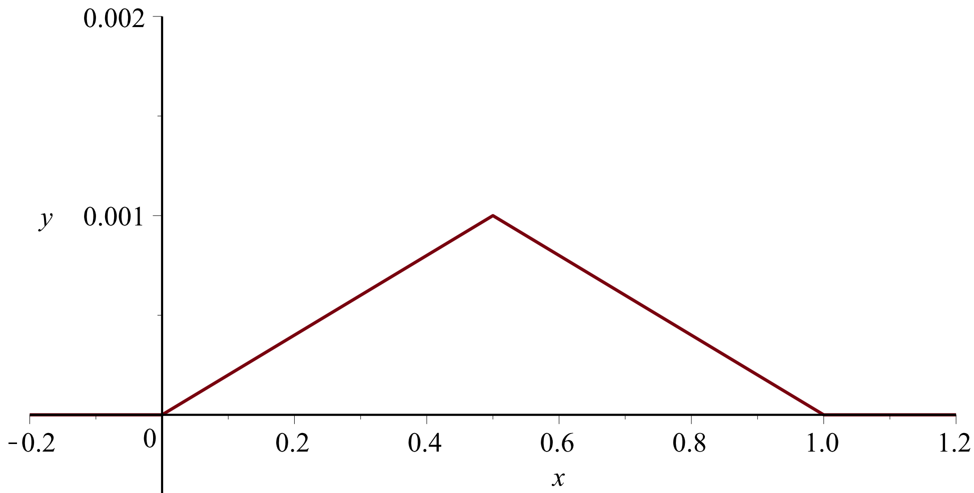
> $SolucionParticularInicialX := F(x) = eval(subs(t = 0, rhs(SolucionGeneral)))$

$$SolucionParticularInicialX := F(x) = \sum_{n=1}^{\infty} \sin(n\pi x)\, b_n \quad \textbf{(26)}$$

> $L := \frac{5}{10}$

$$L := \frac{1}{2} \quad \textbf{(27)}$$

> $plot(rhs(CondIniTray), x = -0.2..1.2, y = -0.002..0.002)$



> $b[n] := \left(\frac{1}{L}\right) \cdot int(rhs(CondIniTray) \cdot \sin(n \cdot \mathrm{Pi} \cdot x), x = 0 ..1)$

$$b_n := \frac{1}{250} \frac{-\sin(n\pi) + 2\sin\left(\frac{1}{2} n\pi\right)}{n^2 \pi^2} \tag{28}$$

> $a[n] := 0$

$$a_n := 0 \tag{29}$$

> $SolucionParticular := SolucionGeneral$

$$SolucionParticular := y(x, t) = \sum_{n=1}^{\infty} \frac{1}{250} \frac{\sin(n \pi x) \left(-\sin(n\pi) + 2\sin\left(\frac{1}{2} n\pi\right)\right) \cos(n \pi t)}{n^2 \pi^2} \tag{30}$$

> $Solucion[500] := y(x, t) = sum\left(\frac{1}{250} \frac{\sin(n \pi x) \left(-\sin(n\pi) + 2\sin\left(\frac{1}{2} n\pi\right)\right) \cos(n \pi t)}{n^2 \pi^2}, n = 1 ..500\right) :$

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>
> with(plots) :
> animate(rhs(Solucion[500]), x = 0 ..1, t = 0 ..4, frames = 150, view = [0 ..1, -0.002 ..0.002])
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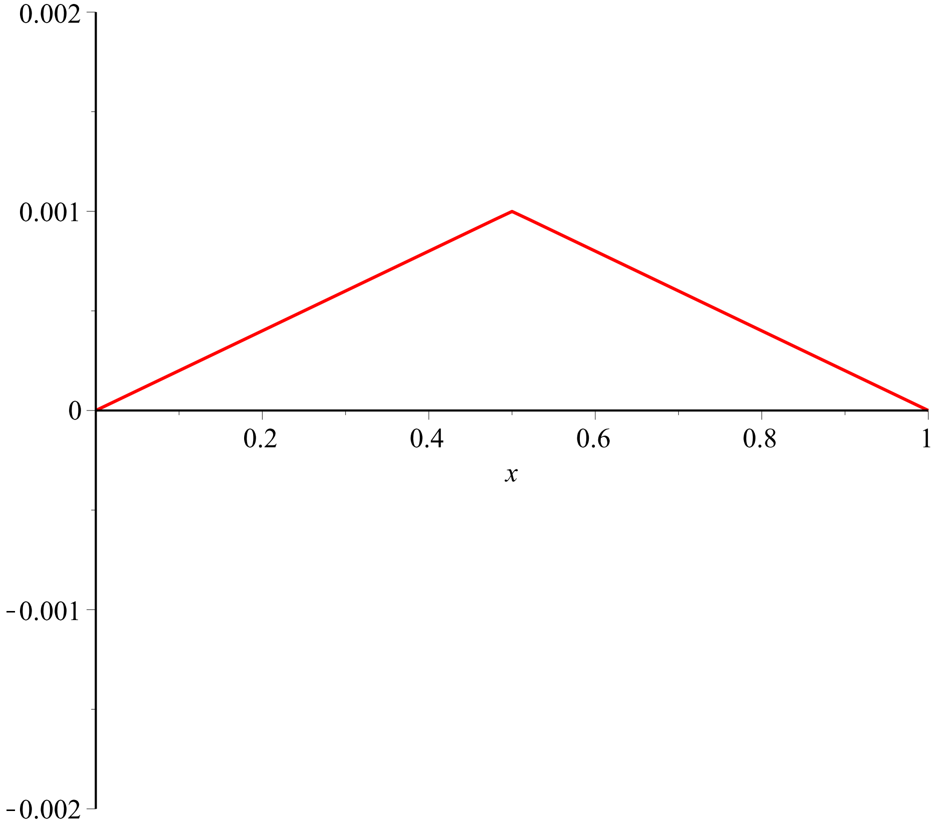


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