

## SERIE 2020-1-2 (capítulo 2)

SEMESTRE 2020-1

`> restart :`

`> restart :`

**3) DADO EL SIGUIENTE PROBLEMA DE CONDICIONES INICIALES & UTILIZANDO EXCLUSIVAMENTE EL MÉTODO DE VARIACIÓN DE PARÁMETROS (sin utilizar dsolve)**

a) OBTENER SU SOLUCIÓN PARTICULAR

b) GRAFICAR EL RESULTADO DEL INCISO a) EN UN INTERVALO  $0 < t < 1$

$$\begin{aligned} > \text{Ecua} := \frac{d^4}{dt^4} y(t) + 5 \left( \frac{d^2}{dt^2} y(t) \right) - 4 y(t) = 5 e^{-3t} \cos(2t) \\ \text{Ecua} := \frac{d^4}{dt^4} y(t) + 5 \left( \frac{d^2}{dt^2} y(t) \right) - 4 y(t) = 5 e^{-3t} \cos(2t) \end{aligned} \quad (1)$$

$$\begin{aligned} > \text{Cond} := y(0) = -2, D(y)(0) = 0, D^{(2)}(y)(0) = 7, D^{(3)}(y)(0) = -5 \\ \text{Cond} := y(0) = -2, D(y)(0) = 0, D^{(2)}(y)(0) = 7, D^{(3)}(y)(0) = -5 \end{aligned} \quad (2)$$

$$\begin{aligned} > \text{rhs}(\text{Cond}[1]) \\ -2 \end{aligned} \quad (3)$$

$$\begin{aligned} > \text{EcuaHom} := \text{lhs}(\text{Ecua}) = 0 \\ \text{EcuaHom} := \frac{d^4}{dt^4} y(t) + 5 \left( \frac{d^2}{dt^2} y(t) \right) - 4 y(t) = 0 \end{aligned} \quad (4)$$

$$\begin{aligned} > \text{EcuaCarac} := m \cdot \cdot 4 + 5 \cdot m \cdot \cdot 2 - 4 = 0 \\ \text{EcuaCarac} := m^4 + 5 m^2 - 4 = 0 \end{aligned} \quad (5)$$

$$\begin{aligned} > \text{Raiz} := \text{solve}(\text{EcuaCarac}) : \text{evalf}(\%, 3) \\ 2.38 \text{ I}, -2.38 \text{ I}, 0.835, -0.835 \end{aligned} \quad (6)$$

$$\begin{aligned} > \text{yy}[1] := \exp(\text{Re}(\text{Raiz}[1]) \cdot t) \cdot \cos(\text{Im}(\text{Raiz}[1]) \cdot t); \text{evalf}(\%, 3) \\ \text{yy}_1 := \cos\left(\frac{1}{2} \sqrt{2 \sqrt{41} + 10} t\right) \\ \cos(2.38 t) \end{aligned} \quad (7)$$

$$\begin{aligned} > \text{yy}[2] := \exp(\text{Re}(\text{Raiz}[1]) \cdot t) \cdot \sin(\text{Im}(\text{Raiz}[1]) \cdot t); \text{evalf}(\%, 3) \\ \text{yy}_2 := \sin\left(\frac{1}{2} \sqrt{2 \sqrt{41} + 10} t\right) \\ \sin(2.38 t) \end{aligned} \quad (8)$$

$$\begin{aligned} > \text{yy}[3] := \exp(\text{Raiz}[3] \cdot t); \text{evalf}(\%, 3) \\ \text{yy}_3 := e^{\frac{1}{2} \sqrt{-10 + 2 \sqrt{41}} t} \\ e^{0.835 t} \end{aligned} \quad (9)$$

> yy[4] := exp(Raiz[4]·t); evalf(%, 3)

$$yy_4 := e^{-\frac{1}{2} \sqrt{-10 + 2\sqrt{41}} t} e^{-0.835 t} \quad (10)$$

> SolHom := y(t) = C[1]·yy[1] + C[2]·yy[2] + C[3]·yy[3] + C[4]·yy[4]

$$SolHom := y(t) = C_1 \cos\left(\frac{1}{2} \sqrt{2\sqrt{41} + 10} t\right) + C_2 \sin\left(\frac{1}{2} \sqrt{2\sqrt{41} + 10} t\right) + C_3 e^{\frac{1}{2} \sqrt{-10 + 2\sqrt{41}} t} + C_4 e^{-\frac{1}{2} \sqrt{-10 + 2\sqrt{41}} t} \quad (11)$$

> SolNoHom := y(t) = A·yy[1] + B·yy[2] + DD·yy[3] + E·yy[4]

$$SolNoHom := y(t) = A \cos\left(\frac{1}{2} \sqrt{2\sqrt{41} + 10} t\right) + B \sin\left(\frac{1}{2} \sqrt{2\sqrt{41} + 10} t\right) + DD e^{\frac{1}{2} \sqrt{-10 + 2\sqrt{41}} t} + E e^{-\frac{1}{2} \sqrt{-10 + 2\sqrt{41}} t} \quad (12)$$

> with(linalg) :

> WW := wronskian([yy[1], yy[2], yy[3], yy[4]], t) :

> BB := array([0, 0, 0, rhs(Ecua)])

$$BB := \begin{bmatrix} 0 & 0 & 0 & 5 e^{-3t} \cos(2t) \end{bmatrix} \quad (13)$$

> ParaDer := linsolve(WW, BB) : Aprima := simplify(ParaDer[1]); evalf(%, 3); Bprima := simplify(ParaDer[2]); evalf(%, 3); Dprima := simplify(ParaDer[3]); evalf(%, 3); Eprima := simplify(ParaDer[4]); evalf(%, 3)

$$Aprima := \frac{10}{41} \frac{\sin\left(\frac{1}{2} \sqrt{2\sqrt{41} + 10} t\right) \sqrt{41} e^{-3t} \cos(2t)}{\sqrt{2\sqrt{41} + 10}}$$

$$0.327 \sin(2.38 t) e^{-3 \cdot t} \cos(2 \cdot t)$$

$$Bprima := -\frac{10}{41} \frac{\sqrt{41} \cos\left(\frac{1}{2} \sqrt{2\sqrt{41} + 10} t\right) e^{-3t} \cos(2t)}{\sqrt{2\sqrt{41} + 10}}$$

$$-0.327 \cos(2.38 t) e^{-3 \cdot t} \cos(2 \cdot t)$$

$$Dprima := \frac{5}{41} \frac{\sqrt{41} e^{-\frac{1}{2} t (\sqrt{-10 + 2\sqrt{41}} + 6)} \cos(2t)}{\sqrt{-10 + 2\sqrt{41}}}$$

$$0.467 e^{-3.84 t} \cos(2 \cdot t)$$

$$Eprima := -\frac{5}{41} \frac{\sqrt{41} e^{\frac{1}{2} t (\sqrt{-10 + 2\sqrt{41}} - 6)} \cos(2t)}{\sqrt{-10 + 2\sqrt{41}}}$$

$$-0.467 e^{-2.16 t} \cos(2 \cdot t)$$

(14)

> A := int(Aprima, t) + C[1]; B := int(Bprima, t) + C[2]; DD := int(Dprima, t) + C[3]; E

$$:= \text{int}(Eprima, t) + C[4]$$

$A :=$

$$\begin{aligned} & \frac{1}{41} \frac{1}{\sqrt{2\sqrt{41}+10}} \left( 5\sqrt{41} \left( \frac{1}{9 + \left( \frac{1}{2} \sqrt{2\sqrt{41}+10} + 2 \right)^2} \left( \left( -\frac{1}{2} \sqrt{2\sqrt{41}+10} - 2 \right) e^{-3t} \cos \left( \left( \frac{1}{2} \sqrt{2\sqrt{41}+10} + 2 \right) t \right) \right) \right. \right. \\ & \left. \left. - \frac{3 e^{-3t} \sin \left( \left( \frac{1}{2} \sqrt{2\sqrt{41}+10} + 2 \right) t \right)}{9 + \left( \frac{1}{2} \sqrt{2\sqrt{41}+10} + 2 \right)^2} \right) \right. \\ & \left. + 5\sqrt{41} \left( \frac{\left( -\frac{1}{2} \sqrt{2\sqrt{41}+10} + 2 \right) e^{-3t} \cos \left( \left( \frac{1}{2} \sqrt{2\sqrt{41}+10} - 2 \right) t \right)}{9 + \left( \frac{1}{2} \sqrt{2\sqrt{41}+10} - 2 \right)^2} \right. \right. \\ & \left. \left. - \frac{3 e^{-3t} \sin \left( \left( \frac{1}{2} \sqrt{2\sqrt{41}+10} - 2 \right) t \right)}{9 + \left( \frac{1}{2} \sqrt{2\sqrt{41}+10} - 2 \right)^2} \right) \right) \right) + C_1 \end{aligned}$$

$$\begin{aligned} B := & \frac{1}{41} \frac{1}{\sqrt{2\sqrt{41}+10}} \left( -5\sqrt{41} \left( -\frac{3 e^{-3t} \cos \left( \left( \frac{1}{2} \sqrt{2\sqrt{41}+10} - 2 \right) t \right)}{9 + \left( \frac{1}{2} \sqrt{2\sqrt{41}+10} - 2 \right)^2} \right. \right. \\ & \left. \left. - \frac{\left( -\frac{1}{2} \sqrt{2\sqrt{41}+10} + 2 \right) e^{-3t} \sin \left( \left( \frac{1}{2} \sqrt{2\sqrt{41}+10} - 2 \right) t \right)}{9 + \left( \frac{1}{2} \sqrt{2\sqrt{41}+10} - 2 \right)^2} \right) - 5\sqrt{41} \left( \right. \right. \\ & \left. \left. - \frac{3 e^{-3t} \cos \left( \left( \frac{1}{2} \sqrt{2\sqrt{41}+10} + 2 \right) t \right)}{9 + \left( \frac{1}{2} \sqrt{2\sqrt{41}+10} + 2 \right)^2} \right. \right. \\ & \left. \left. - \frac{\left( -\frac{1}{2} \sqrt{2\sqrt{41}+10} - 2 \right) e^{-3t} \sin \left( \left( \frac{1}{2} \sqrt{2\sqrt{41}+10} + 2 \right) t \right)}{9 + \left( \frac{1}{2} \sqrt{2\sqrt{41}+10} + 2 \right)^2} \right) \right) \right) + C_2 \end{aligned}$$

$DD :=$

$$\begin{aligned} & \frac{5}{41} \frac{1}{\sqrt{-10+2\sqrt{41}}} \left( \sqrt{41} \left( \frac{1}{\left( -\frac{1}{2} \sqrt{-10+2\sqrt{41}} - 3 \right)^2 + 4} \left( \left( -\frac{1}{2} \sqrt{-10+2\sqrt{41}} - 3 \right) e^{\left( -\frac{1}{2} \sqrt{-10+2\sqrt{41}} - 3 \right) t} \cos(2t) \right) \right. \right. \\ & \left. \left. + \frac{2 e^{\left( -\frac{1}{2} \sqrt{-10+2\sqrt{41}} - 3 \right) t} \sin(2t)}{\left( -\frac{1}{2} \sqrt{-10+2\sqrt{41}} - 3 \right)^2 + 4} \right) \right) + C_3 \end{aligned}$$

$E :=$

(15)

$$\begin{aligned} & -\frac{5}{41} \frac{1}{\sqrt{-10+2\sqrt{41}}} \left( \sqrt{41} \left( \frac{1}{\left( \frac{1}{2} \sqrt{-10+2\sqrt{41}} - 3 \right)^2 + 4} \left( \left( \frac{1}{2} \sqrt{-10+2\sqrt{41}} - 3 \right) e^{\left( \frac{1}{2} \sqrt{-10+2\sqrt{41}} - 3 \right) t} \cos(2t) \right) \right. \right. \\ & \left. \left. + \frac{2 e^{\left( \frac{1}{2} \sqrt{-10+2\sqrt{41}} - 3 \right) t} \sin(2t)}{\left( \frac{1}{2} \sqrt{-10+2\sqrt{41}} - 3 \right)^2 + 4} \right) \right) + C_4 \end{aligned}$$

$\gg \text{evalf}(\text{simplify}(\text{SolNoHom}), 2)$

(16)

$$\begin{aligned} y(t) = & -0.026 \sin(4.4 t) e^{-3 \cdot t} \sin(2.4 t) - 0.017 \sin(4.4 t) e^{-3 \cdot t} \cos(2.4 t) \\ & + 0.054 e^{-3 \cdot t} \sin(2.4 t) \cos(0.40 t) - 0.0072 e^{-3 \cdot t} \sin(2.4 t) \sin(0.40 t) \\ & + 0.017 e^{-3 \cdot t} \sin(2.4 t) \cos(4.4 t) - 0.0072 e^{-3 \cdot t} \cos(0.40 t) \cos(2.4 t) \\ & - 0.054 e^{-3 \cdot t} \sin(0.40 t) \cos(2.4 t) - 0.026 e^{-3 \cdot t} \cos(4.4 t) \cos(2.4 t) + 0.97 e^{-0.85 t} C_4 \\ & + 0.97 e^{0.85 t} C_3 + 0.97 \cos(2.4 t) C_1 + 0.97 \sin(2.4 t) C_2 + 0.020 e^{-3 \cdot t} \cos(2. t) \\ & - 0.058 \sin(2. t) e^{-3 \cdot t} \end{aligned}$$

$\gg \text{DerSolNoHom} := \text{diff}(\text{SolNoHom}, t) : \text{DerDosSolNoHom} := \text{diff}(\text{SolNoHom}, t\$2) :$   
 $\text{DerTresSolNoHom} := \text{diff}(\text{SolNoHom}, t\$3) : \text{DerCuatroSolNoHom} := \text{diff}(\text{SolNoHom}, t$   
 $\$4) :$

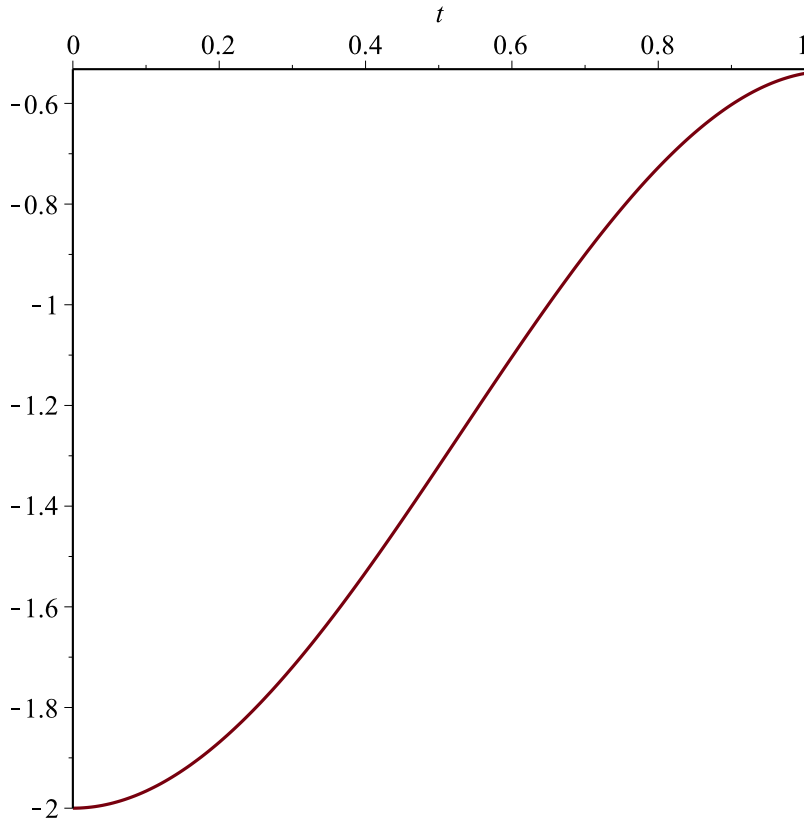
$\gg \text{Sist} := \text{subs}(t=0, \text{rhs}(\text{SolNoHom}) = \text{rhs}(\text{Cond}[1])), \text{subs}(t=0, \text{rhs}(\text{DerSolNoHom})$   
 $= \text{rhs}(\text{Cond}[2])), \text{subs}(t=0, \text{rhs}(\text{DerDosSolNoHom}) = \text{rhs}(\text{Cond}[3])), \text{subs}(t=0,$   
 $\text{rhs}(\text{DerTresSolNoHom}) = \text{rhs}(\text{Cond}[4])) :$

$\gg \text{Para} := \text{solve}(\{\text{Sist}\}, \{C[1], C[2], C[3], C[4]\}) :$

$\gg \text{SolPart} := \text{subs}(C[1] = \text{rhs}(\text{Para}[1]), C[2] = \text{rhs}(\text{Para}[2]), C[3] = \text{rhs}(\text{Para}[3]), C[4]$

$$\begin{aligned}
 &= rhs(Para[4]), SolNoHom) : evalf(%, 2) \\
 y(t) = & \left( -0.024 e^{-3.4t} \cos(4.4t) - 0.018 e^{-3.4t} \sin(4.4t) - 0.0070 e^{-3.4t} \cos(0.4t) \right. \\
 & \left. - 0.050 e^{-3.4t} \sin(0.4t) - 1.3 \right) \cos(2.4t) + \left( 0.050 e^{-3.4t} \cos(0.4t) \right. \\
 & \left. - 0.0070 e^{-3.4t} \sin(0.4t) + 0.018 e^{-3.4t} \cos(4.4t) - 0.024 e^{-3.4t} \sin(4.4t) + 0.27 \right) \\
 & \sin(2.4t) + \left( -0.096 e^{-3.8t} \cos(2.4t) + 0.050 e^{-3.8t} \sin(2.4t) - 0.63 \right) e^{0.85t} \\
 & + \left( 0.11 e^{-2.2t} \cos(2.4t) - 0.10 e^{-2.2t} \sin(2.4t) - 0.21 \right) e^{-0.85t}
 \end{aligned} \tag{17}$$

> plot(rhs(SolPart), t=0..1)



$$\begin{aligned}
 &> CompUno := simplify(eval(subs(y(t) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0))) \\
 & \quad \quad \quad CompUno := 0 = 0
 \end{aligned} \tag{18}$$

$$\begin{aligned}
 &> CompDos := simplify(subs(t=0, rhs(SolPart))) \\
 & \quad \quad \quad CompDos := -2
 \end{aligned} \tag{19}$$

$$\begin{aligned}
 &> CompTres := simplify(subs(t=0, rhs(diff(SolPart, t)))) \\
 & \quad \quad \quad CompTres := 0
 \end{aligned} \tag{20}$$

$$\begin{aligned}
 &> CompCuatro := simplify(subs(t=0, rhs(diff(SolPart, t$2)))) \\
 & \quad \quad \quad CompCuatro := 7
 \end{aligned} \tag{21}$$

$$> CompCinco := simplify(subs(t=0, rhs(diff(SolPart, t$3))))$$

(22)

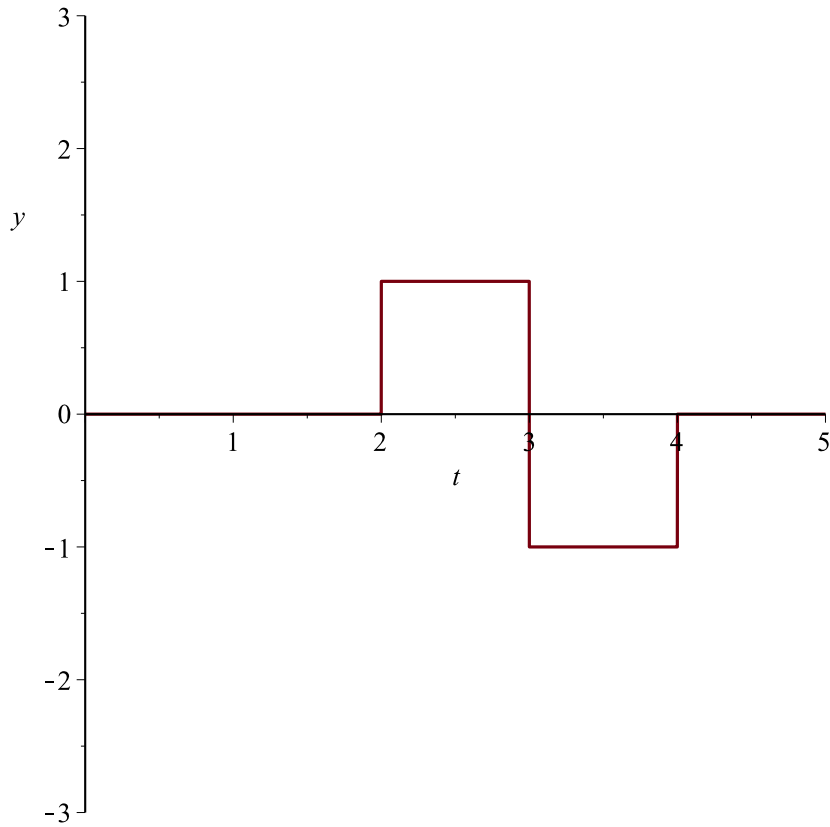
[illegible]

```
> restart
```

```
> f := Heaviside(t-2) - 2*Heaviside(t-3) + Heaviside(t-4)
```

$$f := \text{Heaviside}(t - 2) - 2 \text{Heaviside}(t - 3) + \text{Heaviside}(t - 4)$$

```
> plot(f, t=0..5, y=-3..3)
```



> *with(inttrans)* :

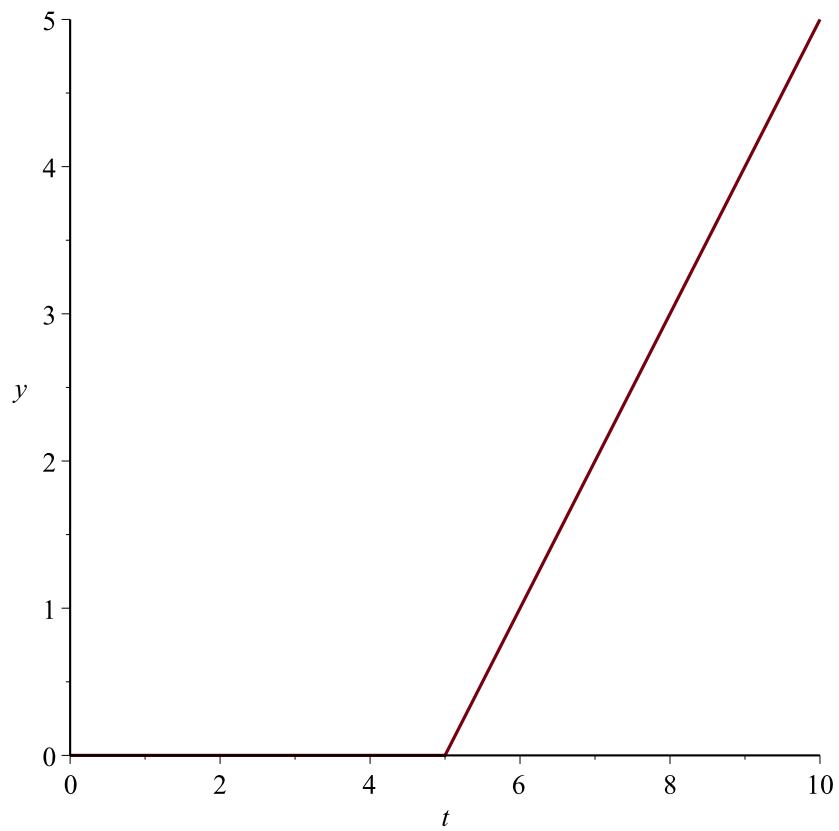
```
> F := laplace(f, t, s)
```

$$F := \frac{e^{-2s} - 2e^{-3s} + e^{-4s}}{s}$$

```
> g := (t - 5) * Heaviside(t - 5)
```

$$g := (t - 5) \text{ Heaviside}(t - 5)$$

```
> plot(g, t=0..10, y=0..5)
```



```
> G := laplace(g, t, s)
```

$$G := \frac{e^{-5s}}{s^2} \tag{26}$$

```
> h := Dirac(t - 5)
```

$$h := \text{Dirac}(t - 5) \tag{27}$$

```
> H := laplace(h, t, s)
```

$$H := e^{-5s} \tag{28}$$

```
>
```

```
>
```

```
>
```

```
>
```