> *RESUELTA*

SERIE 2020-1-2 (capítulo 2)

>

SEMESTRE 2020-1

> *restart* :

1) DADA LA ECUACIÓN DIFERENCIAL

$$\frac{d^2}{dx^2}\, y(x) - 4\, y(x) = 2\, e^{2x} + 5\, e^{-2x} \qquad \textbf{(1)}$$

a) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL UTILIZANDO EXCLUSIVAMENTE EL MÉTODO DE VARIACIÓN DE PARÁMETROS **(sin utilizar dsolve)**
b) CON LA SOLUCIÓN GENERAL OBTENIDA EN EL INCISO a) Y DADAS LAS CONDICIONES INICIALES **y(0) = -6 &**
y'(0) = 8 OBTENER LA SOLUCIÓN PARTICULAR **(sin utilizar dsolve)**
c) GRAFIQUE (JUNTAS) LA SOLUCIÓN PARTICULAR OBTENIDA EN EL INCISO b) Y LA PRIMERA DERIVADA DE ÉSTA, CONSIDERNADO UN INTERVALO $0 < x < 1$

> *restart*

> $Ecua := \frac{d^2}{dx^2}\, y(x) - 4\, y(x) = 2\, e^{2x} + 5\, e^{-2x}$

$$Ecua := \frac{d^2}{dx^2}\, y(x) - 4\, y(x) = 2\, e^{2x} + 5\, e^{-2x} \qquad \textbf{(2)}$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2}\, y(x) - 4\, y(x) = 0 \qquad \textbf{(3)}$$

> $Q := rhs(Ecua)$

$$Q := 2\, e^{2x} + 5\, e^{-2x} \qquad \textbf{(4)}$$

> $EcuaCarac := m \cdot\cdot 2 - 4 = 0$

$$EcuaCarac := m^2 - 4 = 0 \qquad \textbf{(5)}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 2, -2 \qquad \textbf{(6)}$$

> $yy[1] := \exp(Raiz[1]\cdot x); yy[2] := \exp(Raiz[2]\cdot x)$

$$yy_1 := e^{2x}$$
$$yy_2 := e^{-2x} \qquad \textbf{(7)}$$

> $SolHom := y(x) = C[1]\cdot yy[1] + C[2]\cdot yy[2]$

$$SolHom := y(x) = e^{2x}\, C_1 + e^{-2x}\, C_2 \qquad \textbf{(8)}$$

> $with(linalg)$:

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^{2x} & e^{-2x} \\ 2\,e^{2x} & -2\,e^{-2x} \end{bmatrix} \tag{9}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & 2\,e^{2x} + 5\,e^{-2x} \end{bmatrix} \tag{10}$$

> $Para := linsolve(WW, BB)$

$$Para := \begin{bmatrix} \frac{1}{4}\,\frac{2\,e^{2x} + 5\,e^{-2x}}{e^{2x}} & -\frac{1}{4}\,\frac{2\,e^{2x} + 5\,e^{-2x}}{e^{-2x}} \end{bmatrix} \tag{11}$$

> $Aprima := expand(Para[1]); Bprima := expand(Para[2])$

$$Aprima := \frac{1}{2} + \frac{5}{4\,(e^{x})^{4}}$$

$$Bprima := -\frac{1}{2}\,(e^{x})^{4} - \frac{5}{4} \tag{12}$$

> $A := int(Aprima, x) + C[1]; B := int(Bprima, x) + C[2]$

$$A := \frac{1}{2}\,x - \frac{5}{16\,(e^{x})^{4}} + C_1$$

$$B := -\frac{1}{8}\,(e^{x})^{4} - \frac{5}{4}\,x + C_2 \tag{13}$$

> $SolNoHom := y(x) = simplify(A \cdot yy[1] + B \cdot yy[2])$

$$SolNoHom := y(x) = \frac{1}{2}\,e^{2x}\,x - \frac{5}{16}\,e^{-2x} + e^{2x}\,C_1 - \frac{1}{8}\,e^{2x} - \frac{5}{4}\,e^{-2x}\,x + e^{-2x}\,C_2 \tag{14}$$

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b) SolPart

> $Sist := eval(subs(x = 0, rhs(SolNoHom) = -6)), eval(subs(x = 0, rhs(diff(SolNoHom, x)) = 8)) : Sist[1]; Sist[2]$

$$-\frac{7}{16} + C_1 + C_2 = -6$$

$$-\frac{3}{8} + 2\,C_1 - 2\,C_2 = 8 \tag{15}$$

> $Param := solve(\{Sist\}, \{C[1], C[2]\})$

$$Param := \left\{C_1 = -\frac{11}{16}, C_2 = -\frac{39}{8}\right\} \tag{16}$$

> $SolPart := subs(C[1] = rhs(Param[1]), C[2] = rhs(Param[2]), SolNoHom)$

$$SolPart := y(x) = \frac{1}{2}\,e^{2x}\,x - \frac{83}{16}\,e^{-2x} - \frac{13}{16}\,e^{2x} - \frac{5}{4}\,e^{-2x}\,x \tag{17}$$

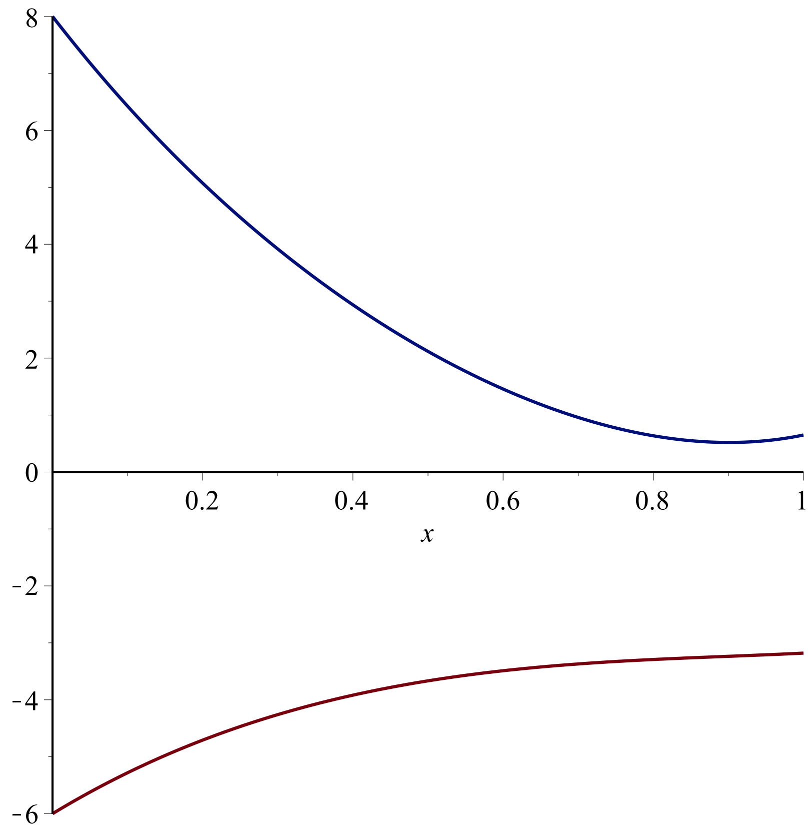
> $CondUno := \mathrm{eval}(\mathrm{subs}(x = 0, SolPart))$

$$CondUno := y(0) = -6 \tag{18}$$

> $CondDos := eval(subs(x = 0, rhs(diff(SolPart, x))))$

$$CondDos := 8 \tag{19}$$

> $plot([rhs(SolPart), rhs(diff(SolPart, x))], x = 0 ..1)$



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> *restart* :

2) OBTENGA Y GRAFIQUE { EN EL INTERVALO - 1..1 } LA SOLUCIÓN PARTICULAR DE LOS SIGUIENTES PROBLEMAS **(sin utilizar dsolve):**

a) CON CONDICIONES EN LA FRONTERA

$$\frac{d^3}{dx^3}y(x) + \frac{d^2}{dx^2}y(x) + \frac{d}{dx}y(x) + y(x) = 0$$
$$y(0) = -3$$
$$y\left(\frac{1}{2}\pi\right) = 3$$
$$y\left(\frac{3}{2}\pi\right) = 9 \qquad \textbf{(20)}$$

> *restart* :

b) CON CONDICIONES INICIALES

$$\frac{d^2}{dt^2} x(t) - 7 \left(\frac{d}{dt} x(t) \right) + 12 x(t) = \cos(3 t) + t^2$$
$$x(1) = 2$$
$$D(x)(1) = -2 \qquad \textbf{(21)}$$

> *restart* :

c) CON CONDICIONES INICIALES

$$\frac{d^2}{dx^2} y(x) + 2 \left(\frac{d}{dx} y(x) \right) + 2 y(x) = 3 e^{2x}$$
$$y(0) = -5$$
$$D(y)(0) = 8 \qquad \textbf{(22)}$$

<<<<<<<<<<<<<<<<<< FIN PREGUNTA 2 >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

> *restart* :

3) DADO EL SIGUIENTE PROBLEMA DE CONDICIONES INICIALES & UTILIZANDO EXCLUSIVAMENTE EL MÉTODO DE VARIACIÓN DE PARÁMETROS **(sin utilizar dsolve)**

$$\frac{d^4}{dt^4} y(t) + 5 \left(\frac{d^2}{dt^2} y(t) \right) - 4 y(t) = 5 e^{-3t} \cos(2 t)$$
$$y(0) = -2$$
$$D(y)(0) = 0$$
$$D^{(2)}(y)(0) = 7$$
$$D^{(3)}(y)(0) = -5 \qquad \textbf{(23)}$$

a) OBTENER SU SOLUCIÓN PARTICULAR

b) GRAFICAR EL RESULTADO DEL INCISO a) EN UN INTERVALO $0 < t < 1$

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<<<<<<<<<<<<<<<<<<<<<<<<FIN PREGUNTA 3 >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

> *restart*

4) SI SABEMOS QUE LA SOLUCIÓN GENERAL

$$y(x) = \frac{C_1}{x^2} + C_2 x \qquad \textbf{(24)}$$

SATISFACE LA ECUACIÓN DIFERENCIAL HOMOGÉNEA SIGUIENTE

$$-2 y(x) + \left(\frac{d^2}{dx^2} y(x) \right) x^2 + 2 x \left(\frac{d}{dx} y(x) \right) = 0 \qquad \textbf{(25)}$$

OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN NO HOMOGÉNEA UTILIZANDO EXCLUSIVAMENTE EL MÉTODO DE VARIACIÓN DE PARÁMETROS **(sin utilizar dsolve)**

$$-2 y(x) + \left(\frac{d^2}{dx^2} y(x) \right) x^2 + 2 x \left(\frac{d}{dx} y(x) \right) = 32 x^2 \qquad \textbf{(26)}$$

> *restart*

> $SolGral := y(x) = \frac{C_1}{x^2} + C_2\, x$

$$SolGral := y(x) = \frac{C_1}{x^2} + C_2\, x \tag{27}$$

> $EcuaHom := -2\, y(x) + \left(\frac{d^2}{dx^2}\, y(x)\right) x^2 + 2\, x \left(\frac{d}{dx}\, y(x)\right) = 0$

$$EcuaHom := -2\, y(x) + \left(\frac{d^2}{dx^2}\, y(x)\right) x^2 + 2\, x \left(\frac{d}{dx}\, y(x)\right) = 0 \tag{28}$$

> $EcuaNoHom := -2\, y(x) + \left(\frac{d^2}{dx^2}\, y(x)\right) x^2 + 2\, x \left(\frac{d}{dx}\, y(x)\right) = 32\, x^2$

$$EcuaNoHom := -2\, y(x) + \left(\frac{d^2}{dx^2}\, y(x)\right) x^2 + 2\, x \left(\frac{d}{dx}\, y(x)\right) = 32\, x^2 \tag{29}$$

> $EcuaHomEst := expand\left(\frac{lhs(EcuaHom)}{x^{\cdot\cdot}2} = \frac{rhs(EcuaHom)}{x^{\cdot\cdot}2}\right)$

$$EcuaHomEst := -\frac{2\, y(x)}{x^2} + \frac{d^2}{dx^2}\, y(x) + \frac{2\left(\frac{d}{dx}\, y(x)\right)}{x} = 0 \tag{30}$$

> $CompUno := simplify(eval(subs(y(x) = rhs(SolGral), EcuaHomEst)))$

$$CompUno := 0 = 0 \tag{31}$$

> $EcuaNoHomEst := expand\left(\frac{lhs(EcuaNoHom)}{x^{\cdot\cdot}2} = \frac{rhs(EcuaNoHom)}{x^{\cdot\cdot}2}\right)$

$$EcuaNoHomEst := -\frac{2\, y(x)}{x^2} + \frac{d^2}{dx^2}\, y(x) + \frac{2\left(\frac{d}{dx}\, y(x)\right)}{x} = 32 \tag{32}$$

> $SolGralNoHom := y(x) = \frac{A}{x^{\cdot\cdot}2} + B\cdot x$

$$SolGralNoHom := y(x) = \frac{A}{x^2} + B\, x \tag{33}$$

> $with(linalg):$

> $yy[1] := \frac{1}{x^{\cdot\cdot}2};\, yy[2] := x$

$$yy_1 := \frac{1}{x^2}$$

$$yy_2 := x \tag{34}$$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} \frac{1}{x^2} & x \\ -\frac{2}{x^3} & 1 \end{bmatrix} \tag{35}$$

> $BB := array([0, rhs(EcuaNoHomEst)])$

$$BB := \begin{bmatrix} 0 & 32 \end{bmatrix} \quad (36)$$

> $Para := linsolve(WW, BB)$

$$Para := \begin{bmatrix} -\frac{32}{3}x^3 & \frac{32}{3} \end{bmatrix} \quad (37)$$

> $Aprima := Para[1]; Bprima := Para[2]$

$$Aprima := -\frac{32}{3}x^3$$

$$Bprima := \frac{32}{3} \quad (38)$$

> $A := int(Aprima, x) + C[1]; B := int(Bprima, x) + C[2]$

$$A := -\frac{8}{3}x^4 + C_1$$

$$B := \frac{32}{3}x + C_2 \quad (39)$$

> $SolFinal := expand(SolGralNoHom)$

$$SolFinal := y(x) = 8x^2 + \frac{C_1}{x^2} + C_2 x \quad (40)$$

> $CompDos := simplify(eval(subs(y(x) = rhs(SolFinal), lhs(EcuaNoHomEst) - rhs(EcuaNoHomEst) = 0)))$

$$CompDos := 0 = 0 \quad (41)$$

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<<<<<<<<<<<<<<<<<<<<<<<<<FIN PREGUNTA 4 >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

> $restart$

> $F := \frac{s}{(s\cdot\cdot 2 + 9)\cdot\cdot 2}$

$$F := \frac{s}{(s^2 + 9)^2} \quad (42)$$

> $with(inttrans):$

> $f := invlaplace(F, s, t)$

$$f := \frac{1}{6}t\sin(3t) \quad (43)$$

> $G := \frac{\exp(-2\cdot s)}{s\cdot\cdot 2 + s + 1}$

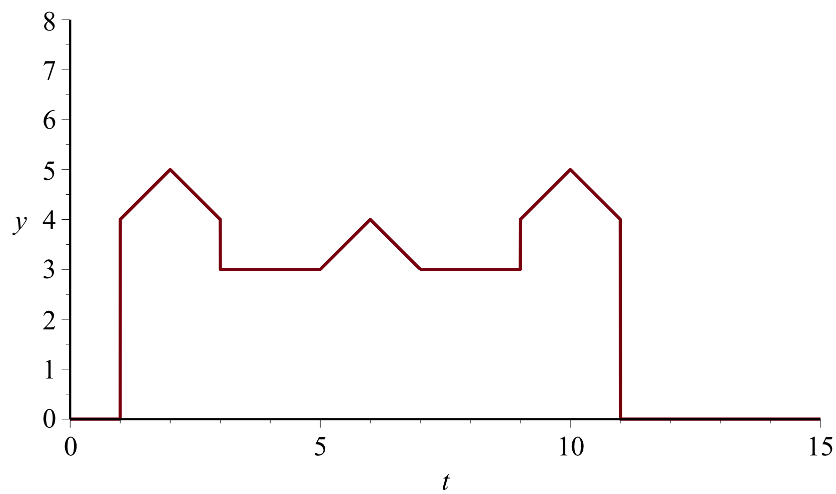
$$G := \frac{e^{-2s}}{s^2 + s + 1} \quad (44)$$

> $g := invlaplace(G, s, t)$

$$g := \frac{2}{3}\,\mathrm{Heaviside}(t-2)\,\sqrt{3}\,\mathrm{e}^{-\frac{1}{2}t+1}\,\sin\left(\frac{1}{2}\sqrt{3}\,(t-2)\right) \quad \textbf{(45)}$$

> $restart$

> $f := 4\cdot\mathrm{Heaviside}(t-1) + (t-1)\cdot\mathrm{Heaviside}(t-1) - 2\cdot(t-2)\cdot\mathrm{Heaviside}(t-2) + (t-3)\cdot\mathrm{Heaviside}(t-3) - \mathrm{Heaviside}(t-3) + (t-5)\cdot\mathrm{Heaviside}(t-5) - 2\cdot(t-6)\cdot\mathrm{Heaviside}(t-6) + (t-7)\cdot\mathrm{Heaviside}(t-7) + \mathrm{Heaviside}(t-9) + (t-9)\cdot\mathrm{Heaviside}(t-9) - 2\cdot(t-10)\cdot\mathrm{Heaviside}(t-10) + (t-11)\cdot\mathrm{Heaviside}(t-11) - 4\cdot\mathrm{Heaviside}(t-11) : plot(f, t=0..15, y=0..8, scaling=CONSTRAINED)$

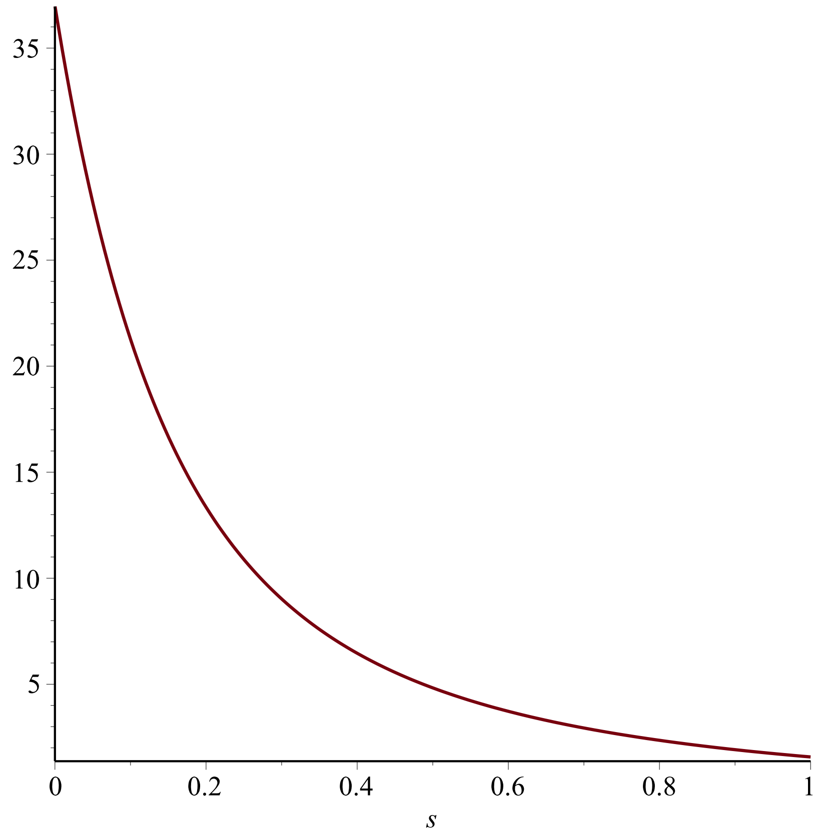


> $with(inttrans) :$

> $F := laplace(f, t, s)$

$$F := \frac{\mathrm{e}^{-s} + \mathrm{e}^{-11s} - 2\,\mathrm{e}^{-10s} + \mathrm{e}^{-9s} + \mathrm{e}^{-7s} - 2\,\mathrm{e}^{-6s} + \mathrm{e}^{-5s} + \mathrm{e}^{-3s} - 2\,\mathrm{e}^{-2s}}{s^2} + \frac{4\,\mathrm{e}^{-s} - 4\,\mathrm{e}^{-11s} + \mathrm{e}^{-9s} - \mathrm{e}^{-3s}}{s} \quad \textbf{(46)}$$

> $plot(F, s=0..1)$



> $restart$

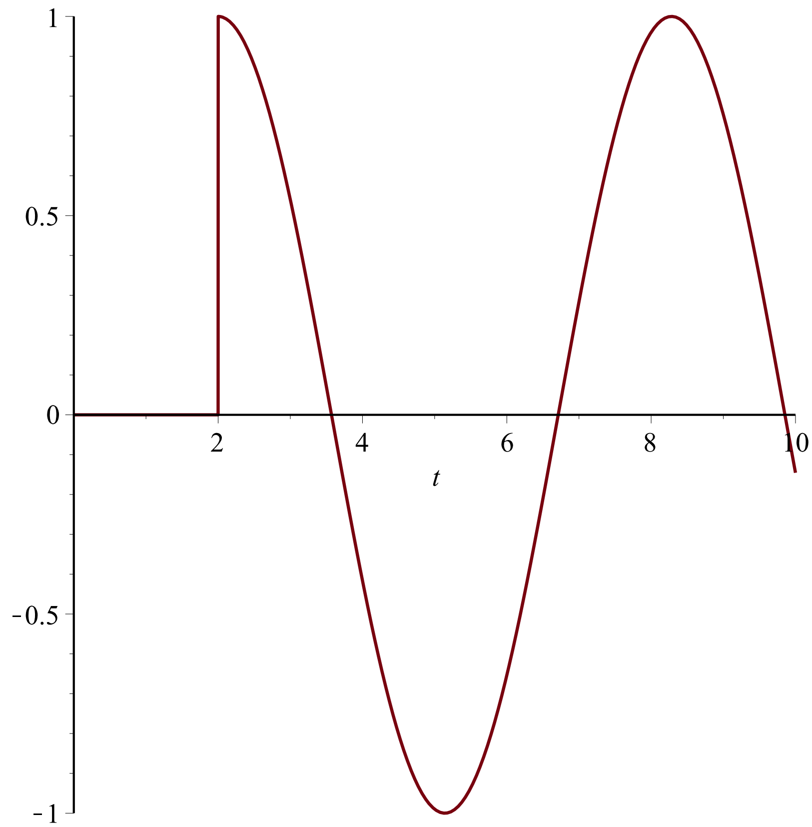
> $Ecua := diff(y(t), t\$3) + diff(y(t), t\$2) + diff(y(t), t) + y(t) = \cos(t-2) \cdot \mathrm{Heaviside}(t-2)$

$$Ecua := \frac{d^3}{dt^3}\, y(t) + \frac{d^2}{dt^2}\, y(t) + \frac{d}{dt}\, y(t) + y(t) = \cos(t-2)\ \mathrm{Heaviside}(t-2) \quad \textbf{(47)}$$

> $Q := rhs(Ecua)$

$$Q := \cos(t-2)\ \mathrm{Heaviside}(t-2) \quad \textbf{(48)}$$

> $plot(Q, t = 0 ..10)$



> $Cond := y(0) = 0, D(y)(0) = 0, D(D(y))(0) = 0$

$$Cond := y(0) = 0, D(y)(0) = 0, D^{(2)}(y)(0) = 0 \tag{49}$$

> $with(inttrans):$

> $EcuaLap := subs(Cond, laplace(Ecua, t, s))$

$$EcuaLap := s^3\, laplace(y(t), t, s) + s^2\, laplace(y(t), t, s) + s\, laplace(y(t), t, s) + laplace(y(t), t, s) = \frac{e^{-2s}\, s}{s^2 + 1} \tag{50}$$

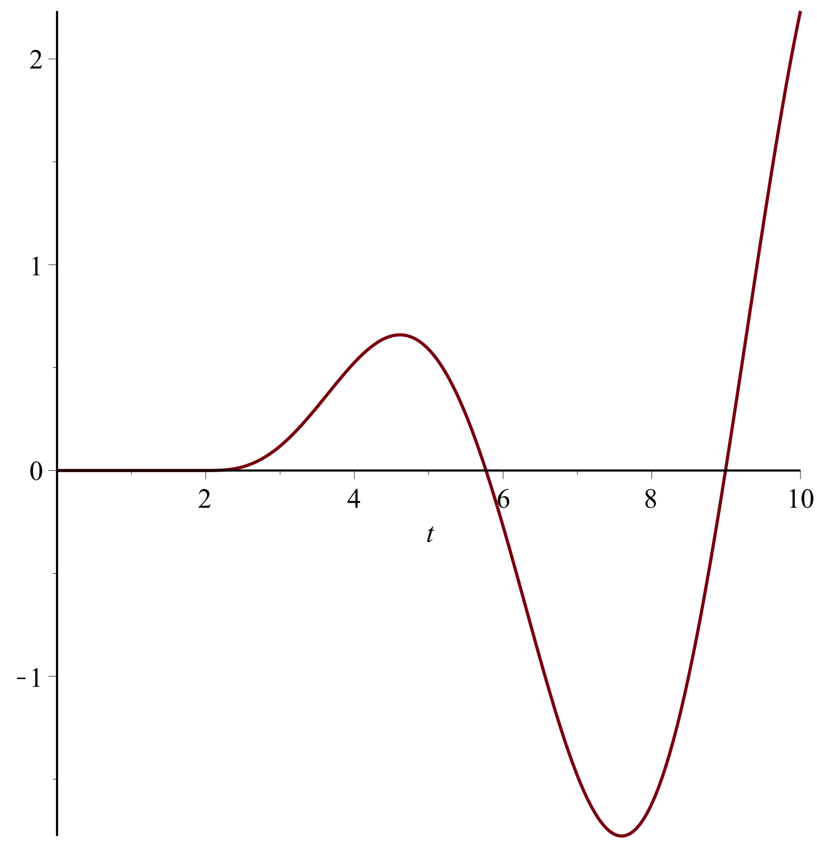
> $SolLap := isolate(EcuaLap, laplace(y(t), t, s))$

$$SolLap := laplace(y(t), t, s) = \frac{e^{-2s}\, s}{(s^2 + 1)\,(s^3 + s^2 + s + 1)} \tag{51}$$

> $SolPart := invlaplace(SolLap, s, t)$

$$SolPart := y(t) = \frac{1}{4}\left(-e^{-t+2} - \cos(t-2)\,(t-3) + \sin(t-2)\,(t-2)\right) \mathrm{Heaviside}(t-2) \tag{52}$$

> $plot(rhs(SolPart), t = 0..10)$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