Continuando TL.

convolución

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$$\mathcal{L}^{-1}\{F(s)\cdot G(s)\} = f(t) * g(t)$$

$$f(t) * g(t) = \int_0^t f(\tau)\cdot g(t-\tau)\,d\tau.$$

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$$\mathcal{L}^{-1}\{aF(s) + b\,G(s)\} = af(t) + bg(t)$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+9)^2}\right\}$$

$$\mathcal{L}^{-1}\left\{\underset{F(s)}{\frac{s}{s^2+9}}\cdot\underset{G(s)}{\frac{1}{s^2+9}}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+3^2}\right\}=\cos(3t)$$

$$\mathcal{L}^{-1}\left\{\frac{3}{s^2+3^2}\right\}=\operatorname{sen}(3t)$$

$$\frac{1}{3}\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\cdot\frac{3}{s^2+9}\right\}=\frac{1}{3}\left(\cos(3t)*\operatorname{sen}(3t)\right)$$

$$\frac{1}{3}\left(\cos(3t)*\operatorname{sen}(3t)\right)=\frac{1}{3}\int_0^t\cos(3z)\cdot\operatorname{sen}\left(3(t-z)\right)dz.$$

$$=\frac{1}{3}\int_0^t\cos(3z)\left[\operatorname{sen}(3t)\cos(3z)-\cos(3t)\operatorname{sen}(3z)\right]dz$$

$$=\frac{1}{3}\left(\operatorname{sen}(3t)\int_0^t\cos^2(3z)dz-\cos(3t)\int_0^t\cos(3z)\operatorname{sen}(3z)dz\right)$$

$$=\frac{\operatorname{sen}(3t)}{3}\left[\int\cos^2(3z)dz\right]_0^t-\frac{\cos(3t)}{3}\left[\int\cos(3z)\operatorname{sen}(3z)dz\right]_0^t$$

$$=\frac{\operatorname{sen}(3t)}{3}\left[\int\left(\frac{1}{2}+\frac{1}{2}\cos(6z)\right)dz\right]_0^t-\frac{\cos(3t)}{9}\left[\int\cos(3z)\operatorname{sen}(3z)3dz\right]_0^t$$

$$=\frac{\operatorname{sen}(3t)}{6}\int dz\Big|_0^t+\frac{\operatorname{sen}(3t)}{36}\int\cos(6z)6dz\Big|_0^t-\frac{\cos(3t)}{9}\left[\frac{\operatorname{sen}^2(3z)}{2}\right]_0^t$$

$$=\frac{\operatorname{sen}(3t)}{6}z\Big|_0^t+\frac{\operatorname{sen}(3t)}{36}\operatorname{sen}(6z)\Big|_0^t-\frac{\cos(3t)}{18}\left[\operatorname{sen}^2(3z)\right]_0^t$$

$$=\frac{t\operatorname{sen}(3t)}{6}+\frac{\operatorname{sen}(3t)\operatorname{sen}(6t)}{36}-\frac{\cos(3t)}{18}\operatorname{sen}^2(3t)$$

$$\boxed{=\frac{t\operatorname{sen}(3t)}{6}}+\cancel{\frac{\operatorname{sen}(3t)\left(2\operatorname{sen}(3t)\cos(3t)\right)}{36}}-\cancel{\frac{\cos(3t)\operatorname{sen}^2(3t)}{18}}$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^2+s+1}\right\} = \frac{2}{\sqrt{3}} e^{-\frac{1}{2}(t-2)} \operatorname{sen}\left(\frac{\sqrt{3}}{2}(t-2)\right) \cdot \mu(t-2).$$

(7) $$\mathcal{L}^{-1}\left\{e^{-as}F(s)\right\} = \begin{cases} 0 & ; t<a \\ f(t-a) & ; t\geq a \end{cases}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+s+1}\right\}$$

$$= f(t-a)\,\mu(t-a)$$

$$\mu(t-a) = \begin{cases} 0 & ; t<a \\ 1 & ; t\geq a \end{cases}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s^2+s)+1}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{\left(s^2+s+\frac{1}{4}\right)+\frac{3}{4}}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right\}$$

$$\frac{2}{\sqrt{3}}\mathcal{L}^{-1}\left\{\frac{\frac{\sqrt{3}}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right\} = \frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \operatorname{sen}\left(\frac{\sqrt{3}}{2}t\right)$$

(8) $$\mathcal{L}\left\{e^{at}f(t)\right\} = F(s-a)$$

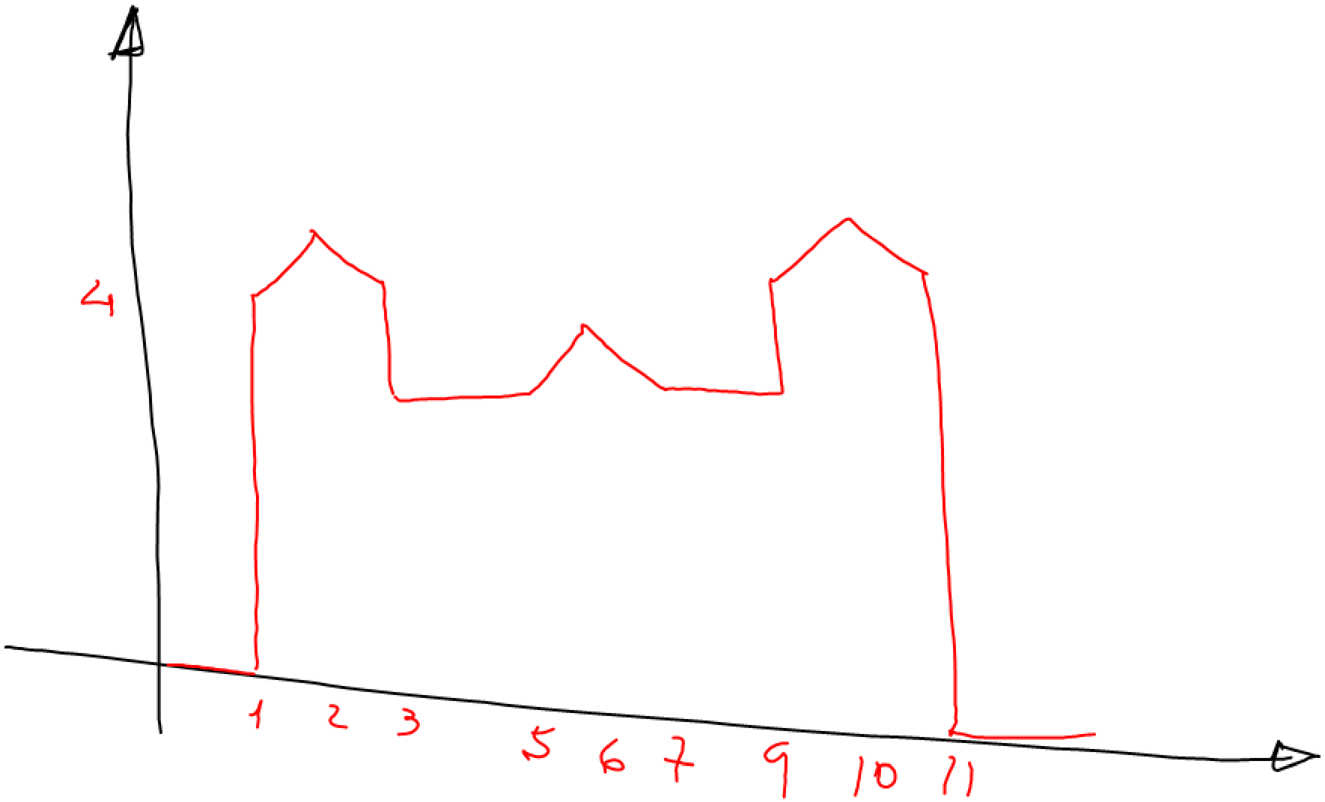
$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+2s+2}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+2s+1)+1}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s+1-1}{(s+1)^2+(1)^2}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s+1)^2+(1)^2}\right\}=\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+1^2}\right\}-\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+(1)}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s+1)^2+(1)^2}\right\}=e^{-t}\cos(t)-e^{-t}\operatorname{sen}(t).$$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