Clase 2019-10-22

5) DADA LA ECUACIÓN DIFERENCIAL DE CUARTO ORDEN SIGUIENTE:

$$\frac{d^4}{dt^4} y(t) + 5\left(\frac{d^2}{dt^2} y(t)\right) - 4\, y(t) = 5\, e^{-2t} \sin(3\, t)$$

$$y(0) = -5$$

$$D(y)(0) = -3$$

$$D^{(2)}(y)(0) = 4$$

$$D^{(3)}(y)(0) = 2$$

a) OBTENER UN SISTEMA DE ECUACIONES DIFERENCIALES EQUIVALENTE (CON TODO Y CONDICIONES INICIALES)

b) MOSTRAR LA REPRESENTACIÓN MATRICIAL DEL MISMO SISTEMA

c) OBTENER LA MATRIZ EXPONENCIAL QUE NOS PERMITA RESOLVERLO

d) OBTENER LA SOLUCIÓN PARTICULAR DADAS LAS CONDICIONES SEÑALADAS UTILIZANDO EL MÉTODO DE MATRIZ EXPONENCIAL

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> restart:
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SOLUCIÓN

> $Ecua := \frac{d^4}{dt^4} y(t) + 5\left(\frac{d^2}{dt^2} y(t)\right) - 4\, y(t) = 5\, e^{-2t} \sin(3\, t)$

$$Ecua := \frac{d^4}{dt^4} yy_1(t) + 5\left(\frac{d^2}{dt^2} yy_1(t)\right) - 4\, yy_1(t) = 5\, e^{-2t} \sin(3\, t) \quad \textbf{(1)}$$

> $y(t) := yy[1](t)$

$$y(t) := yy_1(t) \quad \textbf{(2)}$$

> $Sist[1] := diff(yy[1](t), t) = yy[2](t)$

$$Sist_1 := \frac{d}{dt} yy_1(t) = yy_2(t) \quad \textbf{(3)}$$

> $Sist[2] := diff(yy[2](t), t) = yy[3](t)$

$$Sist_2 := \frac{d}{dt} yy_2(t) = yy_3(t) \quad \textbf{(4)}$$

> $Sist[3] := diff(yy[3](t), t) = yy[4](t)$

$$Sist_3 := \frac{d}{dt} yy_3(t) = yy_4(t) \quad \textbf{(5)}$$

> $Sist[4] := diff(yy[4](t), t) = 4\cdot yy[1](t) - 5\cdot yy[3](t) + rhs(Ecua)$

$$Sist_4 := \frac{d}{dt} yy_4(t) = 4\, yy_1(t) - 5\, yy_3(t) + 5\, e^{-2t} \sin(3\, t) \quad \textbf{(6)}$$

> $Cond := y(0) = -5, D(y)(0) = -3, D^{(2)}(y)(0) = 4, D^{(3)}(y)(0) = 2$

$$Cond := y(0) = -5, D(y)(0) = -3, D^{(2)}(y)(0) = 4, D^{(3)}(y)(0) = 2 \quad \textbf{(7)}$$

> $with(LinearAlgebra):$

> $AA := Matrix([[0, 1, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1], [4, 0, -5, 0]])$

$$AA := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 4 & 0 & -5 & 0 \end{bmatrix} \tag{8}$$

> $MatExp := MatrixExponential(AA, t) : MatExp[1, 1]$

$$\frac{1}{2}\cos\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\,t\right) - \frac{5}{82}\sqrt{41}\cos\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\,t\right) + \frac{5}{164}\sqrt{41}\,e^{-\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,t} + \frac{5}{164}\sqrt{41}\,e^{\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,t} + \frac{1}{4}e^{-\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,t} + \frac{1}{4}e^{\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,t}$$

$$-\frac{5}{164}\sqrt{41}\,e^{-\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,t} + \frac{5}{82}\sqrt{41}\cos\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\,t\right) - \frac{5}{164}\sqrt{41}\,e^{\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,t} + \frac{1}{4}e^{-\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,t} + \frac{1}{2}\cos\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\,t\right) + \frac{1}{4}e^{\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,t} \tag{9}$$

> $MatExp[4, 4]$

$$-\frac{5}{164}\sqrt{41}\,e^{-\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,t} + \frac{5}{82}\sqrt{41}\cos\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\,t\right) - \frac{5}{164}\sqrt{41}\,e^{\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,t} + \frac{1}{4}e^{-\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,t} + \frac{1}{2}\cos\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\,t\right) + \frac{1}{4}e^{\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,t} \tag{10}$$

> $Identidad := map(rcurry(eval, t='0'), MatExp)$

$$Identidad := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

> $Xcero := array([-5, -3, 4, 2])$

$$Xcero := \begin{bmatrix} -5 & -3 & 4 & 2 \end{bmatrix} \tag{12}$$

> $SolHom := evalm(MatExp \&* Xcero) : SolHom[1]$

$$-\frac{5}{2}\cos\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\;t\right)+\frac{25}{82}\sqrt{41}\cos\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\;t\right)$$
$$-\frac{25}{164}\sqrt{41}\,e^{-\frac{1}{2}\sqrt{-10+2\sqrt{41}}\;t}-\frac{25}{164}\sqrt{41}\,e^{\frac{1}{2}\sqrt{-10+2\sqrt{41}}\;t}-\frac{5}{4}\,e^{-\frac{1}{2}\sqrt{-10+2\sqrt{41}}\;t}$$
$$-\frac{5}{4}\,e^{\frac{1}{2}\sqrt{-10+2\sqrt{41}}\;t}-\frac{1}{\sqrt{2\sqrt{41}+10}}\Bigg(3\Bigg($$
$$-\frac{1}{8}\sin\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\;t\right)\sqrt{-10+2\sqrt{41}}\;\sqrt{2\sqrt{41}+10}$$
$$-\frac{5}{328}\sqrt{41}\sin\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\;t\right)\sqrt{-10+2\sqrt{41}}\;\sqrt{2\sqrt{41}+10}$$
$$-\frac{33}{164}\sqrt{41}\,e^{-\frac{1}{2}\sqrt{-10+2\sqrt{41}}\;t}+\frac{33}{164}\sqrt{41}\,e^{\frac{1}{2}\sqrt{-10+2\sqrt{41}}\;t}$$
$$+2\sin\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\;t\right)-\frac{5}{4}\,e^{-\frac{1}{2}\sqrt{-10+2\sqrt{41}}\;t}+\frac{5}{4}\,e^{\frac{1}{2}\sqrt{-10+2\sqrt{41}}\;t}\Bigg)\Bigg)$$
$$+\frac{2}{41}\sqrt{41}\left(e^{-\frac{1}{2}\sqrt{-10+2\sqrt{41}}\;t}-2\cos\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\;t\right)+e^{\frac{1}{2}\sqrt{-10+2\sqrt{41}}\;t}\right)$$
$$-\frac{1}{164}\sqrt{41}\Bigg(2\sqrt{-10+2\sqrt{41}}\;\sin\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\;t\right)$$
$$+\sqrt{2\sqrt{41}+10}\;e^{-\frac{1}{2}\sqrt{-10+2\sqrt{41}}\;t}-\sqrt{2\sqrt{41}+10}\;e^{\frac{1}{2}\sqrt{-10+2\sqrt{41}}\;t}\Bigg) \tag{13}$$

> $CondIni := map(rcurry(eval, t='0'), SolHom)$

$$CondIni := \begin{bmatrix} -5 & -3 & 4 & 2 \end{bmatrix} \tag{14}$$

> $BB := array([0, 0, 0, rhs(Ecua)])$

$$BB := \begin{bmatrix} 0 & 0 & 0 & 5\,e^{-2t}\sin(3\,t) \end{bmatrix} \tag{15}$$

> $BBtau := map(rcurry(eval, t='tau'), BB)$

$$BBtau := \begin{bmatrix} 0 & 0 & 0 & 5\,e^{-2\tau}\sin(3\,\tau) \end{bmatrix} \tag{16}$$

> $MatExpTau := map(rcurry(eval, t='t - tau'), MatExp) : MatExpTau[4, 4]$

$$-\frac{5}{164}\sqrt{41}\,e^{-\frac{1}{2}\sqrt{-10+2\sqrt{41}}\;(t-\tau)}+\frac{5}{82}\sqrt{41}\cos\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\;(t-\tau)\right) \tag{17}$$
$$-\frac{5}{164}\sqrt{41}\,e^{\frac{1}{2}\sqrt{-10+2\sqrt{41}}\;(t-\tau)}+\frac{1}{4}\,e^{-\frac{1}{2}\sqrt{-10+2\sqrt{41}}\;(t-\tau)}$$

$$+\frac{1}{2}\cos\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\,(t-\tau)\right)+\frac{1}{4}\,e^{\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,(t-\tau)}$$

> *ProdTau* := *evalm*(*MatExpTau* &* *BBtau*) : *ProdTau*[4]

$$5\left(-\frac{5}{164}\sqrt{41}\,e^{-\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,(t-\tau)}+\frac{5}{82}\sqrt{41}\cos\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\,(t-\tau)\right)\right.$$
$$-\frac{5}{164}\sqrt{41}\,e^{\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,(t-\tau)}+\frac{1}{4}\,e^{-\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,(t-\tau)}$$
$$\left.+\frac{1}{2}\cos\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\,(t-\tau)\right)+\frac{1}{4}\,e^{\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,(t-\tau)}\right)e^{-2\tau}\sin(3\,\tau)$$

> *SolNoHom* := *map*(*int*, *ProdTau*, tau = 0 ..*t*) : *SolNoHom*[4]

$$-\frac{5}{2091328}\left(2715\,e^{-\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,t+2t}\sqrt{-10+2\sqrt{41}}\,\sqrt{41}\right.$$
$$-5430\sin\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\,t\right)\sqrt{2\sqrt{41}+10}\,\sqrt{41}\,e^{2t}$$
$$-2715\,e^{\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,t+2t}\sqrt{-10+2\sqrt{41}}\,\sqrt{41}$$
$$-21279\,e^{-\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,t+2t}\sqrt{-10+2\sqrt{41}}+8934\,e^{-\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,t+2t}\sqrt{41}$$
$$-42558\sin\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\,t\right)\sqrt{2\sqrt{41}+10}\,e^{2t}$$
$$+21279\,e^{\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,t+2t}\sqrt{-10+2\sqrt{41}}+8934\,e^{\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,t+2t}\sqrt{41}$$
$$-17868\sqrt{41}\cos\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\,t\right)e^{2t}-83886\,e^{-\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,t+2t}$$
$$-83886\,e^{\frac{1}{2}\sqrt{-10+2\sqrt{41}}\,t+2t}-167772\cos\left(\frac{1}{2}\sqrt{2\sqrt{41}+10}\,t\right)e^{2t}$$
$$\left.+2055904\sin(t)\cos(t)^2+1342176\cos(t)^3-513976\sin(t)-1006632\cos(t)\right)e^{-2t}$$

> *Ceros* := *evalf*(*map*(*rcurry*(*eval*, *t* ='0'), *SolNoHom*))

$$Ceros := \left[\begin{array}{cccc} -2.3\;10^{-12} & 0. & 0. & 0. \end{array}\right] \qquad \textbf{(18)}$$

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> restart;
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> $Ecua := diff(z(x,y), x\$2) + diff(z(x,y), x, y) = z(x,y)$

$$Ecua := \frac{\partial^2}{\partial x^2} z(x,y) + \frac{\partial^2}{\partial y\,\partial x} z(x,y) = z(x,y) \tag{19}$$

> $with(PDEtools):$

> $SolGral := pdsolve(Ecua)$

$$SolGral := (z(x,y) = \_F1(\_\xi 1)\,\_F2(\_\xi 2)) \ \&\text{where} \left[\left\{\frac{d}{d\_\xi 1}\_F1(\_\xi 1) = \_c_1\,\_F1(\_\xi 1), \frac{d}{d\_\xi 2}\_F2(\_\xi 2) = \frac{\_F2(\_\xi 2)}{\_c_1}\right\}, \{\_\xi 1 = y, \_\xi 2 = x - y\}\right] \tag{20}$$

> $SolGralFinal := build(SolGral)$

$$SolGralFinal := z(x,y) = \_C1\, e^{\_c_1 y}\, \_C2\, e^{\frac{x-y}{\_c_1}} \tag{21}$$

> $Comp := eval(subs(z(x,y) = rhs(SolGralFinal), Ecua))$

$$Comp := \_C1\, e^{\_c_1 y}\, \_C2\, e^{\frac{x-y}{\_c_1}} = \_C1\, e^{\_c_1 y}\, \_C2\, e^{\frac{x-y}{\_c_1}} \tag{22}$$

> $SolGralDos := z(x,y) = \exp(\_c_1 \cdot y) \cdot \left(C[10] \cdot \exp\left(\frac{(-\_c_1 + \text{sqrt}(\_c_1 \cdot\cdot 2 + 4))}{2} \cdot x \right) + C[20] \cdot \exp\left(\frac{(-\_c_1 - \text{sqrt}(\_c_1 \cdot\cdot 2 + 4))}{2} \cdot x \right) \right)$

$$SolGralDos := z(x,y) = e^{\_c_1 y} \left(C_{10}\, e^{\frac{1}{2}\left(-\_c_1 + \sqrt{\_c_1^2 + 4}\right)x} + C_{20}\, e^{\frac{1}{2}\left(-\_c_1 - \sqrt{\_c_1^2 + 4}\right)x} \right) \tag{23}$$

> $Comp := simplify(eval(subs(z(x,y) = rhs(SolGralDos), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comp := 0 = 0 \tag{24}$$

> $SolGralTres := z(x,y) = e^{-\_c_1 y} \left(C_{10}\, e^{\frac{1}{2}\left(\_c_1 + \sqrt{\_c_1^2 + 4}\right)x} + C_{20}\, e^{\frac{1}{2}\left(\_c_1 - \sqrt{\_c_1^2 + 4}\right)x} \right)$

$$SolGralTres := z(x,y) = e^{-\_c_1 y} \left(C_{10}\, e^{\frac{1}{2}\left(\_c_1 + \sqrt{\_c_1^2 + 4}\right)x} + C_{20}\, e^{\frac{1}{2}\left(\_c_1 - \sqrt{\_c_1^2 + 4}\right)x} \right) \tag{25}$$

> $CompTres := simplify(eval(subs(z(x,y) = rhs(SolGralTres), lhs(Ecua) - rhs(Ecua) = 0)))$

$$CompTres := 0 = 0 \tag{26}$$

> restart

>

SERIE TRIGONOMETRICA DE FOURIER

> $f := x \cdot\cdot 2 - 3 \cdot x$

$$f := x^2 - 3\,x \tag{27}$$

> $C := \frac{1}{4} \cdot int(f, x = -2..2)$

$$C := \frac{4}{3} \tag{28}$$

> $a[n] := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^{\wedge} n, \frac{1}{2} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{2}\right), x = -2..2\right)\right)$

$$a_n := \frac{16\,(-1)^n}{n^2\,\pi^2} \tag{29}$$

> $b[n] := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^{\wedge} n, \frac{1}{2} \cdot int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{2}\right), x = -2..2\right)\right)$

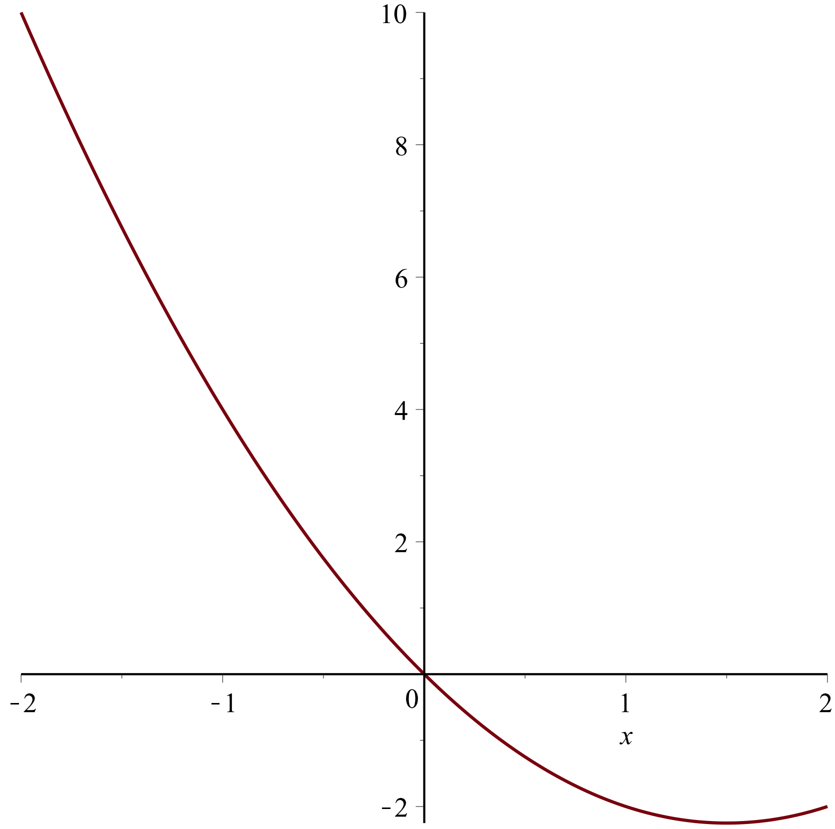
$$b_n := \frac{12\,(-1)^n}{n\,\pi} \tag{30}$$

> $STF := C + Sum\left(a[n] \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{2}\right) + b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{2}\right), n = 1..infinity\right)$

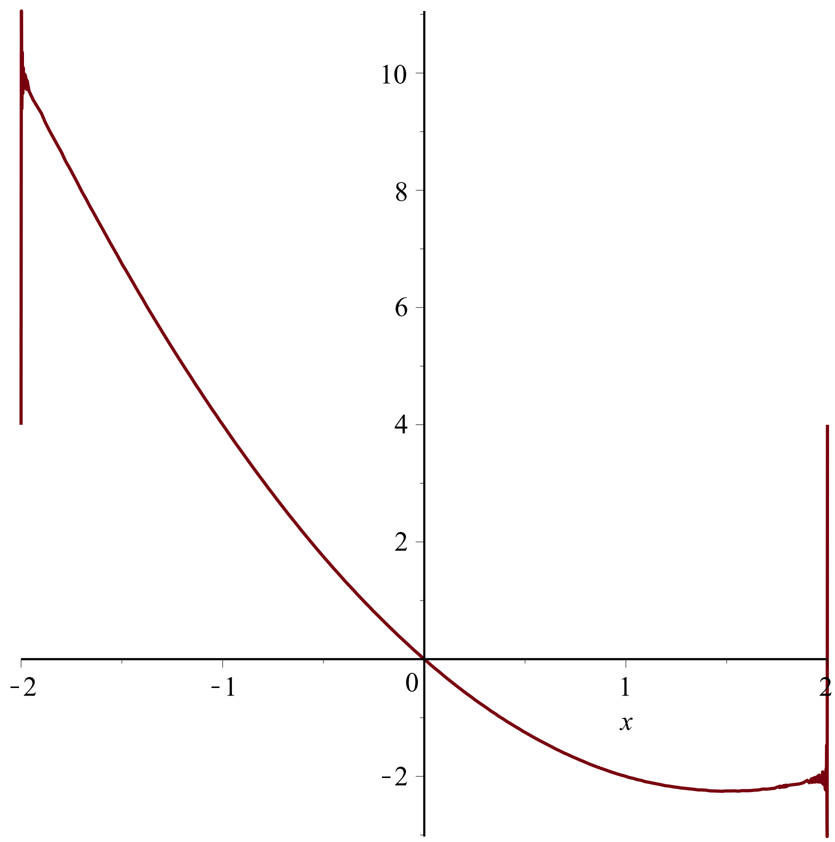
$$STF := \frac{4}{3} + \sum_{n=1}^{\infty} \left(\frac{16\,(-1)^n \cos\left(\frac{1}{2}\,n\,\pi\,x\right)}{n^2\,\pi^2} + \frac{12\,(-1)^n \sin\left(\frac{1}{2}\,n\,\pi\,x\right)}{n\,\pi} \right) \tag{31}$$

> $STF1000 := C + Sum\left(a[n] \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{2}\right) + b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{2}\right), n = 1..1000\right):$

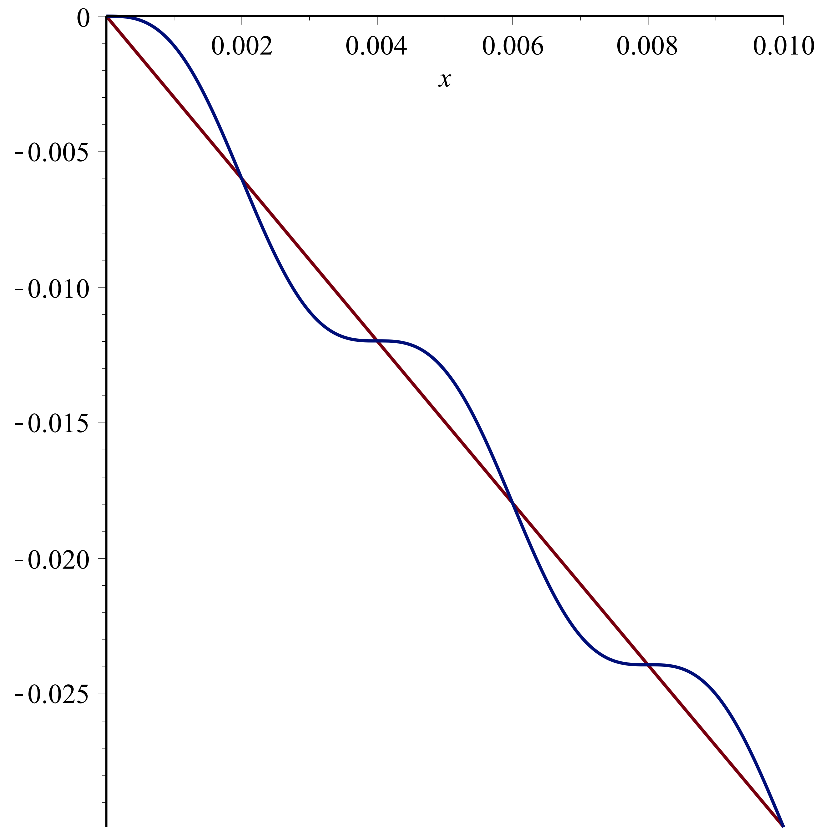
> $plot(f, x = -2..2)$



> $plot(STF1000, x = -2\,..2)$

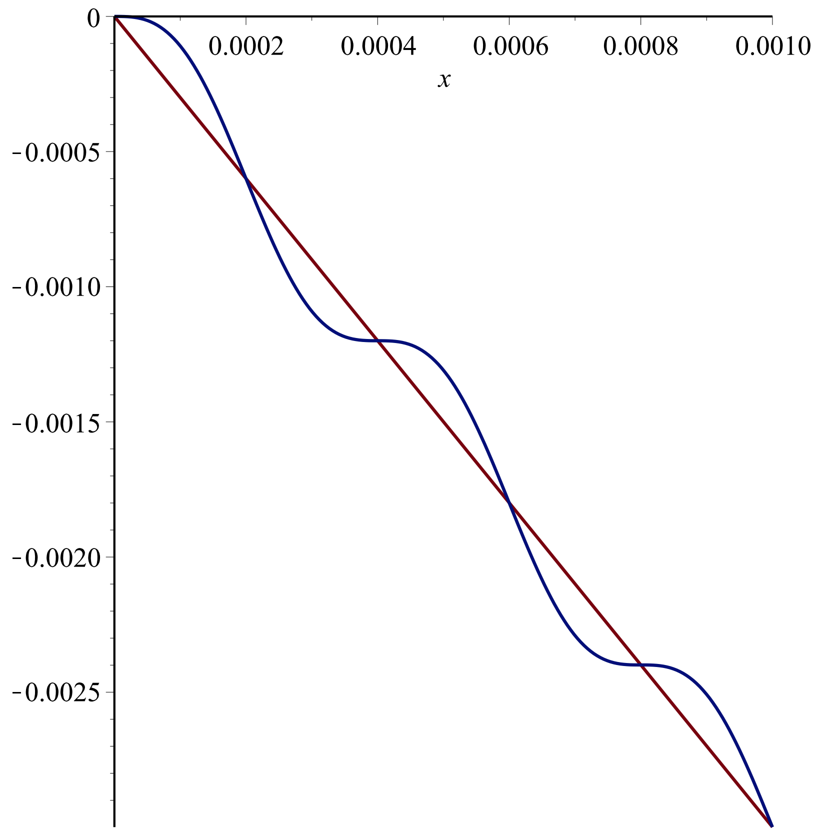


> $plot([f, STF1000], x = 0 ..0.01)$

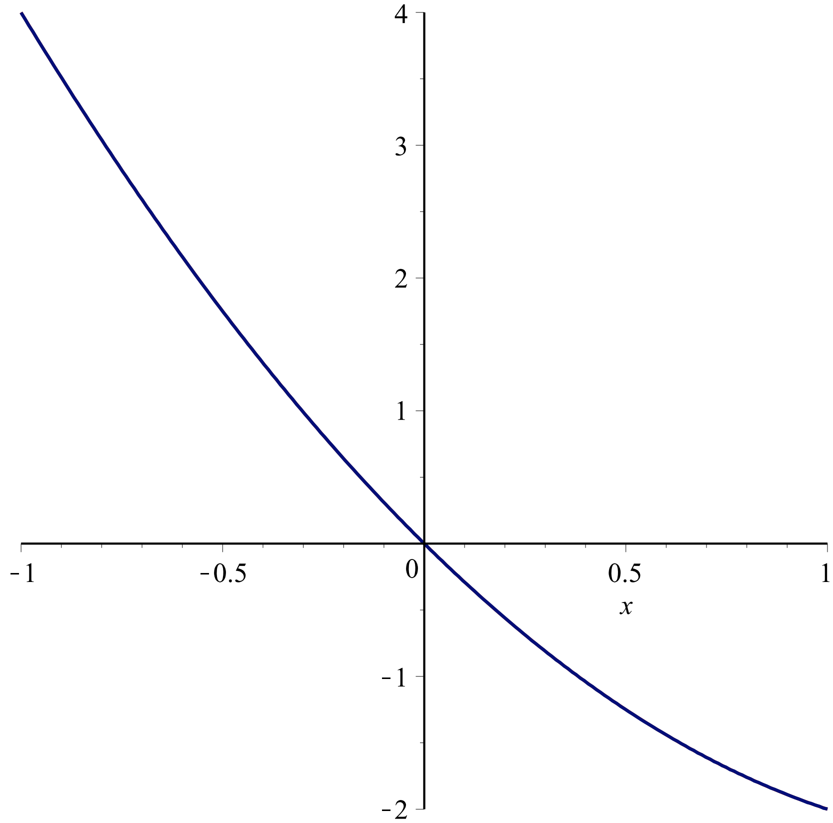


> $STF10000 := C + Sum\left(a[n]\cdot\cos\left(\frac{n\cdot\mathrm{Pi}\cdot x}{2}\right) + b[n]\cdot\sin\left(\frac{n\cdot\mathrm{Pi}\cdot x}{2}\right), n = 1\,..10000\right):$

> $plot([f, STF10000], x = 0\,..0.001)$



> $plot([f, STF1000], x = -1..1)$


4
3
2
1
-1
-2
-1
-0.5
0
0.5
1
x

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