

> restart

> Ecua := diff(z(x, y), x\$2) + 4·diff(z(x, y), x, y) - diff(z(x, y), y) = z(x, y)

$$Ecua := \frac{\partial^2}{\partial x^2} z(x, y) + 4 \left( \frac{\partial^2}{\partial y \partial x} z(x, y) \right) - \left( \frac{\partial}{\partial y} z(x, y) \right) = z(x, y) \quad (1)$$

> SolGral := pdsolve(Ecua)

$$SolGral := (z(x, y) = \_F1(\_ \xi 1) \_F2(\_ \xi 2)) \&where \left[ \left\{ \frac{d}{d\_ \xi 1} \_F1(\_ \xi 1) = \_c1 \_F1(\_ \xi 1), \right. \right. \\ \left. \left. \frac{d}{d\_ \xi 2} \_F2(\_ \xi 2) = \frac{\_F2(\_ \xi 2) + \_c1 \_F2(\_ \xi 2)}{4 \_c1 + \frac{1}{4}} \right\}, \left\{ \_ \xi 1 = y, \_ \xi 2 = x - \frac{1}{4} y \right\} \right] \quad (2)$$

> with(PDEtools) :

> SolGralDiez := build(SolGral)

$$SolGralDiez := z(x, y) = \_C1 \left( e^{\frac{-c_1^2 y}{16 \_c1 + 1}} \right)^{16} e^{\frac{-c_1 y}{16 \_c1 + 1}} \left( e^{\frac{\left(x - \frac{1}{4} y\right) \_c1}{16 \_c1 + 1}} \right)^4 \left( e^{\frac{x - \frac{1}{4} y}{16 \_c1 + 1}} \right)^4 \_C2 \quad (3)$$

> SolucionGeneral := simplify(subs(\\_c1 = -2, build(SolGral))); evalf(%, 3)

$$SolucionGeneral := z(x, y) = \_C1 e^{-\frac{63}{31} y + \frac{4}{31} x} \_C2 \\ z(x, y) = \_C1 e^{-2.03 y + 0.129 x} \_C2 \quad (4)$$

> ComprobacionUno := eval(subs(z(x, y) = rhs(SolucionGeneral), lhs(Ecua) - rhs(Ecua) = 0))  
ComprobacionUno := 0 = 0

POR SEPARACIÓN DE VARIABLES

> EcuaDos := eval(subs(z(x, y) = F(x)·G(y), Ecua))

$$EcuaDos := \left( \frac{d^2}{dx^2} F(x) \right) G(y) + 4 \left( \frac{d}{dx} F(x) \right) \left( \frac{d}{dy} G(y) \right) - F(x) \left( \frac{d}{dy} G(y) \right) \\ = F(x) G(y) \quad (6)$$

$$> EcuaTres := simplify \left( \frac{\left( lhs(EcuaDos) - \left( \frac{d^2}{dx^2} F(x) \right) G(y) \right)}{(4 \cdot diff(F(x), x) - F(x)) \cdot G(y)} \right)$$

$$= simplify \left( \frac{\left( rhs(EcuaDos) - \left( \frac{d^2}{dx^2} F(x) \right) G(y) \right)}{(4 \cdot diff(F(x), x) - F(x)) \cdot G(y)} \right)$$

$$EcuaTres := \frac{\frac{d}{dy} G(y)}{G(y)} = - \frac{\frac{d^2}{dx^2} F(x) - F(x)}{4 \left( \frac{d}{dx} F(x) \right) - F(x)} \quad (7)$$

> EcuaX := rhs(EcuaTres) = -2; EcuaY := lhs(EcuaTres) = -2

$$EcuaX := - \frac{\frac{d^2}{dx^2} F(x) - F(x)}{4 \left( \frac{d}{dx} F(x) \right) - F(x)} = -2$$

$$EcuaY := \frac{\frac{d}{dy} G(y)}{G(y)} = -2 \quad (8)$$

> SolX := dsolve(EcuaX); SolY := dsolve(EcuaY)

$$SolX := F(x) = \_C1 e^{(4 + \sqrt{15})x} + \_C2 e^{(-4 + \sqrt{15})x}$$

$$SolY := G(y) = \_C1 e^{-2y} \quad (9)$$

> SolGral := z(x, y) = simplify(subs(\\_C1 = 1, rhs(SolY)) \cdot rhs(SolX)); evalf(%, 3)

$$SolGral := z(x, y) = e^{-2y} \left( \_C1 e^{(4 + \sqrt{15})x} + \_C2 e^{(-4 + \sqrt{15})x} \right)$$

$$z(x, y) = e^{-2y} \left( \_C1 e^{7.87x} + \_C2 e^{0.13x} \right) \quad (10)$$

> ComprobacionDos := simplify(eval(subs(z(x, y) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0)))

$$ComprobacionDos := 0 = 0 \quad (11)$$

> SolucionGeneral; evalf(%, 3)

$$z(x, y) = \_C1 e^{-\frac{63}{31}y + \frac{4}{31}x} \_C2$$

$$z(x, y) = \_C1 e^{-2.03y + 0.129x} \_C2 \quad (12)$$

> restart

> Ecua := 2 \cdot diff(z(x, y), x\$2, y) - 2 \cdot diff(z(x, y), x, y) = z(x, y)

$$Ecua := 2 \left( \frac{\partial^3}{\partial y \partial x^2} z(x, y) \right) - 2 \left( \frac{\partial^2}{\partial y \partial x} z(x, y) \right) = z(x, y) \quad (13)$$

>

> EcuaDos := eval(subs(z(x, y) = F(x) \cdot G(y), Ecua))

$$EcuaDos := 2 \left( \frac{d^2}{dx^2} F(x) \right) \left( \frac{d}{dy} G(y) \right) - 2 \left( \frac{d}{dx} F(x) \right) \left( \frac{d}{dy} G(y) \right) = F(x) G(y) \quad (14)$$

> EcuaTres := simplify\left( \frac{lhs(EcuaDos)}{F(x) \cdot diff(G(y), y)} \right) = \frac{rhs(EcuaDos)}{F(x) \cdot diff(G(y), y)}

$$EcuaTres := \frac{2 \left( \frac{d^2}{dx^2} F(x) - \left( \frac{d}{dx} F(x) \right) \right)}{F(x)} = \frac{G(y)}{\frac{d}{dy} G(y)} \quad (15)$$

> EcuaX := lhs(EcuaTres) = -1; EcuaY := rhs(EcuaTres) = -1

$$EcuaX := \frac{2 \left( \frac{d^2}{dx^2} F(x) - \left( \frac{d}{dx} F(x) \right) \right)}{F(x)} = -1$$

$$EcuaY := \frac{G(y)}{\frac{d}{dy} G(y)} = -1 \quad (16)$$

>  $SolX := dsolve(EcuaX); SolY := dsolve(EcuaY)$

$$SolX := F(x) = \_C1 e^{\frac{1}{2}x} \sin\left(\frac{1}{2}x\right) + \_C2 e^{\frac{1}{2}x} \cos\left(\frac{1}{2}x\right)$$

$$SolY := G(y) = \_C1 e^{-y} \quad (17)$$

>  $SolGral := z(x, y) = subs(\_C1 = 1, rhs(SolY)) \cdot rhs(SolX); evalf(%, 3)$

$$SolGral := z(x, y) = e^{-y} \left( \_C1 e^{\frac{1}{2}x} \sin\left(\frac{1}{2}x\right) + \_C2 e^{\frac{1}{2}x} \cos\left(\frac{1}{2}x\right) \right)$$

$$z(x, y) = e^{-1 \cdot y} (\_C1 e^{0.500x} \sin(0.500x) + \_C2 e^{0.500x} \cos(0.500x)) \quad (18)$$

> *restart*

>  $Ecua := diff(z(x, y), x\$2) + 4 \cdot diff(z(x, y), x, y) - diff(z(x, y), y) = z(x, y)$

$$Ecua := \frac{\partial^2}{\partial x^2} z(x, y) + 4 \left( \frac{\partial^2}{\partial y \partial x} z(x, y) \right) - \left( \frac{\partial}{\partial y} z(x, y) \right) = z(x, y) \quad (19)$$

>  $SolGral := pdsolve(Ecua)$

$$SolGral := (z(x, y) = \_F1(\_ \xi1) \_F2(\_ \xi2)) \&where \left\{ \left\{ \frac{d}{d\_ \xi1} \_F1(\_ \xi1) = \_c1 \_F1(\_ \xi1), \right. \right. \quad (20)$$

$$\left. \left. \frac{d}{d\_ \xi2} \_F2(\_ \xi2) = \frac{\_F2(\_ \xi2) + \_c1 \_F2(\_ \xi2)}{4 \_c1 + \frac{1}{4}} \right\}, \left\{ \_ \xi1 = y, \_ \xi2 = x - \frac{1}{4} y \right\} \right\}$$

> *with(PDEtools) :*

>  $SolGralDiez := build(SolGral)$

$$SolGralDiez := z(x, y) = \_C1 \left( e^{\frac{-c_1^2 y}{16 \_c1 + 1}} \right)^{16} e^{\frac{-c_1 y}{16 \_c1 + 1}} \left( e^{\frac{\left(x - \frac{1}{4} y\right) \_c1}{16 \_c1 + 1}} \right)^4 \left( e^{\frac{x - \frac{1}{4} y}{16 \_c1 + 1}} \right)^4 \_C2 \quad (21)$$

>  $SolucionGeneral := simplify(subs(\_c1 = -2, build(SolGral))); evalf(%, 3)$

$$SolucionGeneral := z(x, y) = \_C1 e^{-\frac{63}{31}y + \frac{4}{31}x} \_C2$$

$$z(x, y) = \_C1 e^{-2.03y + 0.129x} \_C2 \quad (22)$$

POR SEPARACIÓN DE VARIABLES

>  $EcuaDos := eval(subs(z(x, y) = F(x) + G(y), Ecua))$

$$EcuaDos := \frac{d^2}{dx^2} F(x) - \left( \frac{d}{dy} G(y) \right) = F(x) + G(y) \quad (23)$$

>  $EcuaTres := simplify\left(\left( lhs(EcuaDos) + \left( \frac{d}{dy} G(y) \right) - F(x) \right)\right) = simplify\left(\left( rhs(EcuaDos) + \left( \frac{d}{dy} G(y) \right) - F(x) \right)\right)$

$$EcuaTres := \frac{d^2}{dx^2} F(x) - F(x) = G(y) + \frac{d}{dy} G(y) \quad (24)$$

>  $EcuaX := rhs(EcuaTres) = -2; EcuaY := lhs(EcuaTres) = -2$

$$EcuaX := G(y) + \frac{d}{dy} G(y) = -2$$

$$EcuaY := \frac{d^2}{dx^2} F(x) - F(x) = -2 \quad (25)$$

> SolX := dsolve(EcuaX); SolY := dsolve(EcuaY)

$$SolX := G(y) = -2 + e^{-y} \_C1$$

$$SolY := F(x) = e^{-x} \_C2 + e^x \_C1 + 2 \quad (26)$$

> SolGral := z(x,y) = simplify(subs(\\_C1 = 1, rhs(SolY)) + rhs(SolX)); evalf(%, 3)

$$SolGral := z(x,y) = e^{-x} \_C2 + e^x + e^{-y} \_C1$$

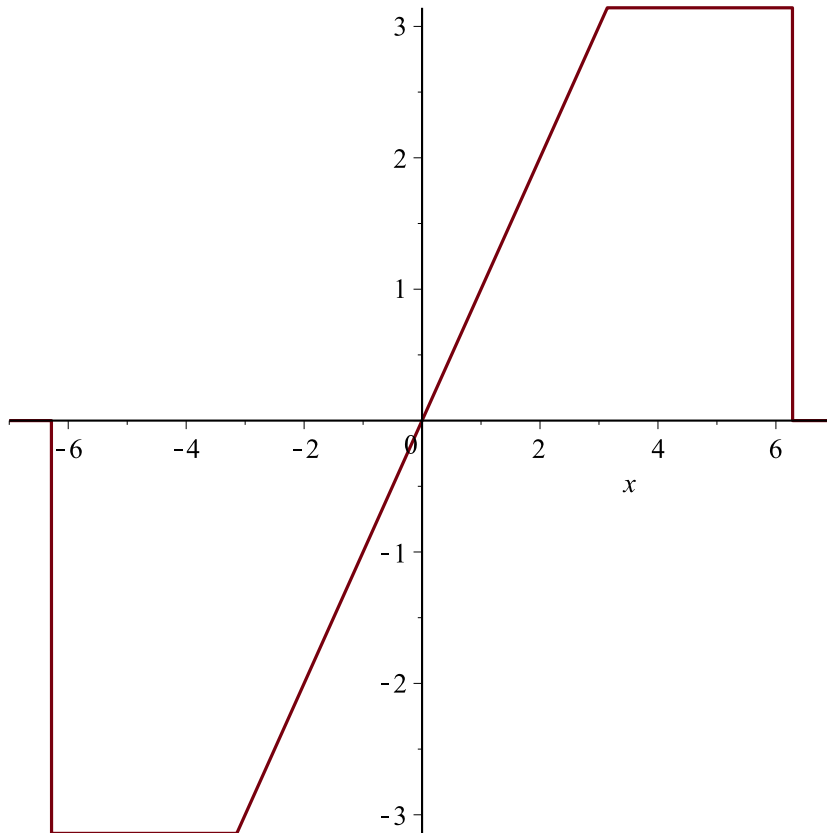
$$z(x,y) = e^{-1 \cdot x} \_C2 + e^x + e^{-1 \cdot y} \_C1 \quad (27)$$

>

> restart

> f := -Pi·Heaviside(x + 2·Pi) + (x + Pi)·Heaviside(x + Pi) - (x - Pi)·Heaviside(x - Pi) - Pi·Heaviside(x - 2·Pi); plot(f, x = -7..7)

$$f := -\pi \text{Heaviside}(x + 2\pi) + (x + \pi) \text{Heaviside}(x + \pi) - (x - \pi) \text{Heaviside}(x - \pi) - \pi \text{Heaviside}(x - 2\pi)$$



> L := 2·Pi

$$L := 2\pi$$

(28)

$$\begin{aligned} &> a[0] := \frac{1}{L} \cdot \text{int}(f, x = -L..L) \\ & \qquad \qquad \qquad a_0 := 0 \end{aligned} \tag{29}$$

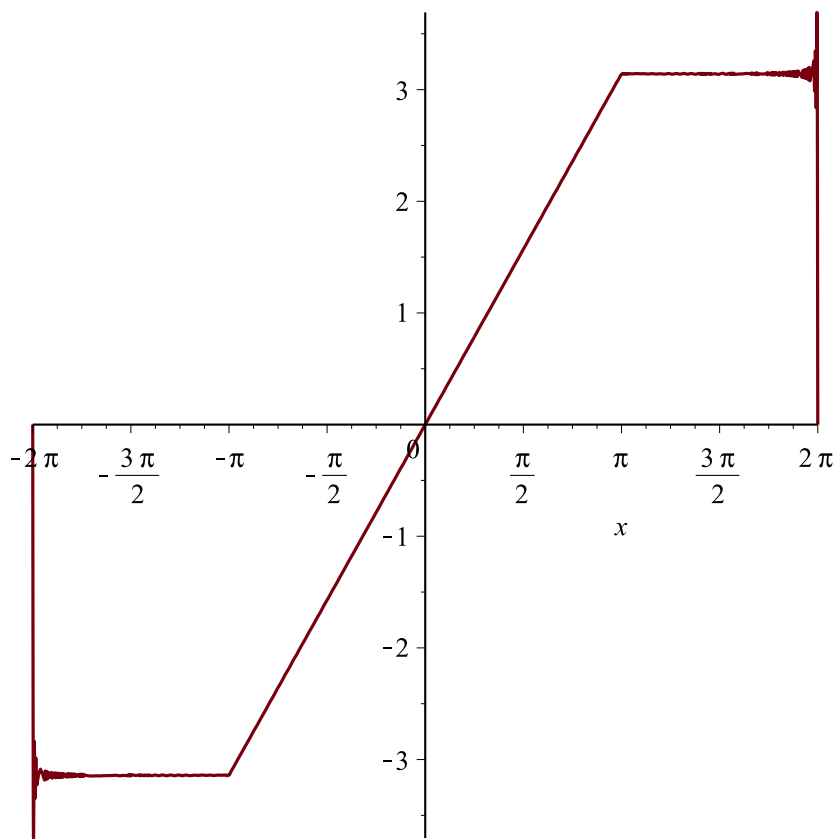
$$\begin{aligned} &> a[n] := \frac{1}{L} \cdot \text{int}\left(f \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right), x = -L..L\right) \\ & \qquad \qquad \qquad a_n := 0 \end{aligned} \tag{30}$$

$$\begin{aligned} &> b[n] := \frac{1}{L} \cdot \text{int}\left(f \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right), x = -L..L\right) \\ & \qquad \qquad \qquad b_n := \frac{1}{2} \frac{-4 n \pi \cos(n \pi) + 8 \sin\left(\frac{1}{2} n \pi\right)}{\pi n^2} \end{aligned} \tag{31}$$

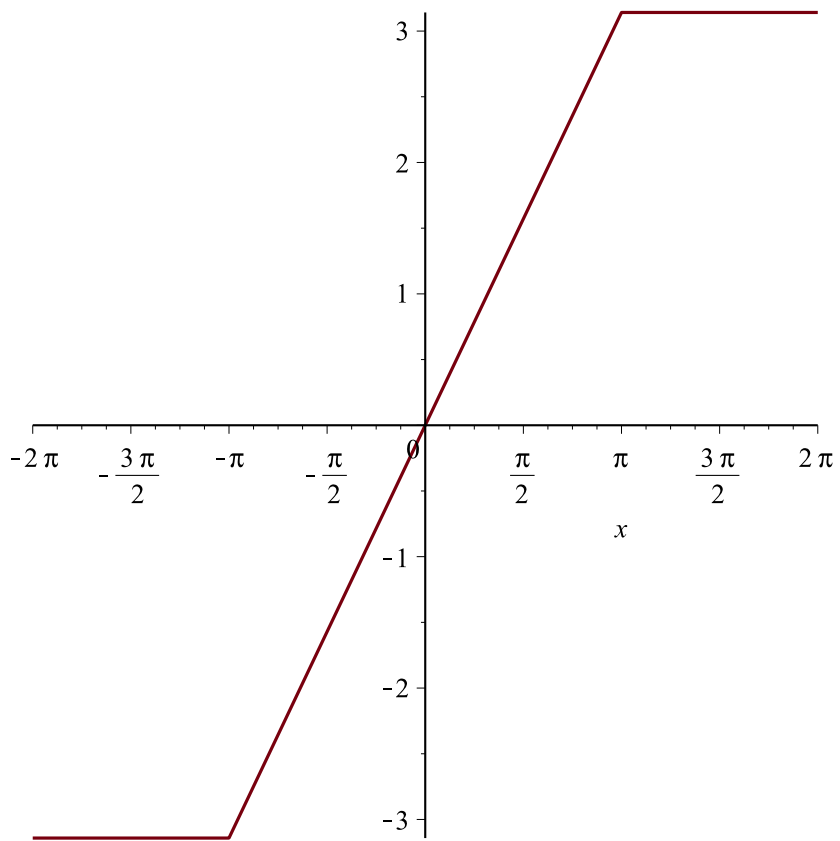
$$\begin{aligned} &> STF := \text{Sum}\left(b[n] \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right), n = 1..infinity\right) \\ & \qquad \qquad \qquad STF := \sum_{n=1}^{\infty} \frac{1}{2} \frac{\left(-4 n \pi \cos(n \pi) + 8 \sin\left(\frac{1}{2} n \pi\right)\right) \sin\left(\frac{1}{2} n x\right)}{\pi n^2} \end{aligned} \tag{32}$$

$$> STF500 := \text{sum}\left(b[n] \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right), n = 1..500\right) :$$

$$> \text{plot}(STF500, x = -2 \cdot \text{Pi}..2 \cdot \text{Pi})$$



=  
> `plot(f, x=-2·Pi..2·Pi)`



=  
> `plot([f, STF500], x=-0.05..0.05)`

