Clase 28/10/2021

Tema 3:- 2ª parte.

Sistemas de EDO primer orden

$$\boxed{\begin{aligned} (1)\quad \frac{dx(t)}{dt} &= 2x(t) + 3y(t) \\ (2)\quad \frac{dy(t)}{dt} &= x(t) + 4y(t) \end{aligned}}$$

Sistema 2 EDOL(1)CCH.

$$(3)\quad x(t) = \frac{dy(t)}{dt} - 4y(t)$$

$$(4)\quad \frac{d}{dt}\left(\quad \frac{dx(t)}{dt} = \frac{d^2y(t)}{dt^2} - 4\frac{dy(t)}{dt}\right.$$

sust. (3) y (4) en (1)

$$(5)\quad \left[\frac{d^2y(t)}{dt^2} - 4\frac{dy(t)}{dt}\right] = 2\left[\frac{dy(t)}{dt} - 4y(t)\right] + 3y(t)$$

$$\frac{d^2y(t)}{dt} + (-4-2)\frac{dy}{dt} + (8-3)\,y(t) = 0$$

$$(6)\quad \boxed{\frac{d^2y(t)}{dt^2} - 6\frac{dy(t)}{dt} + 5y(t) = 0}$$

EDOL(2)CCH.

$$(D^2 - 6D + 5)\,y(t) = 0$$

$$(D-1)(D-5)\,y(t) = 0$$

$$y(t) = C_1e^{t} + C_2e^{5t}$$

$$\frac{dy(t)}{dt} = C_1e^{t} + 5C_2e^{5t}$$

$$x(t) = \left[C_1e^{t} + 5C_2e^{5t}\right] - 4\left[C_1e^{t} + C_2e^{5t}\right]$$

$$\boxed{\begin{aligned} x(t) &= -3C_1e^{t} + C_2e^{5t} \\ y(t) &= C_1e^{t} + C_2e^{5t} \end{aligned}}$$

SOLUCIÓN GENERAL

$x(0) = 3 \qquad y(0) = -4$

$$-3c_1 + c_2 = 3$$
$$-c_1 - c_2 = +4$$
$$-4c_1 + (0)c_2 = 7$$

$$c_1 = -\frac{7}{4}$$

$$\frac{7}{4} - c_2 = 4$$
$$7 - 4c_2 = 16$$
$$-4c_2 = 9$$
$$c_2 = -\frac{9}{4}$$

$$\boxed{\begin{aligned} x(t) &= \frac{21}{4}e^{t} - \frac{9}{4}e^{5t} \\ y(t) &= -\frac{7}{4}e^{t} - \frac{9}{4}e^{5t} \end{aligned}}$$

SOLUCIÓN PARTICULAR

$$x(0) = \frac{21}{4} - \frac{9}{4} \Rightarrow \frac{12}{4} \Rightarrow 3$$
$$y(0) = -\frac{7}{4} - \frac{9}{4} \Rightarrow \frac{-16}{4} \Rightarrow -4$$

$$\underline{S(n)\,EDOL(1)\,cc.H \Leftrightarrow EDOL(n)\,cc\,H.}$$

$$\frac{dx(t)}{dt} = 2x(t) + 3y(t)$$

$$\frac{dy(t)}{dt} = x(t) + 4y(t)$$

$$\bar{x} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \qquad \frac{d}{dt}\bar{x} = \begin{bmatrix} \frac{dx(t)}{dt} \\ \frac{dy(t)}{dt} \end{bmatrix}$$

$$\boxed{\frac{d}{dt}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} * \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}}$$

$$\frac{d}{dt}\bar{x} = \underset{n\times n}{A}\,\bar{x}^{1\times n} \qquad S(n)\,EDOL(1)\,cc\,H.$$

VECTORIAL

$$\bar{x} = \left[e^{At}\right]\bar{x}(0)$$

matriz exponencial

$$\frac{d\bar{x}}{dt} = Ae^{At}\bar{x}(0)$$

$$\left[e^{At}\right]_{t=0} = I$$

$$\frac{d}{dt}\left[e^{At}\right] = Ae^{At}$$

$$\left[e^{At}\right] \times \left[e^{At}\right]^{-1} = I$$

$$\left[e^{At}\right]^{-1} = \left[e^{A(-t)}\right]$$

ESCALAR

$$\frac{dx(t)}{dt} = a\,x(t)$$

$$x(t) = C_1 e^{at}$$

$$x(0) = C_1 \cdot (1)$$

$$x(t) = x(0)\cdot e^{at}$$

$$\frac{dx(t)}{dt} = a e^{at} x(0)$$

$$\frac{dx(t)}{dt} = a\,\bar{x}(t)$$

función escalar $e^t = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \dots + \frac{t^k}{k!} + \dots\dots$

$t = 1 \rightarrow \underbrace{e = 2.72}\dots = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \dots + \frac{1}{k!} + \dots$

$$e^{At} = I + At + A^2\frac{t^2}{2!} + A^3\frac{t^3}{3!} + \dots + A^k\frac{t^k}{k!} + \dots$$

$A \quad \det(A - \lambda I) = 0$ valores característicos

$$\lambda^n + a_1\lambda^{n-1} + \dots + a_{n-1}\lambda + a_n = 0$$

$$A^n + a_1A^{n-1} + \dots + a_{n-1}A + a_nI = [0]$$

$$A^n = \underbrace{-a_nI - a_{n-1}A + \dots + a_1A^{n-1}}_{n \text{ términos}}$$

$$\frac{d^3y(t)}{dt^3} + \frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + y(t) = 0 \quad \text{EDOL}(3)\text{ccH.}$$

$$y(t) = y_1(t)$$

$$\frac{dy(t)}{dt} = y_1'(t) = y_2(t)$$

$$\frac{d^2y(t)}{dt} = y_2'(t) = y_3(t)$$

$$\frac{d^3y(t)}{dt^3} = -y(t) - \frac{dy(t)}{dt} - \frac{d^2y}{dt^2}$$

$$y_3'(t) = -y_1(t) - y_2(t) - y_3(t)$$

$$\left[\begin{array}{ll} \frac{dy_1(t)}{dt} = y_2(t) & y_1(0) \\ \frac{dy_2(t)}{dt} = y_3(t) & y_2(0) \\ \frac{dy_3(t)}{dt} = -y_1(t) - y_2(t) - y_3(t) & y_3(0) \end{array}\right.$$

$$\frac{d}{dt}\begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix}}_{A} \times \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix}$$

$$\bar{y} = e^{A(t-t_0)}\bar{y}(t_0)$$