> restart

>

Vamos a resolver la ecuación diferencial ordinaria que representa la caida libre en Física

> $Ecuacion := diff(y(t), t\$2) =- g$

$$Ecuacion := \frac{d^2}{dt^2} y(t) = -g \quad (1)$$

> $SolucionGeneral := dsolve(Ecuacion)$

$$SolucionGeneral := y(t) = -\frac{1}{2} g t^2 + \_C1\, t + \_C2 \quad (2)$$

> $DerivSol := diff(SolucionGeneral, t)$

$$DerivSol := \frac{d}{dt} y(t) = -g t + \_C1 \quad (3)$$

> $DerivSegSol := diff(DerivSol, t)$

$$DerivSegSol := \frac{d^2}{dt^2} y(t) = -g \quad (4)$$

>

> $Comprobacion := eval(subs(y(t) = rhs(SolucionGeneral), lhs(Ecuacion) - rhs(Ecuacion) = 0))$

$$Comprobacion := 0 = 0 \quad (5)$$

> $Ecuacion$

$$\frac{d^2}{dt^2} y(t) = -g \quad (6)$$

> $SolucionGeneral$

$$y(t) = -\frac{1}{2} g t^2 + \_C1\, t + \_C2 \quad (7)$$

> $Condiciones := y(0) = 20, D(y)(0) = 0$

$$Condiciones := y(0) = 20, D(y)(0) = 0 \quad (8)$$

> $SolPart := dsolve(\{Ecuacion, Condiciones\}, y(t))$

$$SolPart := y(t) = -\frac{1}{2} g t^2 + 20 \quad (9)$$

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> $SolPartFinal := subs\left(g = \frac{98067}{10000}, SolPart\right)$

$$SolPartFinal := y(t) = -\frac{98067}{20000} t^2 + 20 \quad (10)$$

> $TiempoCaida := solve(rhs(SolPartFinal) = 0, t)$

$$TiempoCaida := -\frac{200}{98067}\sqrt{980670}, \frac{200}{98067}\sqrt{980670} \quad (11)$$

en segundos

> $evalf(TiempoCaida[2], 4)$

$$2.019 \quad (12)$$

> $VelocidadFinal := diff(SolPartFinal, t)$

(13)

$$VelocidadFinal := \frac{d}{dt} y(t) = -\frac{98067}{10000} t \quad (13)$$

```
> VelocidadChoque := subs(t = TiempoCaida[2], rhs(VelocidadFinal))
```

$$VelocidadChoque := -\frac{1}{50}\sqrt{980670} \quad (14)$$

Velocidad de choque en metros por segundo (m/s)

```
> evalf(%, 4)
```

$$-19.81 \quad (15)$$

Velocidad de choque en Kilómetros por hora (Km/h)

```
> evalf(%·3.6, 4)
```

$$-71.32 \quad (16)$$

```
> restart
> IntM := Int((x··2)/(1+x), x) = int((x··2)/(1+x), x)
```

$$IntM := \int \frac{x^2}{1+x}\, dx = \frac{1}{2}x^2 - x + \ln(1+x) \quad (17)$$

```
> IntN := Int((y··2)/(1-y), y) = int((y··2)/(1-y), y)
```

$$IntN := \int \frac{y^2}{1-y}\, dy = -y - \frac{1}{2}y^2 - \ln(y-1) \quad (18)$$

```
> restart
> with(DEtools):
> Ecuacion := (y(x)··2 + x·y(x)··2)·diff(y(x), x) + (x··2 - y(x)·x··2) = 0
```

$$Ecuacion := \left(y(x)^2 + x\,y(x)^2\right)\left(\frac{d}{dx}y(x)\right) + x^2 - y(x)\,x^2 = 0 \quad (19)$$

```
> odeadvisor(Ecuacion)
```

$$[\_separable] \quad (20)$$

```
> M := factor(x··2 - y·x··2)
```

$$M := -x^2\,(y-1) \quad (21)$$

```
> P := -x··2; Q := (y - 1)
```

$$P := -x^2$$
$$Q := y - 1 \quad (22)$$

```
> N := factor(y··2 + x·y··2)
```

$$N := y^2\,(x+1) \quad (23)$$

```
> R := (x + 1); S := y··2
```

$$R := x + 1$$
$$S := y^2 \quad (24)$$

```
>
> SolGral := int(P/R, x) + int(S/Q, y) = C
```

$$SolGral := \frac{1}{2}x^2 - x + \ln(1+x) - y - \frac{1}{2}y^2 - \ln(y-1) = C \quad (25)$$

```
> restart
```

> $Int\left(\frac{1}{\mathrm{sqrt}(1-y\cdot\cdot 2)}, y\right) = int\left(\frac{1}{\mathrm{sqrt}(1-y\cdot\cdot 2)}, y\right)$

$$\int \frac{1}{\sqrt{-y^2+1}}\, dy = \arcsin(y) \tag{26}$$

> *restart*

> *with*(*DEtools*) :

> $Ecuacion := (y(x) + \mathrm{sqrt}(x\cdot\cdot 2 - y(x)\cdot\cdot 2)) - x\cdot diff(y(x), x) = 0$

$$Ecuacion := y(x) + \sqrt{x^2 - y(x)^2} - x\left(\frac{d}{dx} y(x)\right) = 0 \tag{27}$$

> $Tipo := odeadvisor(Ecuacion)$

$$Tipo := [[\_homogeneous, class\ A], \_rational, \_dAlembert] \tag{28}$$

>

> $EcuacionDos := expand(subs(y(x) = x\cdot u(x), Ecuacion))$

$$EcuacionDos := \sqrt{x^2 - x^2 u(x)^2} - x^2\left(\frac{d}{dx} u(x)\right) = 0 \tag{29}$$

> $M := factor\left(\sqrt{x^2 - x^2 u^2}\right)$

$$M := \sqrt{-x^2 (u-1)(u+1)} \tag{30}$$

> $N := -x^2$

$$N := -x^2 \tag{31}$$

> $P := x; Q := \sqrt{-(u-1)(u+1)}$

$$P := x$$

$$Q := \sqrt{-(u-1)(u+1)} \tag{32}$$

> $R := -x\cdot\cdot 2; S := 1$

$$R := -x^2$$

$$S := 1 \tag{33}$$

> $SolGral := int\left(\frac{P}{R}, x\right) + int\left(\frac{S}{Q}, u\right) = C$

$$SolGral := -\ln(x) + \arcsin(u) = C \tag{34}$$

> $SolGralDos := subs\left(u = \frac{y(x)}{x}, SolGral\right)$

$$SolGralDos := -\ln(x) + \arcsin\left(\frac{y(x)}{x}\right) = C \tag{35}$$

> $SolGralTres := isolate(SolGralDos, y(x))$

$$SolGralTres := y(x) = \sin(C + \ln(x))\, x \tag{36}$$

> $Sol := dsolve(EcuacionDos)$

$$Sol := -\arctan\left(\frac{u(x)}{\sqrt{-u(x)^2+1}}\right) + \ln(x) - \_C1 = 0 \tag{37}$$

> $SolDos := subs\left(u(x) = \frac{y(x)}{x}, Sol\right)$

(38)

$$SolDos := -\arctan\left(\frac{y(x)}{x\sqrt{-\frac{y(x)^2}{x^2}+1}}\right)+\ln(x)-\_C1=0 \tag{38}$$

```
> SolTres := simplify(isolate(SolDos, y(x)))
```

$$SolTres := y(x) = -x\sin(-\ln(x)+\_C1)\,\mathrm{csgn}(\cos(-\ln(x)+\_C1)) \tag{39}$$

```
>
```