Clase 12 OCTUBRE 2021.

Operador diferencial: método particular

EDOL(n)ccNH.

$y'(x)$ $\quad \frac{dy}{dx}$ $\quad Dy$ $\quad \dot{y}(x)$

Lineal respecto a la suma alg.
y al producto por una constante.

$$(D^2 - 4D + 4)y \Rightarrow (D-2)^2 y$$

$$\Rightarrow (D-2)(D-2)y$$

$$D^{-1}(Dy) \Rightarrow D^0 y = y$$

antiderivada

$$\int y\,dx = D^{-1}y + c_1$$

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$

$$(D^2 - 4D + 4)y = 0 \rightarrow m^2 - 4m + 4 = 0$$

$$m_1 = m_2 = 2$$

$$(m-2)(m-2) = 0$$

$$(D-2)^2 y = 0 \leftarrow (m-2)^2 = 0$$

CASO II $y = c_1 e^{2x} + c_2 x e^{2x}$

$$\frac{d^2y}{dx^2} + 9y = 0$$

$$(D^2 + 9)y = 0 \qquad m^2 + 9 = 0$$

$m_1 = 3i$ CASO III

$m_2 = -3i$

$$y = c_1 \cos(3x) + c_2 \operatorname{Sen}(3x)$$

$$y_g = c_1 e^{5x} + c_2 x e^{5x} + c_3 x^2 e^{5x}$$

$$(D-5)^3 y = 0$$

CASO II - repetido 3 veces

$$(D^3 - 15D^2 + 75D - 125)y = 0$$

$$\frac{d^3y}{dx^3} - 15\frac{d^2y}{dx^2} + 75\frac{dy}{dx} - 125y = 0$$

EDOL(3)CCH.

EDOL(2) cc NH con MOD

$y(x)$	D
1 x x^2 $\vdots$ x^n e^{ax} xe^{ax} $\vdots$ $x^n e^{ax}$	D D^2 D^3 $\vdots$ D^{n+1} $D-a$ $(D-a)^2$ $\vdots$ $(D-a)^{n+1}$
$\left.\begin{matrix}\cos bx\\ \operatorname{sen} bx\end{matrix}\right\}$ -	(D^2+b^2)
$\left.\begin{matrix}e^{ax}\cos(bx)\\ e^{ax}\operatorname{sen}(bx)\end{matrix}\right\}$	$((D-a)^2+b^2)$

$$y_g = C_1 e^{x}\cos(x) + C_2 e^{x}\operatorname{sen}(x) + C_3 x e^{x}\cos(x) + C_4 x e^{x}\operatorname{sen}(x)$$

CASOS II y III

$$\left((D-1)^2+(1)^2\right)^2 y = 0 \qquad EDOL(4)ccH$$

$$(D^2-2D+1+1)^2 y = 0$$

$$(D^2-2D+2)^2 y = 0$$

$$(D^2-2D+2)(D^2-2D+2)y = 0$$

$$(D^4-2D^3+2D^2-2D^3+4D^2-4D+2D^2-4D+4)y=0$$

$$(D^4-4D^3+8D^2-8D+4)y=0$$

$$\frac{d^4y}{dx^4} - 4\frac{d^3y}{dx^3} + 8\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 4y = 0$$

Utilizar el Operador Diferencial para resolver NO HOMOGENEAS.

$$(D^n + a_1 D^{n-1} + a_2 D^{n-2} \dots a_{n-1}D + a_n)y = Q(x)$$

$$\boxed{\begin{array}{ll}(D-a)(D-b)\dots\dots(D-n)y=0 & \text{HOMOGÉNEA ASOCIADA} \\ y_{g/H} = C_1y_1 + C_2y_2 + \dots\dots + C_ny_n & \text{SOL GENAL HA.}\end{array}}$$

$$Q(x) = \begin{cases} x^n & n = 0,1,2\dots \\ e^{ax} & a\in\mathbb{R} \\ \cos(bx) & b\in\mathbb{R}^+ \\ \operatorname{sen}(bx) & \end{cases}$$

Método de Coeficientes Indeterminados

EDOL(n)ccNH. $\rightarrow$ operador diferencial

$$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 4e^{2x} + 5x$$

EDOL(2)cc NH.

$$(D^2 - 7D + 12)y = 4e^{2x} + 5x$$

$$(D^2 - 7D + 12)y = 0$$

$$(D-3)(D-4)y = 0$$

$$y = C_1 e^{3x} + C_2 e^{4x}$$

Q(x)

$$(D-3)(D-4)y = \underbrace{4e^{2x} + 5x}$$

$$(D-3)(D-4)(D-2)_A D^2_P \, y = 0 \quad \text{EDOL(5)ccH.}$$

$$y = C_1 e^{3x} + C_2 e^{4x} + Ae^{2x} + Bx + D$$

$$y_{P/a} = Ae^{2x} + Bx + D \quad \text{coeficientes indeterminados}$$

$$Dy = 2Ae^{2x} + B + (0)$$

$$D^2y = 4Ae^{2x} + (0)$$

$$(4Ae^{2x}) - 7(2Ae^{2x} + B) + 12(Ae^{2x} + Bx + D) = 4e^{2x} + 5x$$

$$(4A - 14A + 12A)e^{2x} + 12Bx + (-7B + 12D)(1) = 4e^{2x} + 5x$$

$$2A = 4 \qquad 12B = 5 \qquad -7B + 12D = 0$$

$$\boxed{A = 2} \qquad \boxed{B = \frac{5}{12}} \qquad -7\left(\frac{5}{12}\right) + 12D = 0$$

$$12D = \frac{35}{12}$$

$$\boxed{D = \frac{35}{144}}$$

$$y_{g/NH} = C_1 e^{3x} + C_2 e^{4x} + 2e^{2x} + \frac{5}{12}x + \frac{35}{144}$$