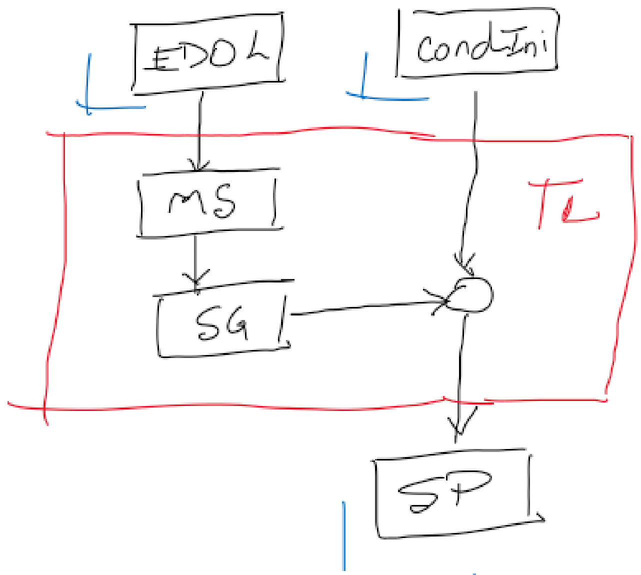
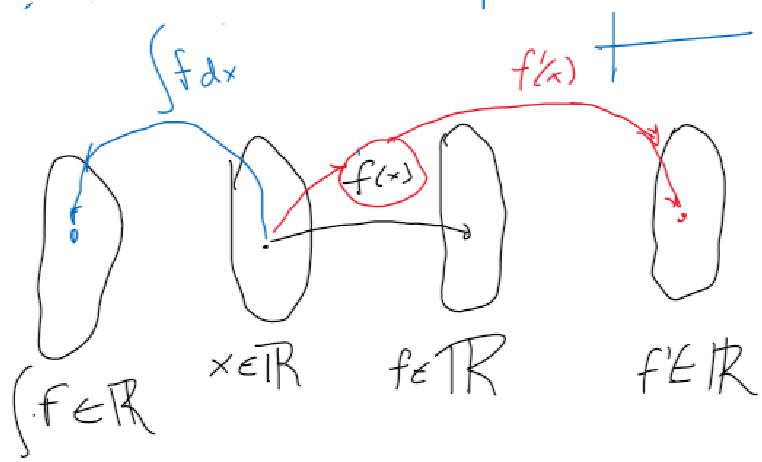
Clase 14 de octubre de 2021.

Van a recibir un correo invitándolos a unirse

https://ursul.yeira.training

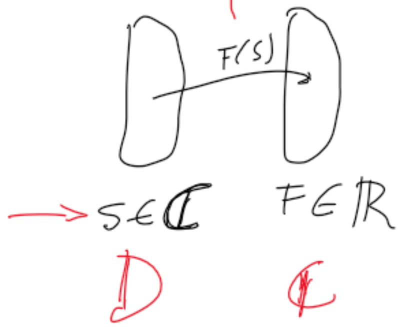
TEMA III - Transformada de Laplace





Cálculo
Diferencial
Integral.

$F(s) = T\{f(x)\}$



$$T\{af+bg\} \longrightarrow aF(s)+bG(s)$$

$a, b \in \mathbb{R}$

$$T\{f'\} \longrightarrow sF(s)$$

fácil

fácil

$$T\left\{\int f\,dx\right\} \longrightarrow \frac{F(s)}{s}$$

ojo

$$T\{f(t)\} = \int_{-\infty}^{\infty} N(s,t)\, f(t)\, dt$$

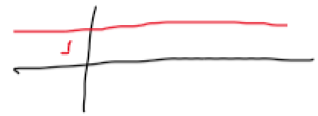
núcleo argumento

$$\boxed{\text{Laplace} \quad N(s,t) = \begin{cases} 0 \;; & -\infty < t < 0 \\ e^{-st} \;; & 0 \le t < \infty \end{cases}}$$

$$\Rightarrow \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\, dt$$

Transformada de Laplace

$f(t) = 1$



$$\mathcal{L}\{1\} = \int_0^\infty e^{-st}\cdot(1)\,dt$$

$$= \left[\int e^{-st}\,dt\right]_0^\infty$$

$$= \left[\frac{e^{-st}}{-s}\right]_0^\infty$$

$$\mathcal{L}\{1\} = \left(-\frac{1}{s}\right)\left[\lim_{t\to\infty} e^{-st} - \underbrace{e^{-s(0)}}_{=1}\right]$$

$$\lim_{a\to\infty} e^{sa}\to\infty \quad \Bigg| \quad \boxed{\lim_{a\to\infty} e^{-sa} = \lim_{a\to\infty}\frac{1}{e^{sa}} = \lim_{b\to\infty}\frac{1}{b} = 0}$$

$$\mathcal{L}\{1\} = \left(-\frac{1}{s}\right)\left[(0) - 1\right] \Rightarrow \frac{1}{s}$$

$$\mathcal{L}\{t\} = \int_0^\infty e^{-st} \cdot t \cdot dt$$

$$= \left[\int t e^{-st} dt \right]_0^\infty$$

$$= \left[\frac{te^{-st}}{-s} - \frac{e^{-st}}{s^2} \right]_0^\infty$$

$$= -\frac{1}{s}\left[te^{-st} \right]_0^\infty - \frac{1}{s^2}\left[e^{-st} \right]_0^\infty$$

$$= -\frac{1}{s}\left[\lim_{t \to \infty} t \cdot \lim_{t \to \infty} e^{-st} - (0)e^{-s(0)} \right] -$$

$$-\frac{1}{s^2}\left[\lim_{t \to \infty} e^{-st} - e^{-s(0)} \right]$$

$$\lim_{t \to \infty} e^{-st} = 0.$$

$$= \frac{1}{s}\left((0) - (0) \right) - \frac{1}{s^2}\left((0) - 1 \right)$$

$$\boxed{\mathcal{L}\{t\} = \frac{1}{s^2}} \qquad \boxed{\mathcal{L}\{1\} = \frac{1}{s}}$$

$f(t)$	$F(s)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^2	$\frac{2}{s^3}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\cos(bt)$	$\frac{s}{s^2+b^2}$
$sen(bt)$	$\frac{b}{s^2+b^2}$

$$\mathcal{L}^{-1}\left\{ F(s) \right\} = \int_{a-i\infty}^{a+i\infty} e^{st} F(s)\,ds$$

$s \in \mathbb{C}$
$f \in \mathbb{R}$

$$\mathcal{L}^{-1}\left\{ \frac{4}{s^2+16} \right\} = sen(4t)$$

1. $\mathcal{L}\{af(t)+bg(t)\} = aF(s) + bG(s) \quad a, b \in \mathbb{R}$

2. $\mathcal{L}\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$

$\mathcal{L}\{f(t)\} = F(s)$

$$\mathcal{L}\{e^{t}\} = \frac{1}{s-1} \qquad \mathcal{L}\{e^{at}\} = \frac{1}{a}\cdot\left(\frac{1}{\frac{s}{a}-1}\right)$$

$$\frac{\frac{a}{b}}{\frac{c}{d}}$$

$$= \frac{1}{a}\left(\frac{1}{\frac{s-a}{a}}\right)$$

$$\frac{a\cdot d}{b\cdot c}$$

$$= \cancel{\frac{1}{a}}\left(\frac{\cancel{a}}{s-a}\right)$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

3.

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

condiciones iniciales

$$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

$$\mathcal{L}\{f'''(t)\} = s^3F(s) - s^2f(0) - sf'(0) - f''(0)$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^nF(s) - \sum_{i=0}^{n-1} s^{n-i}f^{(i)}(0)$$

4. $\mathcal{L}^{-1}\{F'(s)\} = -t\,f(t)$

$\mathcal{L}^{-1}\{F^{(n)}(s)\} = (-1)^n t^n f(t)$

5. $\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{F(s)}{s}$

6. $\mathcal{L}^{-1}\left\{\int_s^\infty F(\sigma)\,d\sigma\right\} = \frac{f(t)}{t}$

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 0 \qquad \begin{array}{l} y(0)=2 \\ y'(0)=-3 \end{array}$$

TL

$$\mathcal{L}\left\{ \frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y \right\} = (0)\mathcal{L}\{1\}$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2}\right\} - 5\mathcal{L}\left\{\frac{dy}{dt}\right\} + 6\mathcal{L}\{y\} = 0$$

$$\left(s^2 Y(s) - s\cdot(2) - (-3)\right) - 5\left(sY(s) - 2\right) + 6Y(s) = 0$$

$$(s^2 - 5s + 6)Y(s) - 2s + 3 + 10 = 0$$

$$(s^2 - 5s + 6)Y(s) = 2s - 13$$

$$Y(s) = \frac{2s-13}{s^2-5s+6} \qquad \begin{array}{l} Y \in \mathbb{R} \\ s \in \mathbb{C} \end{array}$$

$$\frac{2s-13}{s^2-5s+6} = \frac{2s-13}{(s-2)(s-3)}$$

$$\frac{2s-13}{(s-2)(s-3)} = \frac{A}{s-2} + \frac{B}{s-3}$$

$$2s - 13 = A(s-3) + B(s-2)$$

$$= (A+B)s + (-3A - 2B)$$

$$A + B = 2$$

$$-3A - 2B = -13$$

$$3A + 3B = 6$$

$$(0) \quad B = -7$$

$$A - 7 = 2$$

$$A = 9$$

$$Y(s) = \frac{9}{s-2} - \frac{7}{s-3}$$

$$\mathcal{L}^{-1}\{Y(s)\} = 9\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} - 7\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\}$$

$$y(t) = 9e^{2t} - 7e^{3t}$$

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 0 \qquad y(0) = 2$$

$$y'(0) = -3$$