Clase jueves 21 octubre 2021

Transformada de Laplace

$\mathcal{L}\{f(t)\} = F(s)$ $\qquad$ $\mathcal{L}^{-1}\{F(s)\} = f(t)$

es única $\neq$ no es única ∞

En la clase anterior 6 propiedades básicas

7.- $\mathcal{L}\{f(t-a)\} = e^{-as}F(s)$ trasl. de dom. "t"

8.- $\mathcal{L}\{e^{at}f(t)\} = F(s-a)$ taslación del dominio "s"

$$\mathcal{L}\{\cos(bt)\} = \frac{s}{s^2+b^2}$$

$$\mathcal{L}\{e^{3t}\cos(2t)\} = \frac{(s-3)}{(s-3)^2+(2)^2}$$

$$= \frac{s-3}{s^2-6s+9+4}$$

$$\mathcal{L}\{e^{3t}\cos(2t)\} = \frac{s-3}{s^2-6s+13}$$

$$\mathcal{L}\{t^2\} = \frac{3!}{s^4}$$

$$\mathcal{L}\{t^2e^{2t}\} = \frac{6}{(s-2)^4}$$

---

e^{at}, t^n $\}$ $t^n e^{at}$

$\cos bt$, $\text{sen } bt$ $\}$ $t^n\cos(bt)$, $t^n \text{sen}(bt)$

$e^{at}\cos(bt)$ $\qquad$ $n = 0, 1, \ldots \in \mathbb{Z}^{+}$

$e^{at}\text{sen}(bt)$

$t^n e^{at}\cos(bt)$ $\qquad$ $a \in \mathbb{R}$

$t^n e^{at}\text{sen}(bt)$ $\qquad$ $b \in \mathbb{R}^{+}$

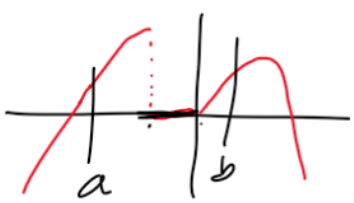
$\mathcal{L}\{f(t)\} = F(s)$ $\qquad$ $f(t)$ clase "A"

1. una función es de clase "A"

1.- Seccionalmente

finito número de discont en $a \leq t \leq b$

$\left.\begin{matrix} \tan(bt) \\ \cot(bt) \\ \sec(bt) \\ \csc(bt) \end{matrix}\right\}$ seccionalmente continuas.



no es de clase "A"

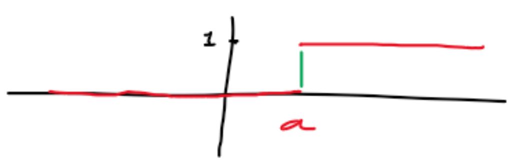
infinito # discont. en $a \leq t \leq b$

2.- orden exponencial

$$|f(t)| \leq Ae^{mt} \quad A, m \in \mathbb{R}$$

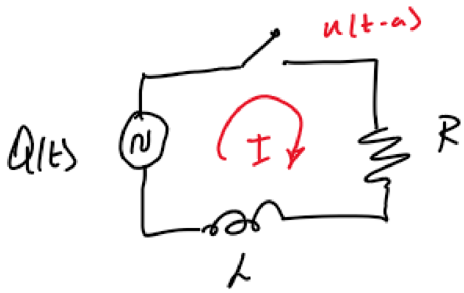
no es DE $\quad e^{t^2}$

Función escalón unitario $\quad u(t-a) \begin{cases} 0 \; ; & t < a \\ 1 \; ; & t \geq a \end{cases}$



clase "A"

$$\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$$

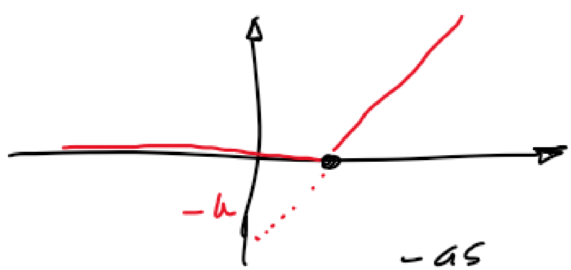


$$L\frac{di}{dt} + Ri = u(t-a)\,Q(t) \qquad \mathcal{L}\{i(t)\} = I(s)$$

$$L\left(sI(s) - i(0)\right) + R\,I(s) = \mathcal{L}\{u(t-a)\,Q(t)\}.$$

Rampa unitaria

$$r(t-a) = \begin{cases} 0 & ; \ t < a \\ (t-a) & ; \ t \geq a \end{cases}$$



$r(t-a) = 0$
$t = a$

$$\mathcal{L}\{r(t-a)\} = \frac{e^{-as}}{s^2}$$

$$\mathcal{L}\left\{\frac{d}{dt} r(t-a)\right\} = s\,\mathcal{L}\{r(t-a)\} - r(t-a)\Big|_{t=0}$$

$$= s\left[\frac{e^{-as}}{s^2}\right] - (0)$$

$$\mathcal{L}\left\{\frac{d}{dt} r(t-a)\right\} = \frac{e^{-as}}{s}$$

$$\mathcal{L}\left\{\frac{d}{dt} r(t-a)\right\} = \mathcal{L}\{u(t-a)\}$$

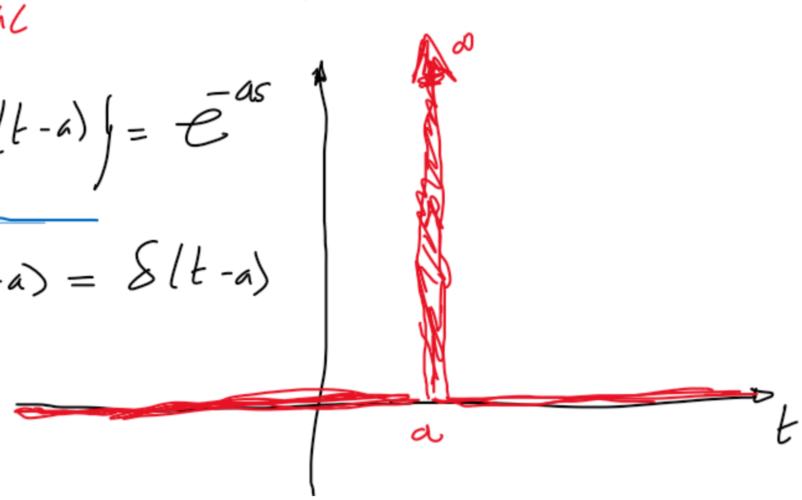
$$\boxed{\frac{d}{dt} r(t-a) = u(t-a)}$$

impulso unitario $\delta(t-a) = \begin{cases} 0 : t \neq a \\ \int_{-\infty}^{\infty} \delta(t-a)\,dt = 1 \end{cases}$

Dirac

$$\mathcal{L}\{\delta(t-a)\} = e^{-as}$$

$$\frac{d}{dt} u(t-a) = \delta(t-a)$$



1.- $\mathcal{L}^{-1}\{F(s)\cdot G(s)\} = f(t) * g(t)$

convolución

$$f(t) * g(t) = \int_0^t f(\tau)\cdot g(t-\tau)\,d\tau$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-3)\cdot(s-2)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s-3}\cdot\frac{1}{s-2}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\} = e^{3t}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} = e^{2t}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-3)(s-2)}\right\} = e^{3t} * e^{2t}$$

$$e^{3t} * e^{2t} = \int_0^t e^{3\tau}\cdot e^{2(t-\tau)}\,d\tau$$

$$= \int_0^t e^{3\tau}\cdot e^{2t}\cdot e^{-2\tau}\,d\tau$$

$$= \left[e^{2t}\int e^{\tau}\,d\tau\right]_0^t$$

$$= e^{2t}\left[e^{\tau}\right]_0^t$$

$$= e^{2t}(e^{t}-1)$$

$$= e^{3t} - e^{2t}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+9)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\cdot\frac{1}{s^2+9}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} * \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{3}{s^2+9}\right\}$$

$$= \cos(3t) * \frac{1}{3}\operatorname{sen}(3t)$$

convolución
de 2 funciones

$$= \frac{1}{3}\int_0^t \cos(3\tau)\operatorname{sen}(3(t-\tau))\,d\tau$$

$$= \frac{1}{6}\,t\operatorname{sen}(3t)$$