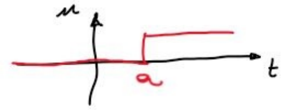
Clase 26-10-2021

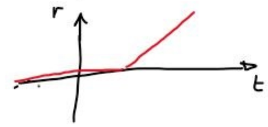
Transformada de Laplace

escalón unitario $\rightarrow u(t-a) = \begin{cases} 0 \ ; & t<a \\ 1 \ ; & t \geq a \end{cases}$



Heaviside

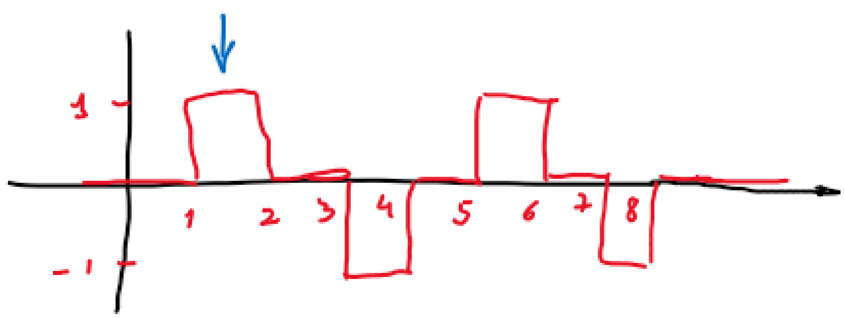
rampa unitaria $\rightarrow r(t-a) = \begin{cases} 0 \ ; & t<a \\ (t-a) \ ; & t \geq a \end{cases}$

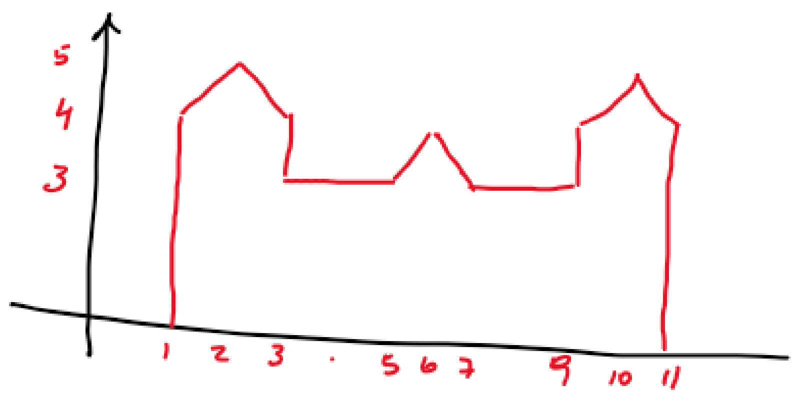


$\hookrightarrow r(t-a) = (t-a) \cdot u(t-a) \leftarrow$

$= (t-a) \cdot \begin{cases} 0 \ ; & t<a \\ 1 \ ; & t \geq a \end{cases}$

Dirac. $\Rightarrow$ impulso unitario





$$\mathcal{L}^{-1}\left\{\frac{e^{-2s}\cdot s}{s^2+2s+2}\right\}=$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+2s+2}\right\}=\mathcal{L}^{-1}\left\{\frac{s}{(s^2+2s)+2}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{s}{(s^2+2s+1)+1}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{s}{(s+1)^2+(1)^2}\right\}$$

$$=\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+(1)^2}-\frac{1}{(s+1)^2+(1)^2}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+2s+2}\right\}=e^{-t}\cos(t)-e^{-t}\operatorname{sen}(t)$$

$$\boxed{\mathcal{L}^{-1}\left\{\frac{e^{-2s}s}{s^2+2s+2}\right\}=e^{-(t-2)}\cos(t-2)\cdot\mu(t-2)-e^{-(t-2)}\operatorname{sen}(t-2)\cdot\mu(t-2).}$$

$$\frac{d^2y}{dt^2}-9\frac{dy}{dt}+20y=2e^{3t}\cos(4t)\qquad \begin{array}{l} y(0)=5\\ y'(0)=-2\end{array}$$

Resolver el problema de cond. iniciales con la transformada de Laplace y graficar la sol. particular.

$$\mathcal{L}\{y''-9y'+20y\}=2\mathcal{L}\{e^{3t}\cos(4t)\}$$

$$\mathcal{L}\{y''\} - 9\mathcal{L}\{y'\} + 20\mathcal{L}\{y\} = 2\left(\frac{s-3}{(s-3)^2+(4)^2}\right)$$

$$\left(s^2\mathcal{L}\{y\} - s(5) - (-2)\right) - 9\left(s\mathcal{L}\{y\} - (5)\right) + 20\mathcal{L}\{y\} = \frac{2(s-3)}{(s-3)^2+(4)^2}$$

$$(s^2-9s+20)\mathcal{L}\{y\} - 5s + 47 = \frac{2(s-3)}{s^2-6s+25}$$

$$(s^2-9s+20)\mathcal{L}\{y\} = \frac{2s-6}{s^2-6s+25} + 5s - 47$$

$$= \frac{2s-6+(5s-47)(s^2-6s+25)}{(s^2-6s+25)}$$

$$\mathcal{L}\{y\} = \frac{5s^3-77s^2+(125+6\times 47+2)s+(25\times 47-6)}{(s^2-9s+20)\left((s-3)^2+4^2\right)}$$

$$= \frac{A}{(s-4)} + \frac{B}{(s-5)} + \frac{Ds+E}{\left((s-3)^2+(4)^2\right)}$$

$$\boxed{y = Ae^{4t} + Be^{5t} + D\left(e^{3t}\cos(4t)\right) + (?)\,e^{3t}\operatorname{sen}(4t)}$$

SolPart := *invlaplace*(*SolTL*, *s*, *t*)

$$SolPart := y(t) = \frac{457}{17}e^{4t} - \frac{109}{5}e^{5t} - \frac{1}{85}e^{3t}\left(7\cos(4t) + 6\sin(4t)\right)$$

plot([*rhs*(*SolPart*), *rhs*(*diff*(*SolPart*, *t*))], *t* = 0 ..0.3, *y* = −50 ..10)