

Clase 26-10-2021

Transformada de Laplace

escalón unitario $\rightarrow u(t-a) = \begin{cases} 0 & ; t < a \\ 1 & ; t \geq a \end{cases}$

rampa unitaria $\rightarrow r(t-a) = \begin{cases} 0 & ; t < a \\ (t-a) & ; t \geq a \end{cases}$

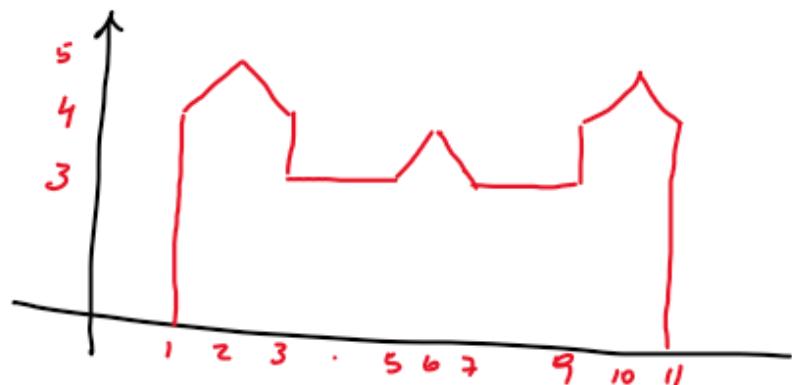
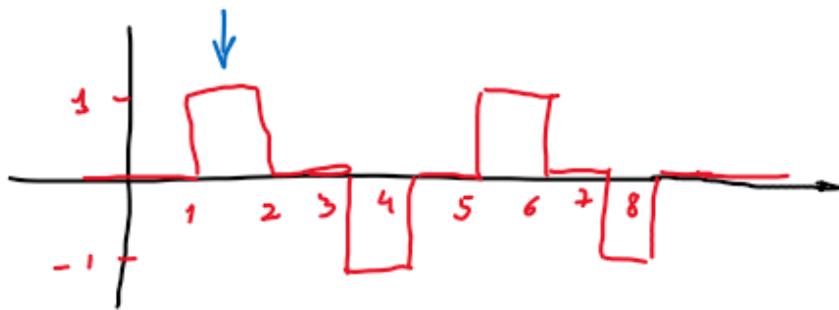
The top graph shows a red Heaviside step function labeled 'u' with a vertical arrow pointing up and a horizontal arrow pointing right, labeled 't'. It jumps from 0 to 1 at time 'a'. The bottom graph shows a red ramp function labeled 'r' with a vertical arrow pointing up and a horizontal arrow pointing right, labeled 't'. It increases linearly from 0 to 1 over the interval [a, a+1] and then remains constant.

$$\hookrightarrow r(t-a) = (t-a) \cdot u(t-a)$$

$$= (t-a) \cdot \begin{cases} 0 & ; t < a \\ 1 & ; t \geq a \end{cases}$$

Dirac \Rightarrow

impulso
unitario



$$\begin{aligned}
 L^{-1} \left\{ \frac{e^{-2s}}{s^2 + 2s + 2} \right\} &= \\
 L^{-1} \left\{ \frac{s}{s^2 + 2s + 2} \right\} &= L^{-1} \left\{ \frac{s}{(s^2 + 2s) + 2} \right\} \\
 &= L^{-1} \left\{ \frac{s}{(s+1)^2 + 1^2} \right\} \\
 &= L^{-1} \left\{ \frac{s+1}{(s+1)^2 + 1^2} - \frac{1}{(s+1)^2 + 1^2} \right\} \\
 L^{-1} \left\{ \frac{s}{s^2 + 2s + 2} \right\} &= e^{-t} \cos(t) - e^{-t} \sin(t)
 \end{aligned}$$

$$L^{-1} \left\{ \frac{e^{-2s}}{s^2 + 2s + 2} \right\} = e^{-(t-2)} \cos(t-2) \cdot u(t-2) - e^{-(t-2)} \sin(t-2) \cdot u(t-2).$$

$$\frac{d^2y}{dt^2} - 9 \frac{dy}{dt} + 20y = 2e^{3t} \cos(4t) \quad y(0) = 5 \\
 y'(0) = -2$$

Resolver el problema de cond. iniciales
 con la transformada de Laplace
 y graficar la sol. particular.

$$L \left\{ y'' - 9y' + 20y \right\} = 2 L \left\{ e^{3t} \cos(4t) \right\}$$

$$\begin{aligned}
& L\{y''\} - 9L\{y'\} + 20L\{y\} = 2 \left(\frac{s-3}{(s-3)^2 + 4^2} \right) \\
& (s^2 L\{y\} - s(s-3) - (-2)) - 9(sL\{y\} - (s)) + 20L\{y\} = \frac{2(s-3)}{(s-3)^2 + 4^2} \\
& (s^2 - 9s + 20)L\{y\} - 5s - 47 = \frac{2(s-3)}{s^2 - 6s + 25} \\
& (s^2 - 9s + 20)L\{y\} = \frac{2s-6}{s^2 - 6s + 25} + 5s - 47 \\
& = \frac{2s-6 + (5s-47)(s^2 - 6s + 25)}{(s^2 - 6s + 25)} \\
& L\{y\} = \frac{5s^3 - 77s^2 + (125 + 6 \cdot 47 + 2)s + (2s \cdot 47 - 6)}{(s^2 - 9s + 20)((s-3)^2 + 4^2)} \\
& = \frac{A}{(s-4)} + \frac{B}{(s-5)} + \frac{Cs + D}{((s-3)^2 + 4^2)}
\end{aligned}$$

$$\begin{aligned}
y = & Ae^{4t} + Be^{5t} + C \left(e^{3t} \cos(4t) \right) + \\
& + (?) e^{3t} \sin(4t)
\end{aligned}$$

SolPart := invlaplace(SolTL, s, t)

$$SolPart := y(t) = \frac{457}{17} e^{4t} - \frac{109}{5} e^{5t} - \frac{1}{85} e^{3t} (7 \cos(4t) + 6 \sin(4t))$$

plot([rhs(SolPart), rhs(diff(SolPart, t))], t=0..0.3, y=-50..10)

