Clase del 4-11-2021

TEMA 3 - Sistemas de Ecuaciones Diferenciales de 1er orden

S(1)EDOL(1)ccH $\longleftrightarrow$ EDOL(n)ccH.

$$\begin{cases} \dfrac{d^2x_1(t)}{dt^2} = -3x_1(t) + 2x_2(t) \\ \dfrac{d^2x_2(t)}{dt^2} = 4x_1(t) - 4x_2(t) \end{cases}$$

$x_1(0) = 3$

$x_1'(0) = 0$

$x_2(0) = -2$

$x_2'(0) = 0$

$$\frac{dx_1(t)}{dt} = x_3(t) \qquad x_3(0) = 0$$

$$\frac{dx_2(t)}{dt} = x_4(t) \qquad x_4(0) = 0$$

$$\frac{dx_1(t)}{dt} = x_3(t)$$

$$\frac{dx_2(t)}{dt} = x_4(t)$$

$$\frac{dx_3(t)}{dt} = -3x_1(t) + 2x_2(t)$$

$$\frac{dx_4(t)}{dt} = 4x_1(t) - 4x_2(t)$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 2 & 0 & 0 \\ 4 & -4 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad \bar{x}(0) = \begin{bmatrix} 3 \\ 0 \\ -2 \\ 0 \end{bmatrix}$$

S(4) EDOL(1)ccH.

$$\bar{x} = \left[e^{At}\right] \cdot \bar{x}(0)$$

S(n) EDOL(1) cc. NH.

$$\frac{d}{dt}\bar{x} = A\cdot\bar{x} + \bar{b}(t)$$

$$\frac{dx_1}{dt} = 3x_1 + 4x_2 + 3e^{t}$$

$$\frac{dx_2}{dt} = 2x_1 + 5x_2 - t^2$$

$$\frac{d}{dt}\bar{x} = A\bar{x} + \bar{b}(t)$$

$$A = \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} \quad \bar{b}(t) = \begin{bmatrix} 3e^{t} \\ -t^2 \end{bmatrix} \quad \bar{x}(0) = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\boxed{\bar{X} = e^{At}\cdot\bar{x}(0) + \underbrace{\int_0^t e^{A(t-\tau)}\cdot\bar{b}(\tau)\,d\tau}_{\text{SolPart}}}$$

MatExp

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MatExpTau:=map(rcurry(eval, t='t-tau'), MatExp)
BBtau:=Map(rcurry(eval, t='tau'), BB)
PPtau:= evalm(MatExpTau &* BBtau)
SolPart:= map(int, PPtau, tau=0..t)
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$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = 5e^{3t} \qquad x(0) = 2 \qquad x'(0) = -2$$

$$\mathcal{L}\{x''\} + \mathcal{L}\{x'\} + \mathcal{L}\{x\} = 5\mathcal{L}\{e^{3t}\}$$

$$\left[s^2\mathcal{L}\{x\} - s(2) - (-2)\right] + \left[s\mathcal{L}\{x\} - (2)\right] + \mathcal{L}\{x\} = \frac{5}{s-3}$$

$$(s^2+s+1)\mathcal{L}\{x\} - 2s = \frac{5}{s-3}$$

$$(s^2+s+1)\mathcal{L}\{x\} = \frac{5}{s-3} + 2s$$

$$= \frac{5 + 2s(s-3)}{s-3}$$

$$(s^2+s+1)\mathcal{L}\{x\} = \frac{5+2s^2-6s}{s-3}$$

$$\mathcal{L}\{x\} = \frac{2s^2-6s+5}{(s-3)(s^2+s+1)}$$

$$\frac{2s^2-6s+5}{(s-3)(s^2+s+1)} = \frac{A}{s-3} + \frac{Bs+D}{s^2+s+1}$$

$$2s^2-6s+5 = A(s^2+s+1) + (Bs+D)(s-3)$$

$$= A(s^2+s+1) + Bs^2 - 3Bs + Ds - 3D$$

$$= (A+B)s^2 + (A-3B+D)s + (A-3D)$$

$$A+B=2 \qquad A = 2-B$$
$$A-3B+D=-6$$
$$A \quad -3D = 5 \qquad A = 5+3D$$
$$2-B = 5+3D$$
$$3D = A-5$$

$$A - 3(2-A) + \left(\frac{A-5}{3}\right) = -6$$
$$3A - 18 + 9A + A - 5 = -18$$
$$13A - 23 = -18$$
$$13A = 5 \qquad \boxed{A = \frac{5}{13}} \qquad B = 2 - \frac{5}{13}$$
$$\boxed{B = \frac{21}{13}}$$

$$3D = A-5$$
$$3D = \frac{5}{13} - 5$$
$$3D = \frac{-60}{13} \qquad \boxed{D = \frac{-60}{39}}$$

$$L\{x\} = \frac{5}{13}\left(\frac{1}{s-3}\right) + \frac{\frac{21}{13}s - \frac{60}{39}}{s^2+s+1}$$

$$\frac{\frac{21}{13}s - \frac{60}{39}}{s^2+s+1} = \frac{21}{13}\left(\frac{s - \frac{60}{3\times 21}}{s^2+s+1}\right)$$

$$= \frac{21}{13}\left(\frac{s}{s^2+s+1}\right) - \left(\frac{\frac{60}{39}}{s^2+s+1}\right)$$

$$= \frac{21}{13}\left(\frac{s+\frac{1}{2}}{s^2+s+\frac{1}{4}+\frac{3}{4}}\right) - \left(\frac{+\frac{21}{13}\cdot\frac{1}{2} + \frac{60}{39}}{s^2+s+\frac{1}{4}+\frac{3}{4}}\right)$$

$$= \frac{21}{13}\left(\frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}\right) - \left(\frac{+\frac{21}{26}+\frac{60}{39}}{\frac{\sqrt{3}}{2}}\right)\left(\frac{\frac{\sqrt{3}}{2}}{(s+\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}\right)$$

$$x = \frac{5}{13}e^{3t} + \frac{21}{13}e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right) - \left(\frac{+\frac{21}{26}+\frac{60}{39}}{\frac{\sqrt{3}}{2}}\right)e^{-\frac{t}{2}}\operatorname{sen}\left(\frac{\sqrt{3}}{2}t\right)$$