

> restart

> AA := array([[0, 0, 1, 0], [0, 0, 0, 1], [-3, 2, 0, 0], [4, -4, 0, 0]])

$$AA := \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 2 & 0 & 0 \\ 4 & -4 & 0 & 0 \end{bmatrix} \quad (1)$$

> Xcero := array([3, 0, -2, 0])

$$Xcero := \begin{bmatrix} 3 & 0 & -2 & 0 \end{bmatrix} \quad (2)$$

> with(linalg) :

> MatExp := exponential(AA, t) : MatExp[1, 1] : ME[1, 1] = evalf(%, 2); MatExp[1, 2] : ME[1, 2] = evalf(%, 2); MatExp[1, 3] : ME[1, 3] = evalf(%, 2); MatExp[1, 4] : ME[1, 4] = evalf(%, 2); MatExp[2, 1] : ME[2, 1] = evalf(%, 2); MatExp[2, 2] : ME[2, 2] = evalf(%, 2); MatExp[2, 3] : ME[2, 3] = evalf(%, 2); MatExp[2, 4] : ME[2, 4] = evalf(%, 2); MatExp[3, 1] : ME[3, 1] = evalf(%, 2); MatExp[3, 2] : ME[3, 2] = evalf(%, 2); MatExp[3, 3] : ME[3, 3] = evalf(%, 2); MatExp[3, 4] : ME[3, 4] = evalf(%, 2); MatExp[4, 1] : ME[4, 1] = evalf(%, 2); MatExp[4, 2] : ME[4, 2] = evalf(%, 2); MatExp[4, 3] : ME[4, 3] = evalf(%, 2); MatExp[4, 4] : ME[4, 4] = evalf(%, 2);

$$ME_{1,1} = 0.56 \cos(-0.75 t) + 0.40 \cos(2.4 t)$$

$$ME_{1,2} = 0.34 \cos(-0.75 t) - 0.34 \cos(2.4 t)$$

$$ME_{1,3} = -0.70 \sin(-0.75 t) + 0.17 \sin(2.4 t)$$

$$ME_{1,4} = -0.43 \sin(-0.75 t) - 0.12 \sin(2.4 t)$$

$$ME_{2,1} = 0.67 \cos(-0.75 t) - 0.67 \cos(2.4 t)$$

$$ME_{2,2} = 0.40 \cos(-0.75 t) + 0.56 \cos(2.4 t)$$

$$ME_{2,3} = -0.84 \sin(-0.75 t) - 0.24 \sin(2.4 t)$$

$$ME_{2,4} = -0.49 \sin(-0.75 t) + 0.23 \sin(2.4 t)$$

$$ME_{3,1} = 0.44 \sin(-0.75 t) - 0.97 \sin(2.4 t)$$

$$ME_{3,2} = 0.26 \sin(-0.75 t) + 0.84 \sin(2.4 t)$$

$$ME_{3,3} = 0.56 \cos(-0.75 t) + 0.40 \cos(2.4 t)$$

$$ME_{3,4} = 0.34 \cos(-0.75 t) - 0.34 \cos(2.4 t)$$

$$ME_{4,1} = 0.48 \sin(-0.75 t) + 1.7 \sin(2.4 t)$$

$$ME_{4,2} = 0.32 \sin(-0.75 t) - 1.4 \sin(2.4 t)$$

$$ME_{4,3} = 0.67 \cos(-0.75 t) - 0.67 \cos(2.4 t)$$

$$ME_{4,4} = 0.40 \cos(-0.75 t) + 0.56 \cos(2.4 t) \quad (3)$$

> Identidad := simplify(map(rcurry(eval, t=0'), MatExp))

(4)

$$Identidad := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

> *SolucionParticular* := evalm(MatExp &* Xcero) :

> *SolPart*[1] := x[1](t) = evalf(*SolucionParticular*[1], 2)
*SolPart*₁ := $x_1(t) = 1.7 \cos(-0.75 t) + 1.2 \cos(2.4 t) + 1.4 \sin(-0.75 t) - 0.34 \sin(2.4 t)$ (5)

> *SolPart*[2] := x[2](t) = evalf(*SolucionParticular*[2], 2)
*SolPart*₂ := $x_2(t) = 2.1 \cos(-0.75 t) - 2.1 \cos(2.4 t) + 1.7 \sin(-0.75 t) + 0.48 \sin(2.4 t)$ (6)

> *SolPart*[3] := x[3](t) = evalf(*SolucionParticular*[3], 2)
*SolPart*₃ := $x_3(t) = 1.3 \sin(-0.75 t) - 2.9 \sin(2.4 t) - 1.1 \cos(-0.75 t) - 0.80 \cos(2.4 t)$ (7)

> *SolPart*[4] := x[4](t) = evalf(*SolucionParticular*[4], 2)
*SolPart*₄ := $x_4(t) = 1.4 \sin(-0.75 t) + 5.1 \sin(2.4 t) - 1.4 \cos(-0.75 t) + 1.4 \cos(2.4 t)$ (8)

> *Cond*[1] := simplify(subs(t=0, *SolucionParticular*[1]))
*Cond*₁ := 3 (9)

> *Cond*[2] := simplify(subs(t=0, *SolucionParticular*[2]))
*Cond*₂ := 0 (10)

> *Cond*[3] := simplify(subs(t=0, *SolucionParticular*[3]))
*Cond*₃ := -2 (11)

> *Cond*[4] := simplify(subs(t=0, *SolucionParticular*[4]))
*Cond*₄ := 0 (12)

> simplify(diff(*SolucionParticular*[1], t) - *SolucionParticular*[3]) = 0
0 = 0 (13)

> simplify(diff(*SolucionParticular*[2], t) - *SolucionParticular*[4]) = 0
0 = 0 (14)

> restart

> *Sistema* := ([diff(x[1](t), t) = 3·x[1](t) + 4·x[2](t) + 3·exp(t), diff(x[2](t), t) = 2·x[1](t) + 5·x[2](t) - t²]) :

> *Sistema*[1]; *Sistema*[2]

$$\begin{aligned} \frac{d}{dt} x_1(t) &= 3 x_1(t) + 4 x_2(t) + 3 e^t \\ \frac{d}{dt} x_2(t) &= 2 x_1(t) + 5 x_2(t) - t^2 \end{aligned} \quad (15)$$

> *AA* := array([[3, 4], [2, 5]])

$$AA := \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} \quad (16)$$

> *BB* := array([3·exp(t), -t²])

$$BB := \begin{bmatrix} 3 e^t & -t^2 \end{bmatrix} \quad (17)$$

$$\begin{aligned} &> Xcero := array([_C1, _C2]) \\ &Xcero := \begin{bmatrix} _C1 & _C2 \end{bmatrix} \end{aligned} \quad (18)$$

\geq with(linalg) :

\geq MatExp := exponential(AA, t)

$$MatExp := \begin{bmatrix} \frac{2}{3} e^t + \frac{1}{3} e^{7t} & \frac{2}{3} e^{7t} - \frac{2}{3} e^t \\ \frac{1}{3} e^{7t} - \frac{1}{3} e^t & \frac{1}{3} e^t + \frac{2}{3} e^{7t} \end{bmatrix} \quad (19)$$

\geq SolHom := expand(evalm(MatExp &* Xcero)) : SolHom[1]; SolHom[2];

$$\begin{aligned} &\left(\frac{2}{3} e^t + \frac{1}{3} e^{7t} \right) _C1 + \left(\frac{2}{3} e^{7t} - \frac{2}{3} e^t \right) _C2 \\ &\left(\frac{1}{3} e^{7t} - \frac{1}{3} e^t \right) _C1 + \left(\frac{1}{3} e^t + \frac{2}{3} e^{7t} \right) _C2 \end{aligned} \quad (20)$$

\geq MatExpTau := map(rcurry(eval, t = tau'), MatExp)

$$MatExpTau := \begin{bmatrix} \frac{2}{3} e^{t-\tau} + \frac{1}{3} e^{7t-7\tau} & \frac{2}{3} e^{7t-7\tau} - \frac{2}{3} e^{t-\tau} \\ \frac{1}{3} e^{7t-7\tau} - \frac{1}{3} e^{t-\tau} & \frac{1}{3} e^{t-\tau} + \frac{2}{3} e^{7t-7\tau} \end{bmatrix} \quad (21)$$

\geq BBtau := map(rcurry(eval, t = tau'), BB)

$$BBtau := \begin{bmatrix} 3 e^\tau & -\tau^2 \end{bmatrix} \quad (22)$$

\geq PPtau := evalm(MatExpTau &* BBtau) : PPtau[1]; PPtau[2];

$$\begin{aligned} &3 \left(\frac{2}{3} e^{t-\tau} + \frac{1}{3} e^{7t-7\tau} \right) e^\tau - \left(\frac{2}{3} e^{7t-7\tau} - \frac{2}{3} e^{t-\tau} \right) \tau^2 \\ &3 \left(\frac{1}{3} e^{7t-7\tau} - \frac{1}{3} e^{t-\tau} \right) e^\tau - \left(\frac{1}{3} e^{t-\tau} + \frac{2}{3} e^{7t-7\tau} \right) \tau^2 \end{aligned} \quad (23)$$

\geq SolPart := map(int, PPtau, tau = 0 .. t) : SolPart[1]; SolPart[2];

$$\begin{aligned} &\frac{335}{2058} e^{7t} + \frac{7}{6} e^t + 2 t e^t - \frac{4}{7} t^2 - \frac{64}{49} t - \frac{456}{343} \\ &\frac{335}{2058} e^{7t} - \frac{5}{6} e^t - t e^t + \frac{3}{7} t^2 + \frac{34}{49} t + \frac{230}{343} \end{aligned} \quad (24)$$

\geq SolGral := evalm(SolHom + SolPart) :

\geq SolucionGeneral[1] := x[1](t) = SolGral[1]

$$\begin{aligned} SolucionGeneral_1 := x_1(t) = &\left(\frac{2}{3} e^t + \frac{1}{3} e^{7t} \right) _C1 + \left(\frac{2}{3} e^{7t} - \frac{2}{3} e^t \right) _C2 + \frac{335}{2058} e^{7t} \\ &+ \frac{7}{6} e^t + 2 t e^t - \frac{4}{7} t^2 - \frac{64}{49} t - \frac{456}{343} \end{aligned} \quad (25)$$

\geq SolucionGeneral[2] := x[2](t) = SolGral[2]

$$\begin{aligned} SolucionGeneral_2 := x_2(t) = &\left(\frac{1}{3} e^{7t} - \frac{1}{3} e^t \right) _C1 + \left(\frac{1}{3} e^t + \frac{2}{3} e^{7t} \right) _C2 + \frac{335}{2058} e^{7t} \\ &- \frac{5}{6} e^t - t e^t + \frac{3}{7} t^2 + \frac{34}{49} t + \frac{230}{343} \end{aligned} \quad (26)$$

$$\begin{aligned} &> \text{CondIni}[1] := \text{simplify}(\text{subs}(t=0, \text{SolucionGeneral}[1])) \\ &\quad \text{CondIni}_1 := x_1(0) = _C1 \end{aligned} \quad (27)$$

$$\begin{aligned} &> \text{CondIni}[2] := \text{simplify}(\text{subs}(t=0, \text{SolucionGeneral}[2])) \\ &\quad \text{CondIni}_2 := x_2(0) = _C2 \end{aligned} \quad (28)$$

$\begin{aligned} &> \text{restart} \end{aligned}$

$$\begin{aligned} &> \text{Ecua} := \text{diff}(x(t), t^2) + \text{diff}(x(t), t) + x(t) = 5 \cdot \exp(3 \cdot t) \\ &\quad \text{Ecua} := \frac{d^2}{dt^2} x(t) + \frac{d}{dt} x(t) + x(t) = 5 e^{3t} \end{aligned} \quad (29)$$

$$\begin{aligned} &> \text{Cond} := x(0) = 2, D(x)(0) = -2 \\ &\quad \text{Cond} := x(0) = 2, D(x)(0) = -2 \end{aligned} \quad (30)$$

$\begin{aligned} &> \text{with(inttrans)} : \end{aligned}$

$$\begin{aligned} &> \text{EcuaTL} := \text{subs}(\text{Cond}, \text{laplace}(\text{Ecua}, t, s)) \\ &\quad \text{EcuaTL} := s^2 \text{laplace}(x(t), t, s) - 2s + s \text{laplace}(x(t), t, s) + \text{laplace}(x(t), t, s) = \frac{5}{s-3} \end{aligned} \quad (31)$$

$$\begin{aligned} &> \text{SolTL} := \text{isolate}(\text{EcuaTL}, \text{laplace}(x(t), t, s)) \\ &\quad \text{SolTL} := \text{laplace}(x(t), t, s) = \frac{\frac{5}{s-3} + 2s}{s^2 + s + 1} \end{aligned} \quad (32)$$

$$\begin{aligned} &> \text{SolPart} := \text{simplify}(\text{invlaplace}(\text{SolTL}, s, t)) \\ &\quad \text{SolPart} := x(t) = -\frac{61}{39} \sqrt{3} e^{-\frac{1}{2}t} \sin\left(\frac{1}{2} \sqrt{3} t\right) + \frac{21}{13} e^{-\frac{1}{2}t} \cos\left(\frac{1}{2} \sqrt{3} t\right) + \frac{5}{13} e^{3t} \end{aligned} \quad (33)$$

$\begin{aligned} &> \text{restart} \end{aligned}$

$\begin{aligned} &> \end{aligned}$