Clase 11 - 11 - 2021

Tema 4 Ecuaciones diferenciales en derivadas parciales

Método de separación de variables

- Prueba y error
- Hipótesis $\longrightarrow$ $F(x, y) = F(x) \cdot G(y)$
- convertir EDenDP $\longrightarrow$ varias EDO
- Varias soluciones generales
  todas serán válidas

$$\frac{\partial^2 F}{\partial x^2} + 4\frac{\partial F}{\partial y} = 6 \cdot F \quad \text{EDenDP(2)}$$

$$\frac{\partial F}{\partial x} = F' \cdot G \qquad \frac{\partial^2 F}{\partial x^2} = F'' \cdot G \qquad \frac{\partial F}{\partial y} = F \cdot G'$$

$$F'' \cdot G + 4 F G' = 6 \cdot F \cdot G$$

1

$$F'' G = -4FG' + 6FG$$
$$= (-4G' + 6G)F$$

$$F'' G = \left(G' - \frac{6}{4} G\right)(-4F)$$

$$\frac{F''}{-4F} = \frac{G' - \frac{6}{4}G}{G}$$

x y

$F(x,y) = F(x) \cdot G(y)$ pudimos separar las variables miembro a miembro.

$$\frac{F''}{-4F} = \alpha \qquad \frac{G' - \frac{6}{4}G}{4} = \alpha \qquad \begin{cases} \alpha = 0 \\ \alpha > 0 \\ \alpha < 0 \end{cases}$$

$\alpha = 0$

$$\frac{F''}{-4F} = 0 \qquad G' - \frac{6}{4}G = 0$$

$$F \neq 0 \qquad m - \frac{6}{4} = 0 \qquad m_1 = \frac{6}{4}$$

$$F'' = 0$$

$$F(x) = c_1 x + c_2 \qquad G(y) = k_1 e^{\frac{6}{4}y}$$

$$F(x,y) = (c_1 x + c_2) k_1 e^{\frac{6}{4}y}$$

1.a) $\underset{\alpha=0}{F(x,y)} = (c_{10} x + c_{20}) e^{\frac{6}{4}y}$

$$\frac{\partial^2 F(x,y)}{\partial x^2} + 4 \frac{\partial F(x,y)}{\partial y} = 6 F(x,y)$$

$$\frac{\partial F}{\partial x} = c_{10} e^{\frac{6}{4}y} \qquad \frac{\partial^2 F}{\partial x^2} = 0 \qquad \frac{\partial F}{\partial y} = \frac{6}{4}(c_{10}x + c_{20}) e^{\frac{6}{4}y}$$

$$(0) + \cancel{4}\left(\frac{6}{\cancel{4}}(c_{10}x + c_{20}) e^{\frac{6}{4}y} = 6 (c_{10}x + c_{20}) e^{\frac{6}{4}y}$$

$$6(c_{10}x + c_{20}) e^{\frac{6}{4}y} = 6(c_{10}x + c_{20}) e^{\frac{6}{4}y}$$

$\alpha > 0$ $\quad \dfrac{F''}{-4F} = \beta^2 \qquad \dfrac{G' - \frac{6}{4}G}{G} = \beta^2 \qquad \alpha = \beta^2 \quad \forall \beta \neq 0$

$$F'' = -4\beta^2 F \qquad G' - \frac{6}{4}G = \beta^2 G$$
$$F'' + 4\beta^2 F = 0 \qquad G' = \left(\frac{6}{4} + \beta^2\right) G$$
$$m^2 + 4\beta^2 = 0 \qquad m = \left(\frac{6}{4} + \beta^2\right)$$
$$m^2 = -4\beta^2 \qquad G(y) = k_1 e^{\left(\frac{6}{4} + \beta^2\right) y}$$
$$m_{1,2} = \pm 2\beta i$$
$$F(x) = C_1 \cos(2\beta x) + C_2 \operatorname{sen}(2\beta x)$$

1.5) $\underset{\alpha > 0}{F(x,y)} = \left(C_1 \cos(2\beta x) + C_2 \operatorname{sen}(2\beta x)\right) e^{\left(\frac{6}{4} + \beta^2\right) y}$

$\alpha < 0 \qquad \alpha = -\beta^2 \quad \forall \beta \neq 0$

$$\frac{F''(x)}{-4F(x)} = -\beta^2$$
$$F''(x) = 4\beta^2 F(x)$$
$$F''(x) - 4\beta^2 F(x) = 0$$
$$m^2 - 4\beta^2 = 0$$
$$(m - 2\beta)(m + 2\beta) = 0$$
$$F(x) = C_1 e^{2\beta x} + C_2 e^{-2\beta x}$$

$$\frac{G'(y) - \frac{6}{4}G(y)}{G(y)} = -\beta^2$$

$$G'(y) - \frac{6}{4}G(y) = -\beta^2 G(y)$$

$$G'(y) = \left(\frac{6}{4} - \beta^2\right)G(y)$$

$$m = \frac{6}{4} - \beta^2$$

$$G(y) = k_1 e^{\left(\frac{6}{4} - \beta^2\right)y}$$

1.c) $$\underset{\alpha<0}{F(x,y)} = \left(c_1 e^{2\beta x} + c_2 e^{-2\beta x}\right) e^{\left(\frac{6}{4} - \beta^2\right)y}$$