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> restart
> Ecua := diff(Z(x,y), x$2) + 4·diff(Z(x,y), y) = 6·Z(x,y)
      Ecua :=  $\frac{\partial^2}{\partial x^2} Z(x,y) + 4 \left( \frac{\partial}{\partial y} Z(x,y) \right) = 6 Z(x,y)$  (1)
> with(PDEtools) :
> SolGralUno := build(pdsolve(Ecua))
      SolGralUno :=  $Z(x,y) = e^{\sqrt{-c_1} x} \_C3 e^{-\frac{1}{4} y - c_1 \frac{3}{2} y} \_C1 + \frac{\_C3 e^{-\frac{1}{4} y - c_1 \frac{3}{2} y} \_C2}{e^{\sqrt{-c_1} x}}$  (2)
> EcuaSep := eval(subs(Z(x,y) = F(x)·G(y), Ecua))
      EcuaSep :=  $\left( \frac{d^2}{dx^2} F(x) \right) G(y) + 4 F(x) \left( \frac{d}{dy} G(y) \right) = 6 F(x) G(y)$  (3)
> EcuaSepUno
      := simplify( $\frac{1}{-4 \cdot F(x) \cdot G(y)} \left( lhs(EcuaSep) - 4 F(x) \left( \frac{d}{dy} G(y) \right) = rhs(EcuaSep) - 4 F(x) \left( \frac{d}{dy} G(y) \right) \right)$ )
      EcuaSepUno :=  $-\frac{1}{4} \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = -\frac{1}{2} \frac{3 G(y) - 2 \left( \frac{d}{dy} G(y) \right)}{G(y)}$  (4)
> EcuaSepUnoX := lhs(EcuaSepUno) = alpha
      EcuaSepUnoX :=  $-\frac{1}{4} \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \alpha$  (5)
> EcuaSepUnoY := rhs(EcuaSepUno) = alpha
      EcuaSepUnoY :=  $-\frac{1}{2} \frac{3 G(y) - 2 \left( \frac{d}{dy} G(y) \right)}{G(y)} = \alpha$  (6)
para alpha=0
> EcuaSepUnoXcero := subs(alpha=0, EcuaSepUnoX)
      EcuaSepUnoXcero :=  $-\frac{1}{4} \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = 0$  (7)
> EcuaSepUnoYcero := subs(alpha=0, EcuaSepUnoY)
      EcuaSepUnoYcero :=  $-\frac{1}{2} \frac{3 G(y) - 2 \left( \frac{d}{dy} G(y) \right)}{G(y)} = 0$  (8)
> SolGralSepUnoXcero := dsolve(EcuaSepUnoXcero)
      SolGralSepUnoXcero :=  $F(x) = \_C1 x + \_C2$  (9)
> SolGralSepUnoYcero := dsolve(EcuaSepUnoYcero)
      SolGralSepUnoYcero :=  $G(y) = \_C1 e^{\frac{3}{2} y}$  (10)

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> $SolGralSepUnoCero := Z(x, y) = rhs(SolGralSepUnoXcero) \cdot subs(_C1 = 1, rhs(SolGralSepUnoYcero))$

$$SolGralSepUnoCero := Z(x, y) = (_C1 x + _C2) e^{\frac{3}{2}y} \quad (11)$$

para alpha positiva

> $EcuaSepUnoXpos := subs(alpha = \beta^2, EcuaSepUnoX)$

$$EcuaSepUnoXpos := -\frac{1}{4} \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \beta^2 \quad (12)$$

> $EcuaSepUnoYpos := subs(alpha = \beta^2, EcuaSepUnoY)$

$$EcuaSepUnoYpos := -\frac{1}{2} \frac{3 G(y) - 2 \left(\frac{d}{dy} G(y) \right)}{G(y)} = \beta^2 \quad (13)$$

> $SolGralSepUnoXpos := dsolve(EcuaSepUnoXpos)$

$$SolGralSepUnoXpos := F(x) = _C1 \sin(2 \beta x) + _C2 \cos(2 \beta x) \quad (14)$$

> $SolGralSepUnoYpos := dsolve(EcuaSepUnoYpos)$

$$SolGralSepUnoYpos := G(y) = _C1 e^{\frac{1}{2}(2\beta^2 + 3)y} \quad (15)$$

> $SolGralSepUnoPos := Z(x, y) = rhs(SolGralSepUnoXpos) \cdot subs(_C1 = 1, rhs(SolGralSepUnoYpos))$

$$SolGralSepUnoPos := Z(x, y) = (_C1 \sin(2 \beta x) + _C2 \cos(2 \beta x)) e^{\frac{1}{2}(2\beta^2 + 3)y} \quad (16)$$

para alpha negativa

> $EcuaSepUnoXneg := subs(alpha = -\beta^2, EcuaSepUnoX)$

$$EcuaSepUnoXneg := -\frac{1}{4} \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = -\beta^2 \quad (17)$$

> $EcuaSepUnoYneg := subs(alpha = -\beta^2, EcuaSepUnoY)$

$$EcuaSepUnoYneg := -\frac{1}{2} \frac{3 G(y) - 2 \left(\frac{d}{dy} G(y) \right)}{G(y)} = -\beta^2 \quad (18)$$

> $SolGralSepUnoXneg := dsolve(EcuaSepUnoXneg)$

$$SolGralSepUnoXneg := F(x) = _C1 e^{-2\beta x} + _C2 e^{2\beta x} \quad (19)$$

> $SolGralSepUnoYneg := dsolve(EcuaSepUnoYneg)$

$$SolGralSepUnoYneg := G(y) = _C1 e^{-\frac{1}{2}(2\beta^2 - 3)y} \quad (20)$$

> $SolGralSepUnoNeg := Z(x, y) = rhs(SolGralSepUnoXneg) \cdot subs(_C1 = 1, rhs(SolGralSepUnoYneg))$

$$SolGralSepUnoNeg := Z(x, y) = (_C1 e^{-2\beta x} + _C2 e^{2\beta x}) e^{-\frac{1}{2}(2\beta^2 - 3)y} \quad (21)$$

> Ecua

(22)

$$\frac{\partial^2}{\partial x^2} Z(x, y) + 4 \left(\frac{\partial}{\partial y} Z(x, y) \right) = 6 Z(x, y) \quad (22)$$

> $EcuaSep := eval(subs(Z(x, y) = F(x) \cdot G(y), Ecua))$

$$EcuaSep := \left(\frac{d^2}{dx^2} F(x) \right) G(y) + 4 F(x) \left(\frac{d}{dy} G(y) \right) = 6 F(x) G(y) \quad (23)$$

SEGUNDA POSIBLE SOLUCIÓN

> $EcuaSepDos := simplify\left(\frac{1}{-4 \cdot F(x) \cdot G(y)} \left(lhs(EcuaSep) - 4 F(x) \left(\frac{d}{dy} G(y) \right) - 6 F(x) G(y) = rhs(EcuaSep) - 4 F(x) \left(\frac{d}{dy} G(y) \right) - 6 F(x) G(y) \right)\right)$

$$EcuaSepDos := \frac{1}{4} \frac{-\left(\frac{d^2}{dx^2} F(x)\right) + 6 F(x)}{F(x)} = \frac{\frac{d}{dy} G(y)}{G(y)} \quad (24)$$

> $EcuaSepDosX := lhs(EcuaSepDos) = \alpha$

$$EcuaSepDosX := \frac{1}{4} \frac{-\left(\frac{d^2}{dx^2} F(x)\right) + 6 F(x)}{F(x)} = \alpha \quad (25)$$

> $EcuaSepDosY := rhs(EcuaSepDos) = \alpha$

$$EcuaSepDosY := \frac{\frac{d}{dy} G(y)}{G(y)} = \alpha \quad (26)$$

para alpha=0

> $EcuaSepDosXcero := subs(alpha=0, EcuaSepDosX)$

$$EcuaSepDosXcero := \frac{1}{4} \frac{-\left(\frac{d^2}{dx^2} F(x)\right) + 6 F(x)}{F(x)} = 0 \quad (27)$$

> $EcuaSepDosYcero := subs(alpha=0, EcuaSepDosY)$

$$EcuaSepDosYcero := \frac{\frac{d}{dy} G(y)}{G(y)} = 0 \quad (28)$$

> $SolGralSepDosXcero := dsolve(EcuaSepDosXcero)$

$$SolGralSepDosXcero := F(x) = _C1 e^{\sqrt{6} x} + _C2 e^{-\sqrt{6} x} \quad (29)$$

> $SolGralSepDosYcero := dsolve(EcuaSepDosYcero)$

$$SolGralSepDosYcero := G(y) = _C1 \quad (30)$$

> $SolGralSepDosCero := Z(x, y) = rhs(SolGralSepDosXcero) \cdot subs(_C1 = 1, rhs(SolGralSepDosYcero))$

$$SolGralSepDosCero := Z(x, y) = _C1 e^{\sqrt{6} x} + _C2 e^{-\sqrt{6} x} \quad (31)$$

para alpha positiva

> $EcuaSepDosXpos := subs(alpha = \beta^2, EcuaSepDosX)$

(32)

$$EcuaSepDosXpos := \frac{1}{4} \frac{-\left(\frac{d^2}{dx^2} F(x)\right) + 6 F(x)}{F(x)} = \beta^2 \quad (32)$$

$$> EcuaSepDosYpos := \text{subs}(\alpha = \beta^2, EcuaSepDosY)$$

$$EcuaSepDosYpos := \frac{\frac{d}{dy} G(y)}{G(y)} = \beta^2 \quad (33)$$

$$> SolGralSepDosXpos := \text{dsolve}(EcuaSepDosXpos)$$

$$SolGralSepDosXpos := F(x) = _C1 \sin\left(\sqrt{4\beta^2 - 6} x\right) + _C2 \cos\left(\sqrt{4\beta^2 - 6} x\right) \quad (34)$$

$$> SolGralSepDosYpos := \text{dsolve}(EcuaSepDosYpos)$$

$$SolGralSepDosYpos := G(y) = _C1 e^{\beta^2 y} \quad (35)$$

$$> SolGralSepDosPos := Z(x, y) = \text{rhs}(SolGralSepDosXpos) \cdot \text{subs}(_C1 = 1, \text{rhs}(SolGralSepDosYpos))$$

$$SolGralSepDosPos := Z(x, y) = \left(_C1 \sin\left(\sqrt{4\beta^2 - 6} x\right) + _C2 \cos\left(\sqrt{4\beta^2 - 6} x\right)\right) e^{\beta^2 y} \quad (36)$$

para alpha negativa

$$> EcuaSepDosXneg := \text{subs}(\alpha = -\beta^2, EcuaSepDosX)$$

$$EcuaSepDosXneg := \frac{1}{4} \frac{-\left(\frac{d^2}{dx^2} F(x)\right) + 6 F(x)}{F(x)} = -\beta^2 \quad (37)$$

$$> EcuaSepDosYneg := \text{subs}(\alpha = -\beta^2, EcuaSepDosY)$$

$$EcuaSepDosYneg := \frac{\frac{d}{dy} G(y)}{G(y)} = -\beta^2 \quad (38)$$

$$> SolGralSepDosXneg := \text{dsolve}(EcuaSepDosXneg)$$

$$SolGralSepDosXneg := F(x) = _C1 \sin\left(\sqrt{-4\beta^2 - 6} x\right) + _C2 \cos\left(\sqrt{-4\beta^2 - 6} x\right) \quad (39)$$

$$> SolGralSepDosYneg := \text{dsolve}(EcuaSepDosYneg)$$

$$SolGralSepDosYneg := G(y) = _C1 e^{-\beta^2 y} \quad (40)$$

$$> SolGralSepDosNeg := Z(x, y) = \text{rhs}(SolGralSepDosXneg) \cdot \text{subs}(_C1 = 1, \text{rhs}(SolGralSepDosYneg))$$

$$SolGralSepDosNeg := Z(x, y) = \left(_C1 \sin\left(\sqrt{-4\beta^2 - 6} x\right) + _C2 \cos\left(\sqrt{-4\beta^2 - 6} x\right)\right) e^{-\beta^2 y} \quad (41)$$

$$> ComprobacionSeis := \text{simplify}(\text{eval}(\text{subs}(Z(x, y) = \text{rhs}(SolGralSepDosNeg), \text{lhs}(Ecua) - \text{rhs}(Ecua) = 0)))$$

$$ComprobacionSeis := 0 = 0 \quad (42)$$

$$> ComprobacionCinco := \text{simplify}(\text{eval}(\text{subs}(Z(x, y) = \text{rhs}(SolGralSepDosPos), \text{lhs}(Ecua) - \text{rhs}(Ecua) = 0)))$$

$$ComprobacionCinco := 0 = 0 \quad (43)$$

$$> ComprobacionCuatro := \text{simplify}(\text{eval}(\text{subs}(Z(x, y) = \text{rhs}(SolGralSepDosCero), \text{lhs}(Ecua) - \text{rhs}(Ecua) = 0)))$$

$$ComprobacionCuatro := 0 = 0 \quad (44)$$

$$\begin{aligned} & \text{ComprobacionTres} := \text{simplify}(\text{eval}(\text{subs}(Z(x, y) = \text{rhs}(\text{SolGralSepUnoNeg}), \text{lhs}(\text{Ecua}) \\ & \quad - \text{rhs}(\text{Ecua}) = 0))) \\ & \text{ComprobacionTres} := 0 = 0 \end{aligned} \tag{45}$$

$$\begin{aligned} & \text{ComprobacionDos} := \text{simplify}(\text{eval}(\text{subs}(Z(x, y) = \text{rhs}(\text{SolGralSepUnoPos}), \text{lhs}(\text{Ecua}) \\ & \quad - \text{rhs}(\text{Ecua}) = 0))) \\ & \text{ComprobacionDos} := 0 = 0 \end{aligned} \tag{46}$$

$$\begin{aligned} & \text{ComprobacionUno} := \text{simplify}(\text{eval}(\text{subs}(Z(x, y) = \text{rhs}(\text{SolGralSepUnoCero}), \text{lhs}(\text{Ecua}) \\ & \quad - \text{rhs}(\text{Ecua}) = 0))) \\ & \text{ComprobacionUno} := 0 = 0 \end{aligned} \tag{47}$$

$$\begin{aligned} & \text{Ecua} \\ & \frac{\partial^2}{\partial x^2} Z(x, y) + 4 \left(\frac{\partial}{\partial y} Z(x, y) \right) = 6 Z(x, y) \end{aligned} \tag{48}$$

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