> *restart*

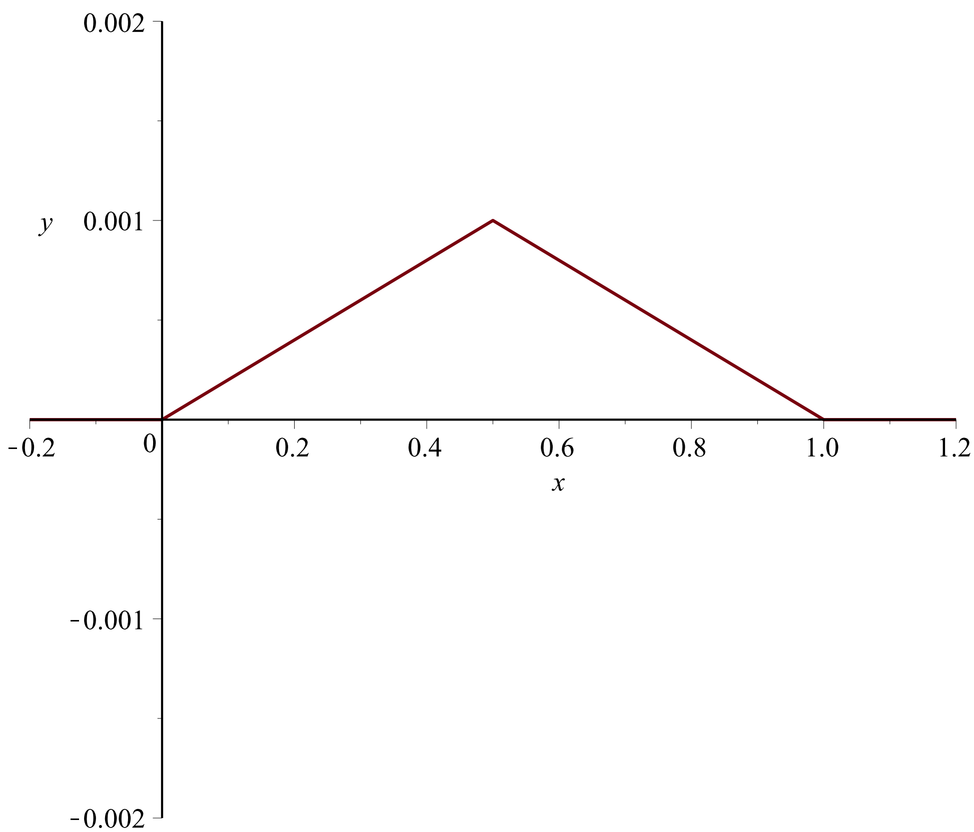
> $Ecua := diff(y(x,t), t\$2) = c^2 \cdot diff(y(x,t), x\$2)$

$$Ecua := \frac{\partial^2}{\partial t^2} y(x,t) = c^2 \left(\frac{\partial^2}{\partial x^2} y(x,t) \right) \quad \textbf{(1)}$$

> $CondInic := f = \frac{\left(\frac{1}{1000}\right)}{\left(\frac{5}{10}\right)} \cdot x \cdot \mathrm{Heaviside}(x) - 2 \cdot \left(\frac{\left(\frac{1}{1000}\right)}{\left(\frac{5}{10}\right)} \cdot \left(x - \frac{5}{10}\right) \cdot \mathrm{Heaviside}\left(\left(x - \frac{5}{10}\right)\right) \right) + \frac{\left(\frac{1}{1000}\right)}{\left(\frac{5}{10}\right)} \cdot (x-1) \cdot \mathrm{Heaviside}(x-1)$

$$CondInic := f = \frac{1}{500} x \,\mathrm{Heaviside}(x) - \frac{1}{250} \left(x - \frac{1}{2}\right) \mathrm{Heaviside}\left(x - \frac{1}{2}\right) + \frac{1}{500} (x - 1) \,\mathrm{Heaviside}(x-1) \quad \textbf{(2)}$$

> $plot(rhs(CondInic), x = -0.2..1.2, y = -0.002..0.002)$



> $CondInicVel := 0$

$$CondInicVel := 0 \quad \textbf{(3)}$$

> $CondFrontera := F(0) = 0, F(1) = 0$

$$CondFrontera := F(0) = 0, F(1) = 0 \qquad \textbf{(4)}$$

> $Ecua$

$$\frac{\partial^2}{\partial t^2} y(x,t) = c^2 \left(\frac{\partial^2}{\partial x^2} y(x,t) \right) \qquad \textbf{(5)}$$

> $Hipotesis := y(x,t) = F(x) \cdot G(t)$

$$Hipotesis := y(x,t) = F(x)\ G(t) \qquad \textbf{(6)}$$

> $EcuaSeparable := eval(subs(y(x,t) = rhs(Hipotesis), Ecua))$

$$EcuaSeparable := F(x) \left(\frac{d^2}{dt^2} G(t) \right) = c^2 \left(\frac{d^2}{dx^2} F(x) \right) G(t) \qquad \textbf{(7)}$$

> $EcuaSeparada := simplify\left(\frac{lhs(EcuaSeparable)}{F(x) \cdot G(t)} \right) = simplify\left(\frac{rhs(EcuaSeparable)}{F(x) \cdot G(t)} \right)$

$$EcuaSeparada := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \frac{c^2 \left(\frac{d^2}{dx^2} F(x) \right)}{F(x)} \qquad \textbf{(8)}$$

> $EcuaX := rhs(EcuaSeparada) = \mathrm{alpha}$

$$EcuaX := \frac{c^2 \left(\frac{d^2}{dx^2} F(x) \right)}{F(x)} = \alpha \qquad \textbf{(9)}$$

> $EcuaT := lhs(EcuaSeparada) = \mathrm{alpha}$

$$EcuaT := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \alpha \qquad \textbf{(10)}$$

Para alpha=0

> $EcuaXcero := subs(\mathrm{alpha} = 0, EcuaX)$

$$EcuaXcero := \frac{c^2 \left(\frac{d^2}{dx^2} F(x) \right)}{F(x)} = 0 \qquad \textbf{(11)}$$

> $SolXcero := dsolve(EcuaXcero)$

$$SolXcero := F(x) = \_C1\ x + \_C2 \qquad \textbf{(12)}$$

> $SolPartXcero := dsolve(\{EcuaXcero, CondFrontera\})$

$$SolPartXcero := F(x) = 0 \qquad \textbf{(13)}$$

Se descarta esta solución para alpha=0 pues si F(x)=0 para toda x entonces no ciumple con las condiciones de frontera

> $EcuaX$

$$\frac{c^2 \left(\frac{d^2}{dx^2} F(x) \right)}{F(x)} = \alpha \qquad \textbf{(14)}$$

Para alpha positiva debemos substituir alpha=beta^2

> $EcuaXpos := subs\left(\mathrm{alpha} = \beta^2, EcuaX\right)$

$$EcuaXpos := \frac{c^2 \left(\frac{d^2}{dx^2} F(x)\right)}{F(x)} = \beta^2 \tag{15}$$

> $SolXpos := dsolve(EcuaXpos)$

$$SolXpos := F(x) = \_C1\, e^{\frac{\beta x}{c}} + \_C2\, e^{-\frac{\beta x}{c}} \tag{16}$$

> $SolPartXpos := dsolve(\{EcuaXpos, CondFrontera\})$

$$SolPartXpos := F(x) = 0 \tag{17}$$

Se descarta esta solución para alpha positiva pues si F(x)=0 para toda x entonces no cumple con las condiciones de frontera

> $EcuaX$

$$\frac{c^2 \left(\frac{d^2}{dx^2} F(x)\right)}{F(x)} = \alpha \tag{18}$$

Para alpha negativa debemos substituir alpha= - beta^2

> $EcuaXneg := subs\left(\text{alpha} = -\beta^2, EcuaX\right)$

$$EcuaXneg := \frac{c^2 \left(\frac{d^2}{dx^2} F(x)\right)}{F(x)} = -\beta^2 \tag{19}$$

> $SolXneg := dsolve(EcuaXneg)$

$$SolXneg := F(x) = \_C1 \sin\left(\frac{\beta x}{c}\right) + \_C2 \cos\left(\frac{\beta x}{c}\right) \tag{20}$$

> $ParaDos := simplify(subs(x = 0, rhs(SolXneg) = 0))$

$$ParaDos := \_C2 = 0 \tag{21}$$

> $SolXnegBis := subs(\_C2 = rhs(ParaDos), SolXneg)$

$$SolXnegBis := F(x) = \_C1 \sin\left(\frac{\beta x}{c}\right) \tag{22}$$

> $\text{beta} := c \cdot n \cdot \text{Pi}$

$$\beta := c\, n\, \pi \tag{23}$$

> $SolPartXneg := SolXnegBis$

$$SolPartXneg := F(x) = \_C1 \sin(n \pi x) \tag{24}$$

> $ParaUno := subs\left(\sin\left(\frac{n \cdot \pi}{c}\right) = 0, subs(x = 1, rhs(SolPartXneg) = 0)\right)$

$$ParaUno := \_C1 \sin(n \pi) = 0 \tag{25}$$

> $\_C1 \neq 0$

$$\_C1 \neq 0 \tag{26}$$

> $EcuaTneg := subs\left(\text{alpha} = -\beta^2, EcuaT\right)$

$$EcuaTneg := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = -c^2 n^2 \pi^2 \tag{27}$$

> $SolTneg := dsolve(EcuaTneg)$

$$SolTneg := G(t) = \_C1 \sin(c\, n\, \pi\, t) + \_C2 \cos(c\, n\, \pi\, t) \quad \textbf{(28)}$$

> $SolUno := y(x, t) = subs(c = 1, \_C1 = 1, rhs(SolPartXneg)) \cdot rhs(SolTneg)$

$$SolUno := y(x, t) = \sin(n\, \pi\, x)\, (\_C1 \sin(c\, n\, \pi\, t) + \_C2 \cos(c\, n\, \pi\, t)) \quad \textbf{(29)}$$

Esta es la solución general para sustituir las condiciones iniciales del problema, pues ya satisafce las de frontera

> $SolGral := y(x, t) = Sum(subs(c = 1, \_C2 = b[n], \_C1 = a[n], rhs(SolUno)), n = 1 ..infinity)$

$$SolGral := y(x, t) = \sum_{n=1}^{\infty} \sin(n\, \pi\, x)\, \left(a_n \sin(n\, \pi\, t) + b_n \cos(n\, \pi\, t)\right) \quad \textbf{(30)}$$

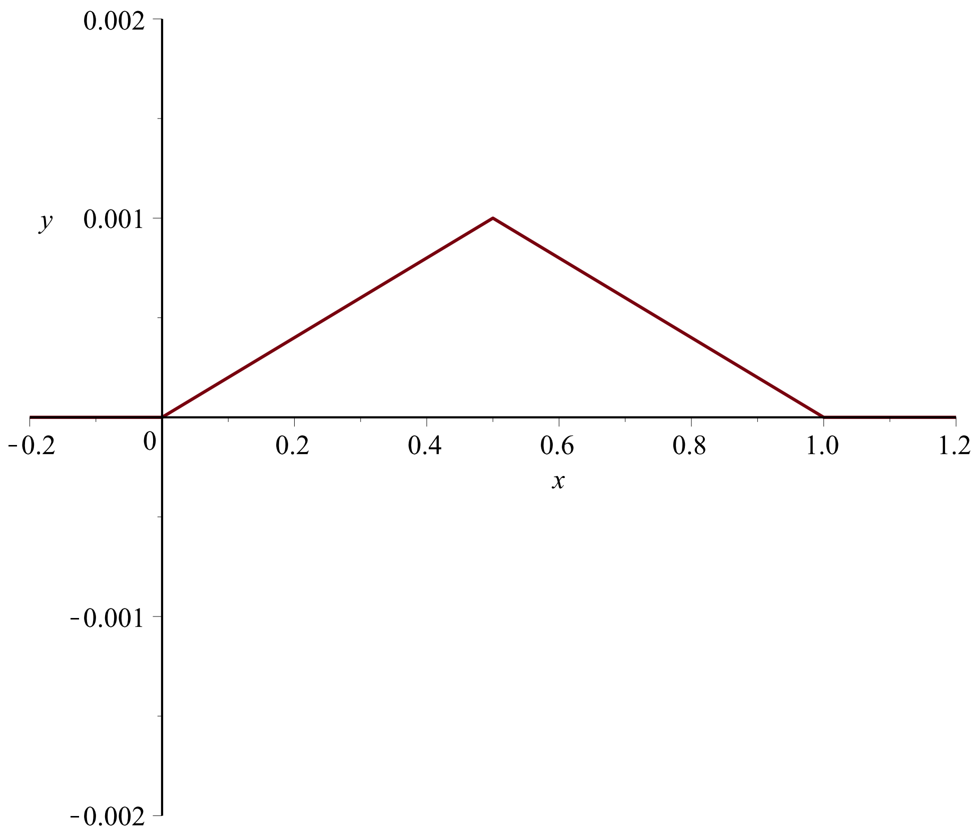
> $SolPartIniX := F(x) = eval(subs(t = 0, rhs(SolGral)))$

$$SolPartIniX := F(x) = \sum_{n=1}^{\infty} \sin(n\, \pi\, x)\, b_n \quad \textbf{(31)}$$

> $L := \frac{5}{10}$

$$L := \frac{1}{2} \quad \textbf{(32)}$$

> $plot(rhs(CondInic), x = -0.2 ..1.2, y = -0.002 ..0.002)$



> $b[n] := \left(\frac{1}{L}\right) \cdot int(rhs(CondInic) \cdot \sin(n \cdot \mathrm{Pi} \cdot x), x = 0..1)$

$$b_n := \frac{1}{250} \frac{-\sin(n\pi) + 2\sin\left(\frac{1}{2} n\pi\right)}{n^2 \pi^2} \tag{33}$$

> $a[n] := 0$

$$a_n := 0 \tag{34}$$

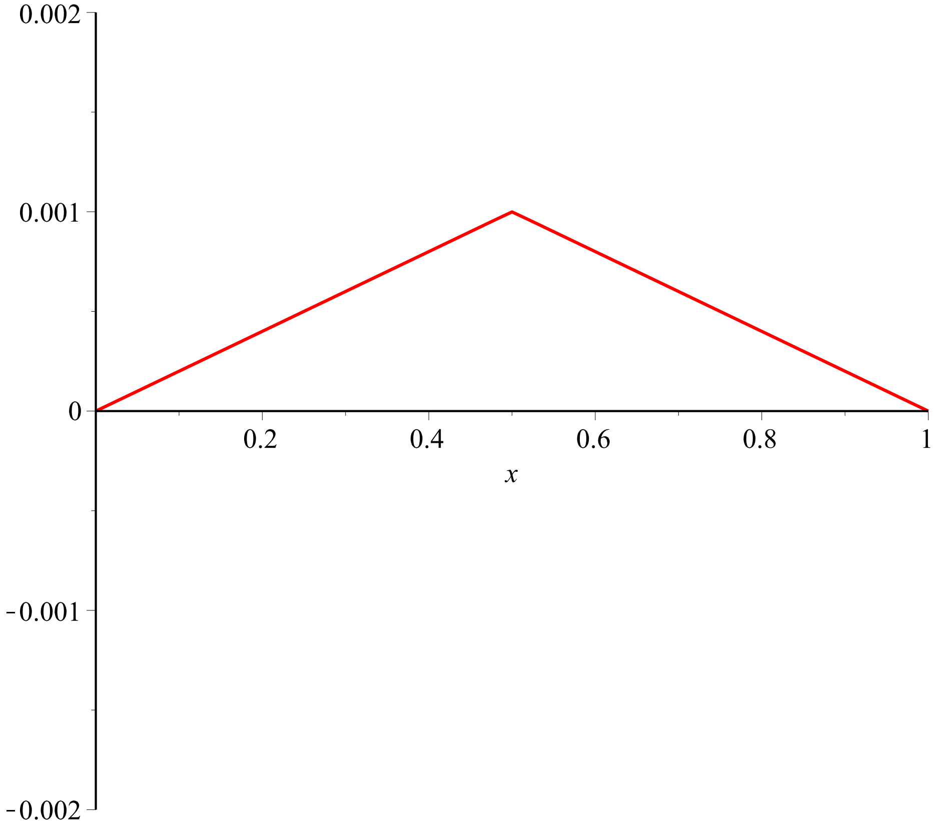
> $SolucionParticular := SolGral$

$$SolucionParticular := y(x,t) = \sum_{n=1}^{\infty} \frac{1}{250} \frac{\sin(n\pi x)\left(-\sin(n\pi) + 2\sin\left(\frac{1}{2} n\pi\right)\right)\cos(n\pi t)}{n^2\pi^2} \tag{35}$$

> $SolPart500 := y(x,t) = sum\left(\frac{1}{250} \frac{\sin(n\pi x)\left(-\sin(n\pi) + 2\sin\left(\frac{1}{2} n\pi\right)\right)\cos(n\pi t)}{n^2\pi^2}, n = 1..500\right):$

> $with(plots):$

> $animate(rhs(SolPart500), x = 0..1, t = 0..4, frames = 150, view = [0..1, -0.002..0.002])$


0.002
0.001
0
-0.001
-0.002
0.2
0.4
0.6
0.8
1
x

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