

```

> restart
> Sistema := diff(x[1](t), t) = x[3](t), diff(x[2](t), t) = x[4](t), diff(x[3](t), t) =
- (k[1] + k[2]) / M[1] * x[1](t) + k[2] / M[1] * x[2](t), diff(x[4](t), t) = k[2] / M[2] * x[1](t) - k[2] / M[2]
· x[2](t) :
> Sistema[1]; Sistema[2]; Sistema[3]; Sistema[4]

$$\frac{d}{dt} x_1(t) = x_3(t)$$


$$\frac{d}{dt} x_2(t) = x_4(t)$$


$$\frac{d}{dt} x_3(t) = -\frac{(k_1 + k_2) x_1(t)}{M_1} + \frac{k_2 x_2(t)}{M_1}$$


$$\frac{d}{dt} x_4(t) = \frac{k_2 x_1(t)}{M_2} - \frac{k_2 x_2(t)}{M_2} \quad (1)$$

> k[1] := 6 : k[2] := 4 : M[1] := 1 : M[2] := 1 : Sistema[1]; Sistema[2]; Sistema[3];
Sistema[4]

$$\frac{d}{dt} x_1(t) = x_3(t)$$


$$\frac{d}{dt} x_2(t) = x_4(t)$$


$$\frac{d}{dt} x_3(t) = -10 x_1(t) + 4 x_2(t)$$


$$\frac{d}{dt} x_4(t) = 4 x_1(t) - 4 x_2(t) \quad (2)$$

> AA := matrix([ [0, 0, 1, 0], [0, 0, 0, 1], [-10, 4, 0, 0], [4, -4, 0, 0] ])

$$AA := \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -10 & 4 & 0 & 0 \\ 4 & -4 & 0 & 0 \end{bmatrix} \quad (3)$$

> Xcero := array([ [4/6 * 1/10, 1/10, 0, 0] ])

$$Xcero := \begin{bmatrix} \frac{1}{15} & \frac{1}{10} & 0 & 0 \end{bmatrix} \quad (4)$$

> with(linalg) :
> MatExp := exponential(AA, t) : evalf(MatExp[1, 1], 3)

$$0.800 \cos(3.46 t) + 0.200 \cos(1.41 t) \quad (5)$$

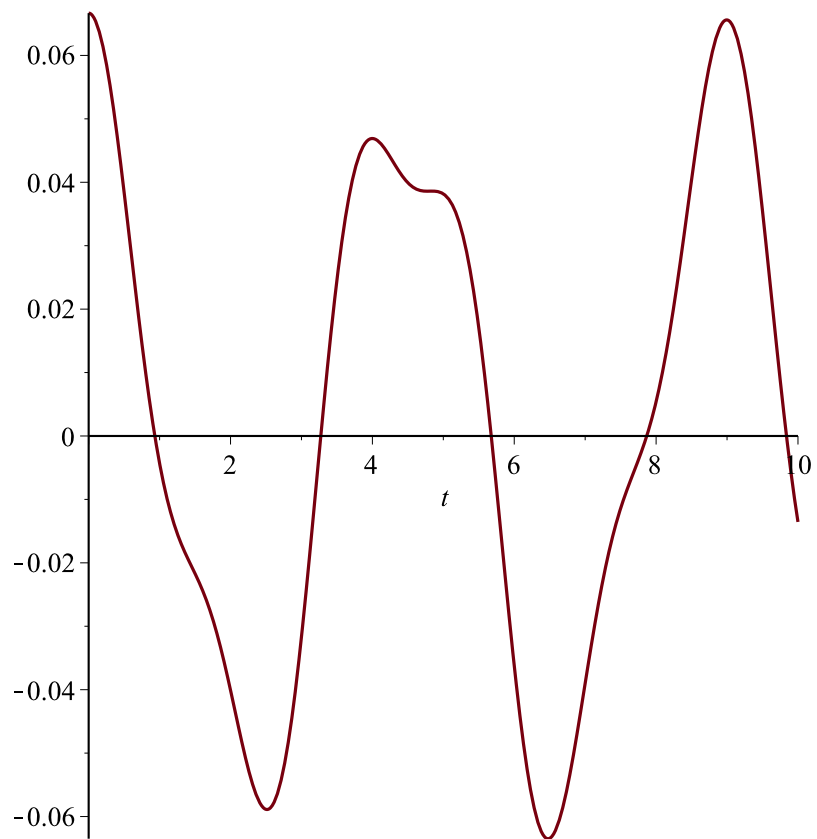
> SolPart := evalm(MatExp &* Xcero) : x[1](t) = evalf(SolPart[1], 3); x[2](t)
= evalf(SolPart[2], 3)

$$x_1(t) = 0.0134 \cos(3.46 t) + 0.0533 \cos(1.41 t)$$

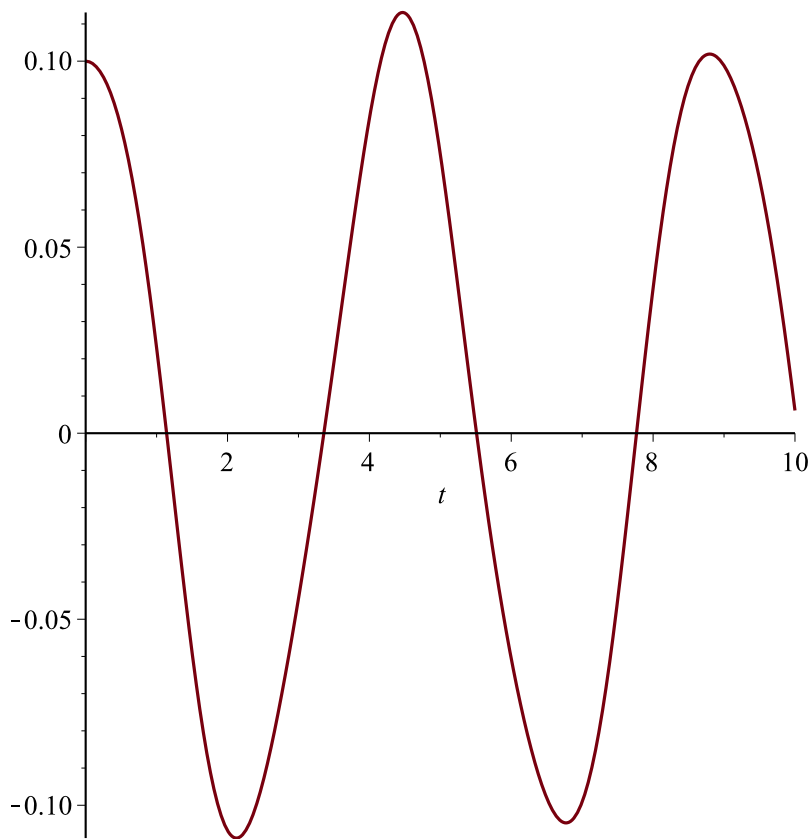

$$x_2(t) = -0.0067 \cos(3.46 t) + 0.107 \cos(1.41 t) \quad (6)$$


```

> `plot(SolPart[1], t = 0..10)`



> `plot(SolPart[2], t = 0..10)`



PRUEBA DOS

> restart

> Sistema := diff(x[1](t), t) = x[3](t), diff(x[2](t), t) = x[4](t), diff(x[3](t), t) =

$$-\frac{(k[1] + k[2])}{M[1]} \cdot x[1](t) + \frac{k[2]}{M[1]} \cdot x[2](t), \text{diff}(x[4](t), t) = \frac{k[2]}{M[2]} \cdot x[1](t) - \frac{k[2]}{M[2]} \cdot x[2](t) :$$

> Sistema[1]; Sistema[2]; Sistema[3]; Sistema[4]

$$\frac{d}{dt} x_1(t) = x_3(t)$$

$$\frac{d}{dt} x_2(t) = x_4(t)$$

$$\frac{d}{dt} x_3(t) = -\frac{(k_1 + k_2) x_1(t)}{M_1} + \frac{k_2 x_2(t)}{M_1}$$

$$\frac{d}{dt} x_4(t) = \frac{k_2 x_1(t)}{M_2} - \frac{k_2 x_2(t)}{M_2}$$

(7)

> k[1] := 4 : k[2] := 6 : M[1] := 1 : M[2] := 1 : Sistema[1]; Sistema[2]; Sistema[3];
 Sistema[4]

$$\frac{d}{dt} x_1(t) = x_3(t)$$

$$\frac{d}{dt} x_2(t) = x_4(t)$$

$$\frac{d}{dt} x_3(t) = -10 x_1(t) + 6 x_2(t)$$

$$\frac{d}{dt} x_4(t) = 6 x_1(t) - 6 x_2(t) \quad (8)$$

```
> AA := matrix([ [0, 0, 1, 0], [0, 0, 0, 1], [-10, 4, 0, 0], [4, -4, 0, 0] ])
```

$$AA := \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -10 & 4 & 0 & 0 \\ 4 & -4 & 0 & 0 \end{bmatrix} \quad (9)$$

```
> Xcero := array([ [ 6/4 * 1/10, 1/10, 0, 0 ] ])
```

$$Xcero := \begin{bmatrix} \frac{3}{20} & \frac{1}{10} & 0 & 0 \end{bmatrix} \quad (10)$$

```
> with(linalg) :
```

```
> MatExp := exponential(AA, t) : evalf(MatExp[1, 1], 3)
```

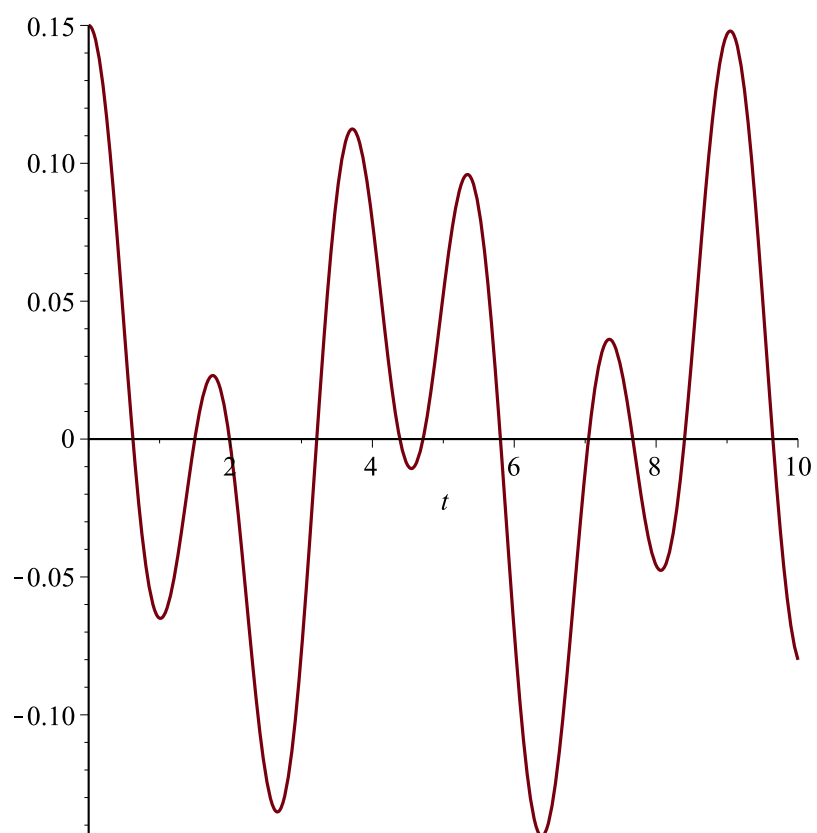
$$0.800 \cos(3.46 t) + 0.200 \cos(1.41 t) \quad (11)$$

```
> SolPart := evalm(MatExp &* Xcero) : x[1](t) = evalf(SolPart[1], 3); x[2](t)
= evalf(SolPart[2], 3)
```

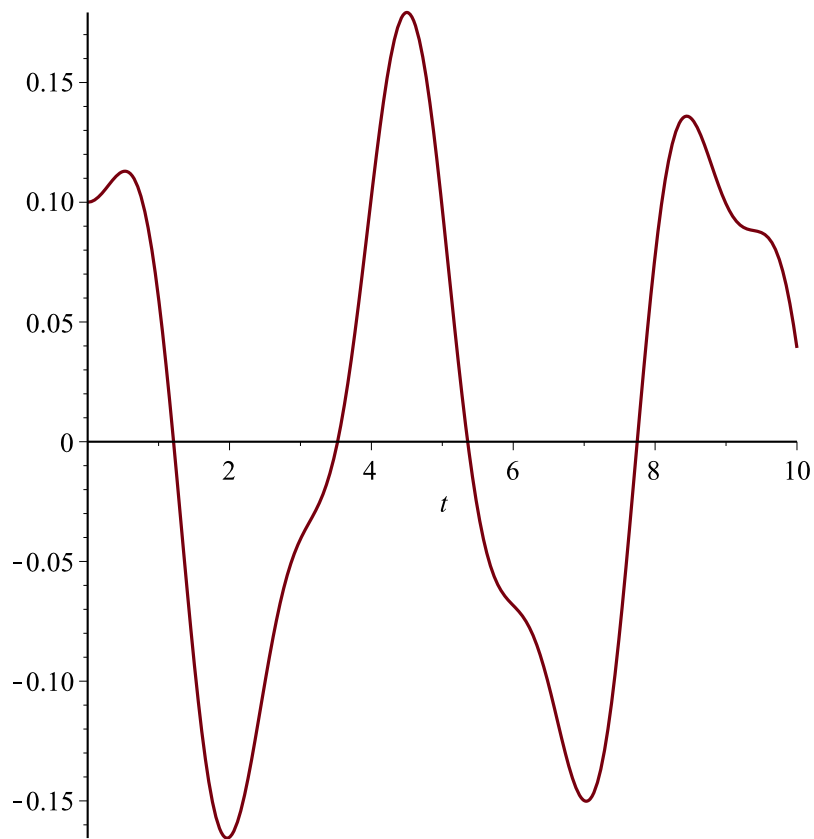
$$x_1(t) = 0.0800 \cos(3.46 t) + 0.0700 \cos(1.41 t)$$

$$x_2(t) = -0.0400 \cos(3.46 t) + 0.140 \cos(1.41 t) \quad (12)$$

```
> plot(SolPart[1], t=0..10)
```



=
> `plot(SolPart[2], t = 0 .. 10)`



PRUEBA TRES

> restart

> Sistema := diff(x[1](t), t) = x[3](t), diff(x[2](t), t) = x[4](t), diff(x[3](t), t) =

$$-\frac{(k[1] + k[2])}{M[1]} \cdot x[1](t) + \frac{k[2]}{M[1]} \cdot x[2](t), \text{diff}(x[4](t), t) = \frac{k[2]}{M[2]} \cdot x[1](t) - \frac{k[2]}{M[2]} \cdot x[2](t) :$$

> Sistema[1]; Sistema[2]; Sistema[3]; Sistema[4]

$$\frac{d}{dt} x_1(t) = x_3(t)$$

$$\frac{d}{dt} x_2(t) = x_4(t)$$

$$\frac{d}{dt} x_3(t) = -\frac{(k_1 + k_2) x_1(t)}{M_1} + \frac{k_2 x_2(t)}{M_1}$$

$$\frac{d}{dt} x_4(t) = \frac{k_2 x_1(t)}{M_2} - \frac{k_2 x_2(t)}{M_2}$$

(13)

> k[1] := 6 : k[2] := 4 : M[1] := 1 : M[2] := $\frac{3}{2}$: Sistema[1]; Sistema[2]; Sistema[3];
 Sistema[4]

$$\frac{d}{dt} x_1(t) = x_3(t)$$

$$\frac{d}{dt} x_2(t) = x_4(t)$$

$$\frac{d}{dt} x_3(t) = -10 x_1(t) + 4 x_2(t)$$

$$\frac{d}{dt} x_4(t) = \frac{8}{3} x_1(t) - \frac{8}{3} x_2(t) \quad (14)$$

> *AA* := *matrix*([[0, 0, 1, 0], [0, 0, 0, 1], [-10, 4, 0, 0], [4, -4, 0, 0]])

$$AA := \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -10 & 4 & 0 & 0 \\ 4 & -4 & 0 & 0 \end{bmatrix} \quad (15)$$

> *Xcero* := *array*([[$\frac{4}{6} \cdot \frac{1}{10}$, $\frac{1}{10}$, 0, 0]])

$$Xcero := \begin{bmatrix} \frac{1}{15} & \frac{1}{10} & 0 & 0 \end{bmatrix} \quad (16)$$

> *with*(*linalg*) :

> *MatExp* := *exponential*(*AA*, *t*) : *evalf*(*MatExp*[1, 1], 3)

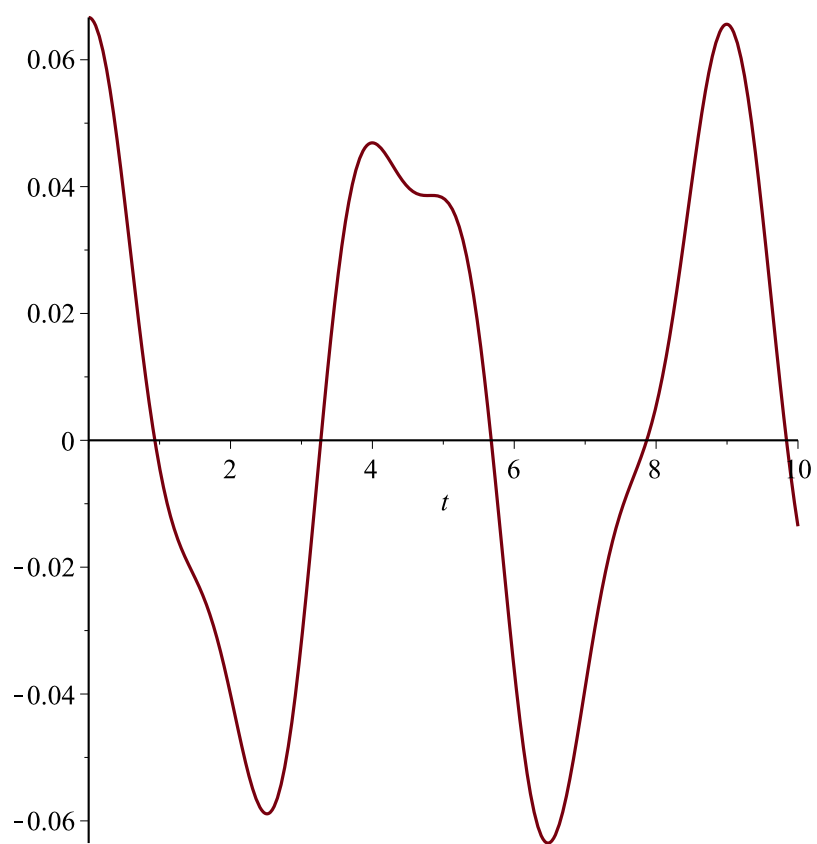
$$0.800 \cos(3.46 t) + 0.200 \cos(1.41 t) \quad (17)$$

> *SolPart* := *evalm*(*MatExp* &* *Xcero*) : *x*[1](*t*) = *evalf*(*SolPart*[1], 3); *x*[2](*t*) = *evalf*(*SolPart*[2], 3)

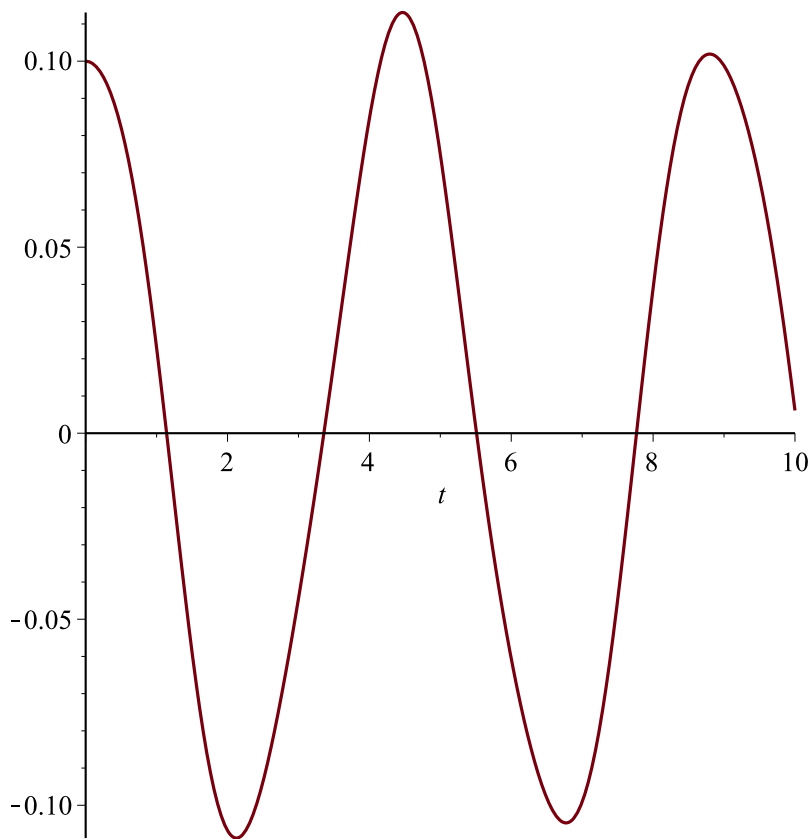
$$x_1(t) = 0.0134 \cos(3.46 t) + 0.0533 \cos(1.41 t)$$

$$x_2(t) = -0.0067 \cos(3.46 t) + 0.107 \cos(1.41 t) \quad (18)$$

> *plot*(*SolPart*[1], *t* = 0..10)



=
> `plot(SolPart[2], t = 0 .. 10)`



PRUEBA CUATRO

> restart

> Sistema := diff(x[1](t), t) = x[3](t), diff(x[2](t), t) = x[4](t), diff(x[3](t), t) =

$$-\frac{(k[1] + k[2])}{M[1]} \cdot x[1](t) + \frac{k[2]}{M[1]} \cdot x[2](t), \text{diff}(x[4](t), t) = \frac{k[2]}{M[2]} \cdot x[1](t) - \frac{k[2]}{M[2]} \cdot x[2](t) :$$

> Sistema[1]; Sistema[2]; Sistema[3]; Sistema[4]

$$\frac{d}{dt} x_1(t) = x_3(t)$$

$$\frac{d}{dt} x_2(t) = x_4(t)$$

$$\frac{d}{dt} x_3(t) = -\frac{(k_1 + k_2) x_1(t)}{M_1} + \frac{k_2 x_2(t)}{M_1}$$

$$\frac{d}{dt} x_4(t) = \frac{k_2 x_1(t)}{M_2} - \frac{k_2 x_2(t)}{M_2}$$

(19)

> k[1] := 4 : k[2] := 6 : M[1] := 1 : M[2] := $\frac{3}{2}$: Sistema[1]; Sistema[2]; Sistema[3];
 Sistema[4]

$$\frac{d}{dt} x_1(t) = x_3(t)$$

$$\frac{d}{dt} x_2(t) = x_4(t)$$

$$\frac{d}{dt} x_3(t) = -10 x_1(t) + 6 x_2(t)$$

$$\frac{d}{dt} x_4(t) = 4 x_1(t) - 4 x_2(t) \quad (20)$$

```
> AA := matrix([ [0, 0, 1, 0], [0, 0, 0, 1], [-10, 4, 0, 0], [4, -4, 0, 0]])
```

$$AA := \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -10 & 4 & 0 & 0 \\ 4 & -4 & 0 & 0 \end{bmatrix} \quad (21)$$

```
> Xcero := array([ [ 6/4 * 1/10, 1/10, 0, 0 ] ])
```

$$Xcero := \begin{bmatrix} \frac{3}{20} & \frac{1}{10} & 0 & 0 \end{bmatrix} \quad (22)$$

```
> with(linalg) :
```

```
> MatExp := exponential(AA, t) : evalf(MatExp[1, 1], 3)
```

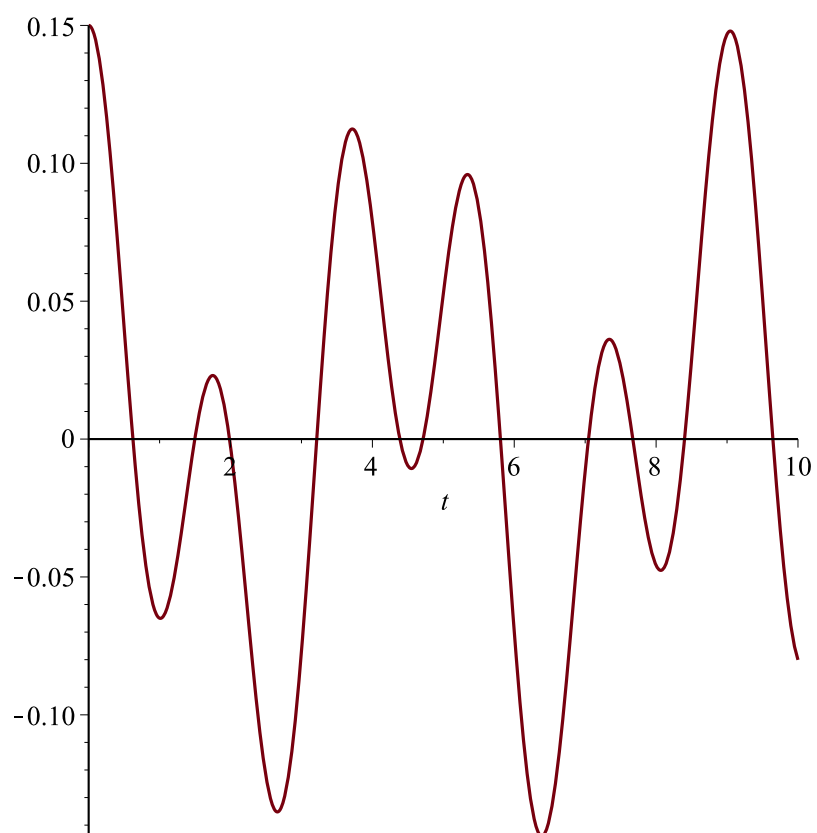
$$0.800 \cos(3.46 t) + 0.200 \cos(1.41 t) \quad (23)$$

```
> SolPart := evalm(MatExp &* Xcero) : x[1](t) = evalf(SolPart[1], 3); x[2](t)
= evalf(SolPart[2], 3)
```

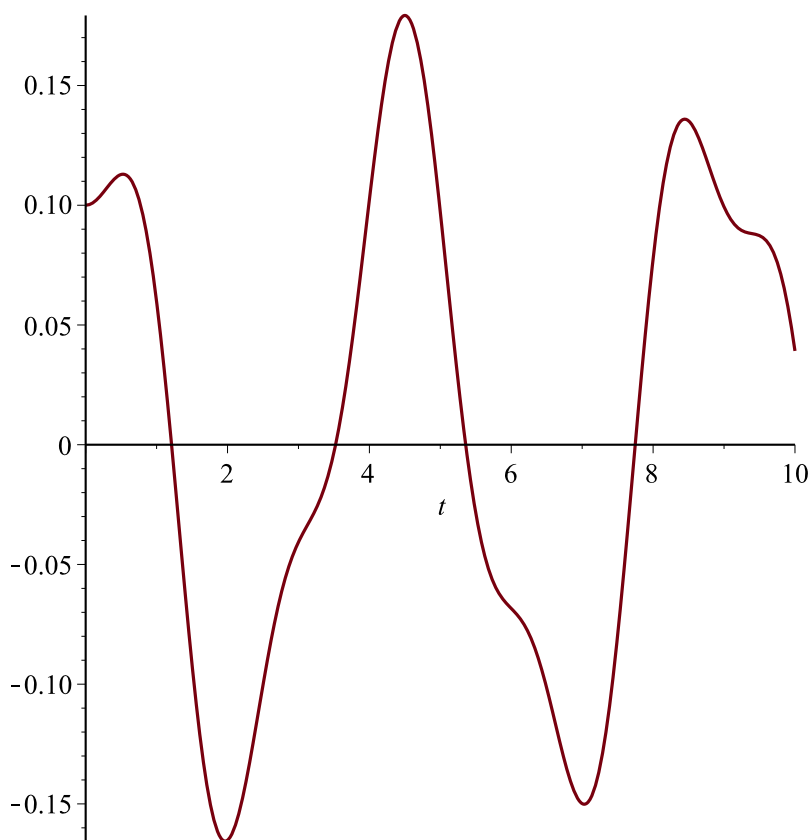
$$x_1(t) = 0.0800 \cos(3.46 t) + 0.0700 \cos(1.41 t)$$

$$x_2(t) = -0.0400 \cos(3.46 t) + 0.140 \cos(1.41 t) \quad (24)$$

```
> plot(SolPart[1], t=0..10)
```



`> plot(SolPart[2], t = 0 .. 10)`



```
> restart
```

```
> Sistema := diff(i[1](t), t) + 50·i[1](t) = 60,  $\frac{50}{10000} \cdot \text{diff}(i[2](t), t) + i[2](t) - i[1](t) = 0$  :  
Sistema[1]; Sistema[2];
```

$$\begin{aligned} \frac{d}{dt} i_1(t) + 50 i_1(t) &= 60 \\ \frac{1}{200} \frac{d}{dt} i_2(t) + i_2(t) - i_1(t) &= 0 \end{aligned} \quad (25)$$

```
> Cond := i[1](0) = 0, i[2](0) = 0  
Cond :=  $i_1(0) = 0, i_2(0) = 0$  (26)
```

```
> with(inttrans) :
```

```
> SistTransLap := subs(Cond, laplace( {Sistema}, t, s))
```

$$\begin{aligned} \text{SistTransLap} := \left\{ s \text{laplace}(i_1(t), t, s) + 50 \text{laplace}(i_1(t), t, s) = \frac{60}{s}, \frac{1}{200} s \text{laplace}(i_2(t), t, \right. \\ \left. s) + \text{laplace}(i_2(t), t, s) - \text{laplace}(i_1(t), t, s) = 0 \right\} \end{aligned} \quad (27)$$

```
> SolUno := isolate(SistTransLap[1], laplace(i[1](t), t, s))
```

(28)

$$SolUno := \text{laplace}(i_1(t), t, s) = \frac{60}{s(s+50)} \quad (28)$$

> $SolDos := \text{isolate}(\text{subs}(\text{laplace}(i[1](t), t, s) = \text{rhs}(SolUno), SistTransLap[2]), \text{laplace}(i[2](t), t, s))$

$$SolDos := \text{laplace}(i_2(t), t, s) = \frac{60}{s(s+50) \left(\frac{1}{200}s + 1 \right)} \quad (29)$$

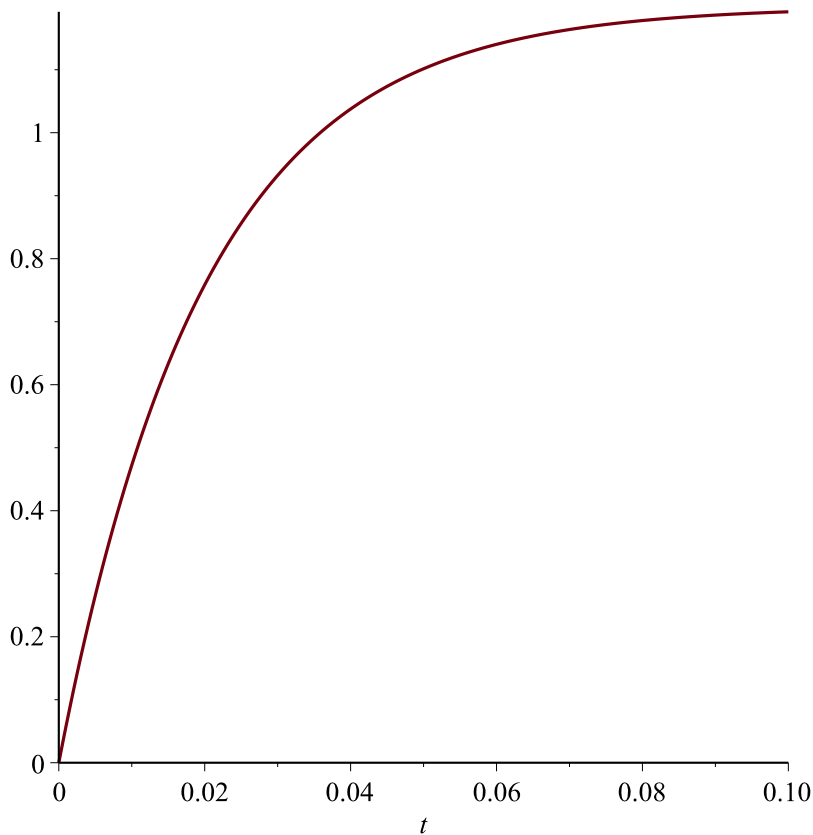
> $SolPartUno := \text{invlaplace}(SolUno, s, t)$

$$SolPartUno := i_1(t) = \frac{6}{5} - \frac{6}{5} e^{-50t} \quad (30)$$

> $SolPartDos := \text{invlaplace}(SolDos, s, t)$

$$SolPartDos := i_2(t) = \frac{2}{5} e^{-200t} + \frac{6}{5} - \frac{8}{5} e^{-50t} \quad (31)$$

> $\text{plot}(\text{rhs}(SolPartUno), t = 0..0.1)$



> $\text{plot}(\text{rhs}(SolPartDos), t = 0..0.1)$

