

$$\begin{aligned}
& \text{restart} \\
& \text{Ecua} := x \cdot \text{sqrt}(1 + y(x)^2) + y(x) \cdot \text{sqrt}(1 + x^2) \cdot \text{diff}(y(x), x) = 0 \\
& \quad \text{Ecua} := x \sqrt{1 + y(x)^2} + y(x) \sqrt{x^2 + 1} \left( \frac{d}{dx} y(x) \right) = 0 \tag{1} \\
& \text{with(DEtools):} \\
& \text{odeadvisor(Ecua)} \\
& \quad \text{[_separable]} \tag{2} \\
& M := x \sqrt{1 + y^2} \\
& \quad M := x \sqrt{y^2 + 1} \tag{3} \\
& N := y \sqrt{x^2 + 1} \\
& \quad N := y \sqrt{x^2 + 1} \tag{4} \\
& P := x; Q := \sqrt{y^2 + 1}; \\
& \quad P := x \\
& \quad Q := \sqrt{y^2 + 1} \tag{5} \\
& R := \sqrt{x^2 + 1}; S := y \\
& \quad R := \sqrt{x^2 + 1} \\
& \quad S := y \tag{6} \\
& Sol := \text{int}\left(\frac{P}{R}, x\right) + \text{int}\left(\frac{S}{Q}, y\right) = \_CI \\
& \quad Sol := \sqrt{x^2 + 1} + \sqrt{y^2 + 1} = \_CI \tag{7} \\
& SolGral := \sqrt{x^2 + 1} + \sqrt{y(x)^2 + 1} = \_CI \\
& \quad SolGral := \sqrt{x^2 + 1} + \sqrt{1 + y(x)^2} = \_CI \tag{8} \\
& Ecua \\
& \quad x \sqrt{1 + y(x)^2} + y(x) \sqrt{x^2 + 1} \left( \frac{d}{dx} y(x) \right) = 0 \tag{9} \\
& DerEcua := \text{isolate}(Ecua, \text{diff}(y(x), x)) \\
& \quad DerEcua := \frac{d}{dx} y(x) = - \frac{x \sqrt{1 + y(x)^2}}{y(x) \sqrt{x^2 + 1}} \tag{10} \\
& DerSol := \text{isolate}(\text{diff}(SolGral, x), \text{diff}(y(x), x)) \\
& \quad DerSol := \frac{d}{dx} y(x) = - \frac{x \sqrt{1 + y(x)^2}}{y(x) \sqrt{x^2 + 1}} \tag{11} \\
& Comp := \text{rhs}(DerSol) - \text{rhs}(DerEcua) = 0 \\
& \quad Comp := 0 = 0 \tag{12} \\
& restart \\
& Ecua := \text{sqrt}(x^2 - y(x)^2) + y(x) - x \cdot \text{diff}(y(x), x) = 0 \\
& \quad Ecua := \sqrt{x^2 - y(x)^2} + y(x) - x \left( \frac{d}{dx} y(x) \right) = 0 \tag{13}
\end{aligned}$$

$$\begin{aligned}
&> \text{with(DEtools)} : \\
&> \text{odeadvisor(Ecua)} \\
&\quad [\text{\_homogeneous, class A}, \text{\_rational}, \text{\_dAlembert}]
\end{aligned} \tag{14}$$

$$\begin{aligned}
&> \text{EcuaDos} := \text{simplify}(\text{isolate}(\text{eval}(\text{subs}(y(x) = x \cdot u(x), \text{Ecua})), \text{diff}(u(x), x))) \\
&\quad \text{EcuaDos} := \frac{d}{dx} u(x) = \frac{\sqrt{-x^2 (u(x)^2 - 1)}}{x^2}
\end{aligned} \tag{15}$$

$$\begin{aligned}
&> \text{EcuaTres} := \text{lhs}(\text{EcuaDos})^2 \cdot x^2 = \text{rhs}(\text{EcuaDos})^2 \cdot x^2 \\
&\quad \text{EcuaTres} := \left( \frac{d}{dx} u(x) \right)^2 x^2 = -u(x)^2 + 1
\end{aligned} \tag{16}$$

$$\begin{aligned}
&> \text{EcuaCuatro} := \text{isolate}(\text{diff}(u(x), x) \cdot x = \text{sqrt}(\text{rhs}(\text{EcuaTres})), \text{diff}(u(x), x)) \\
&\quad \text{EcuaCuatro} := \frac{d}{dx} u(x) = \frac{\sqrt{-u(x)^2 + 1}}{x}
\end{aligned} \tag{17}$$

$$\begin{aligned}
&> \text{odeadvisor(EcuaCuatro)} \\
&\quad [\text{\_separable}]
\end{aligned} \tag{18}$$

$$\begin{aligned}
&> P := \frac{1}{x}; Q := \frac{1}{\sqrt{-u^2 + 1}} \\
&\quad P := \frac{1}{x} \\
&\quad Q := \frac{1}{\sqrt{-u^2 + 1}}
\end{aligned} \tag{19}$$

$$\begin{aligned}
&> \text{Sol} := \text{int}(P, x) - \text{int}(Q, u) = \text{\_CI} \\
&\quad \text{Sol} := \ln(x) - \arcsin(u) = \text{\_CI}
\end{aligned} \tag{20}$$

$$\begin{aligned}
&> \text{SolGral} := \text{subs}\left(u = \frac{y}{x}, \text{Sol}\right) \\
&\quad \text{SolGral} := \ln(x) - \arcsin\left(\frac{y}{x}\right) = \text{\_CI}
\end{aligned} \tag{21}$$

$$\begin{aligned}
&> \text{SolGralDos} := \ln(x) - \arcsin\left(\frac{y(x)}{x}\right) = \text{\_CI} \\
&\quad \text{SolGralDos} := \ln(x) - \arcsin\left(\frac{y(x)}{x}\right) = \text{\_CI}
\end{aligned} \tag{22}$$

$$\begin{aligned}
&> \text{DerSol} := \text{simplify}(\text{isolate}(\text{diff}(\text{SolGralDos}, x), \text{diff}(y(x), x))) \\
&\quad \text{DerSol} := \frac{d}{dx} y(x) = \frac{\sqrt{-\frac{y(x)^2 - x^2}{x^2}} x + y(x)}{x}
\end{aligned} \tag{23}$$

$$\begin{aligned}
&> \text{DerEcua} := \text{isolate}(\text{Ecua}, \text{diff}(y(x), x)) \\
&\quad \text{DerEcua} := \frac{d}{dx} y(x) = -\frac{-\sqrt{x^2 - y(x)^2} - y(x)}{x}
\end{aligned} \tag{24}$$

$$\begin{aligned}
&> \text{Comp} := \text{simplify}(\text{rhs}(\text{DerSol}) - \text{rhs}(\text{DerEcua})) \cdot x = 0 \\
&\quad \text{Comp} := \sqrt{-\frac{y(x)^2 - x^2}{x^2}} x - \sqrt{x^2 - y(x)^2} = 0
\end{aligned} \tag{25}$$

$$\begin{aligned}
 & \text{> } \text{CompDos} := \left( \frac{\text{lhs}(\text{Comp}) + \sqrt{x^2 - y(x)^2}}{x} \right)^2 = \left( \frac{\text{rhs}(\text{Comp}) + \sqrt{x^2 - y(x)^2}}{x} \right)^2 \\
 & \text{CompDos} := - \frac{y(x)^2 - x^2}{x^2} = \frac{x^2 - y(x)^2}{x^2} \tag{26}
 \end{aligned}$$

$$\begin{aligned}
 & \text{> } \text{CompTres} := \text{simplify}(\text{lhs}(\text{CompDos}) - \text{rhs}(\text{CompDos})) = 0 \\
 & \text{CompTres} := 0 = 0 \tag{27}
 \end{aligned}$$

$\text{> restart}$

$\text{>}$