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> restart
> Ecua := ( (x / sqrt(x^2 + y(x)^2) + 1/x + 1/y(x)) + (y(x) / sqrt(x^2 + y(x)^2) + 1/y(x) - x/y(x)^2) )
      ·diff(y(x), x) = 0
Ecua := x / sqrt(x^2 + y(x)^2) + 1/x + 1/y(x) + (y(x) / sqrt(x^2 + y(x)^2) + 1/y(x) - x/y(x)^2) (d/dx y(x)) = 0 (1)

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> with(DEtools):
> odeadvisor(Ecua)
[_exact] (2)

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> M := x / sqrt(x^2 + y^2) + 1/x + 1/y
M := x / sqrt(x^2 + y^2) + 1/x + 1/y (3)

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> N := y / sqrt(x^2 + y^2) + 1/y - x/y^2
N := y / sqrt(x^2 + y^2) + 1/y - x/y^2 (4)

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> Comprobacion := (diff(M, y) - diff(N, x)) = 0
Comprobacion := 0 = 0 (5)

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> IntMx := int(M, x)
IntMx := sqrt(x^2 + y^2) + ln(x) + x/y (6)

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> SolGral := IntMx + int((N - diff(IntMx, y)), y) = _C1
SolGral := sqrt(x^2 + y^2) + ln(x) + x/y + ln(y) = _C1 (7)

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> SolGralFinal := sqrt(x^2 + y(x)^2) + ln(x) + x/y(x) + ln(y(x)) = _C1
SolGralFinal := sqrt(x^2 + y(x)^2) + ln(x) + x/y(x) + ln(y(x)) = _C1 (8)

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> Ecua
x / sqrt(x^2 + y(x)^2) + 1/x + 1/y(x) + (y(x) / sqrt(x^2 + y(x)^2) + 1/y(x) - x/y(x)^2) (d/dx y(x)) = 0 (9)

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> DerSolGral := simplify(isolate(diff(SolGralFinal, x), diff(y(x), x)))
DerSolGral := d/dx y(x) = - y(x) (y(x) x^2 + sqrt(x^2 + y(x)^2) y(x) + x sqrt(x^2 + y(x)^2)) / (x (y(x)^3 + sqrt(x^2 + y(x)^2) y(x) - x sqrt(x^2 + y(x)^2))) (10)

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> DerEcua := simplify(isolate(Ecua, diff(y(x), x)))
DerEcua := d/dx y(x) = - y(x) (y(x) x^2 + sqrt(x^2 + y(x)^2) y(x) + x sqrt(x^2 + y(x)^2)) / (x (y(x)^3 + sqrt(x^2 + y(x)^2) y(x) - x sqrt(x^2 + y(x)^2))) (11)

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> ComprobarDos := rhs(DerEcua) - rhs(DerSolGral) = 0
ComprobarDos := 0 = 0 (12)

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$$\begin{aligned}
&> \text{restart} \\
&> \text{Ecua} := (1 - x^2 \cdot y(x)) + x^2 \cdot (y(x) - x) \cdot \text{diff}(y(x), x) = 0 \\
&\qquad \qquad \qquad \text{Ecua} := 1 - x^2 y(x) + x^2 (y(x) - x) \left(\frac{d}{dx} y(x) \right) = 0 \tag{13} \\
&= \\
&> \text{with(DEtools)} : \\
&> \text{odeadvisor}(\text{Ecua}) \\
&\qquad \qquad \qquad [_{\text{rational}}, [_{\text{1st_order}}, _{\text{with_symmetry_}}[F(x), G(x)]], [_{\text{Abel}}, 2\text{nd type, class B}]] \tag{14} \\
&= \\
&> M := 1 - x^2 y \\
&\qquad \qquad \qquad M := -x^2 y + 1 \tag{15} \\
&= \\
&> N := x^2 \cdot (y - x) \\
&\qquad \qquad \qquad N := x^2 (y - x) \tag{16} \\
&= \\
&> \text{ComprobarExacta} := \text{diff}(M, y) \neq \text{diff}(N, x) \\
&\qquad \qquad \qquad \text{ComprobarExacta} := -x^2 \neq 2x(y - x) - x^2 \tag{17} \\
&= \\
&> f := \frac{(\text{diff}(M, y) - \text{diff}(N, x))}{N} \\
&\qquad \qquad \qquad f := -\frac{2}{x} \tag{18} \\
&= \\
&> \text{IntFact} := \text{isolate}\left(\text{int}\left(\frac{1}{\text{mu}}, \text{mu}\right) = \text{int}(f, x), \text{mu}\right) \\
&\qquad \qquad \qquad \text{IntFact} := \mu = \frac{1}{x^2} \tag{19} \\
&= \\
&> MM := \text{expand}(\text{rhs}(\text{IntFact}) \cdot M) \\
&\qquad \qquad \qquad MM := -y + \frac{1}{x^2} \tag{20} \\
&= \\
&> NN := \text{expand}(\text{rhs}(\text{IntFact}) \cdot N) \\
&\qquad \qquad \qquad NN := y - x \tag{21} \\
&= \\
&> \text{ComprExacta} := \text{diff}(MM, y) = \text{diff}(NN, x) \\
&\qquad \qquad \qquad \text{ComprExacta} := -1 = -1 \tag{22} \\
&= \\
&> \text{EcuaExacta} := MM + NN \cdot \text{diff}(y(x), x) = 0 \\
&\qquad \qquad \qquad \text{EcuaExacta} := -y + \frac{1}{x^2} + (y - x) \left(\frac{d}{dx} y(x) \right) = 0 \tag{23} \\
&= \\
&> \text{IntMMx} := \text{int}(MM, x) \\
&\qquad \qquad \qquad \text{IntMMx} := -yx - \frac{1}{x} \tag{24} \\
&= \\
&> \text{SolGral} := \text{IntMMx} + \text{int}((NN - \text{diff}(\text{IntMMx}, y)), y) = _CI \\
&\qquad \qquad \qquad \text{SolGral} := -yx - \frac{1}{x} + \frac{1}{2} y^2 = _CI \tag{25} \\
&= \\
&> \text{SolGralFinal} := -y(x) x - \frac{1}{x} + \frac{1}{2} y(x)^2 = _CI \\
&\qquad \qquad \qquad \text{SolGralFinal} := -y(x) x - \frac{1}{x} + \frac{1}{2} y(x)^2 = _CI \tag{26} \\
&= \\
&> \text{Ecua}
\end{aligned}$$

$$1 - x^2 y(x) + x^2 (y(x) - x) \left(\frac{d}{dx} y(x) \right) = 0 \quad (27)$$

> DerEcua := isolate(Ecua, diff(y(x), x))

$$DerEcua := \frac{d}{dx} y(x) = \frac{-1 + x^2 y(x)}{x^2 (y(x) - x)} \quad (28)$$

> DerSolGral := isolate(diff(SolGralFinal, x), diff(y(x), x))

$$DerSolGral := \frac{d}{dx} y(x) = \frac{y(x) - \frac{1}{x^2}}{y(x) - x} \quad (29)$$

> ComprobaFinal := simplify(rhs(DerEcua) - rhs(DerSolGral)) = 0

$$ComprobaFinal := 0 = 0 \quad (30)$$

> restart

> y[1] := sin(x - 3·Pi)

$$y_1 := -\sin(x) \quad (31)$$

> y[2] := sen(x - 3·Pi)

$$y_2 := \text{sen}(x - 3 \pi) \quad (32)$$

> y[3] := sin(x - 3·pi)

$$y_3 := -\sin(-x + 3 \pi) \quad (33)$$

> diff(y[1], x)

$$-\cos(x) \quad (34)$$

> diff(y[2], x)

$$D(\text{sen})(x - 3 \pi) \quad (35)$$

> diff(y[3], x)

$$\cos(-x + 3 \pi) \quad (36)$$

> evalf(Pi)

$$3.141592654 \quad (37)$$

> evalf(pi)

$$\pi \quad (38)$$

> evalf(PI)

$$\Pi \quad (39)$$

> restart

> Ecua[1] := y'' - 4·y' + 4·y = 0

$$Ecua_1 := \frac{d^2}{dx^2} y(x) - 4 \left(\frac{d}{dx} y(x) \right) + 4 y(x) = 0 \quad (40)$$

> Sol[1] := dsolve(Ecua[1])

$$Sol_1 := y(x) = _C1 e^{2x} + _C2 e^{2x} x \quad (41)$$

> Ecua[2] := y'' - 3·y' + 3·y = 0

$$Ecua_2 := \frac{d^2}{dx^2} y(x) - 3 \left(\frac{d}{dx} y(x) \right) + 3 y(x) = 0 \quad (42)$$

> Sol[2] := dsolve(Ecua[2])

$$Sol_2 := y(x) = _C1 e^{\frac{3}{2}x} \sin\left(\frac{1}{2} \sqrt{3} x\right) + _C2 e^{\frac{3}{2}x} \cos\left(\frac{1}{2} \sqrt{3} x\right) \quad (43)$$

$$> Ecua[3] := y'' - 2 \cdot y' + 2 \cdot y = 0$$

$$Ecua_3 := \frac{d^2}{dx^2} y(x) - 2 \left(\frac{d}{dx} y(x) \right) + 2 y(x) = 0 \quad (44)$$

$$> Sol[3] := dsolve(Ecua[3])$$

$$Sol_3 := y(x) = _C1 e^x \sin(x) + _C2 e^x \cos(x) \quad (45)$$

$$> Ecua[4] := y'' - y' + y = 0$$

$$Ecua_4 := \frac{d^2}{dx^2} y(x) - \left(\frac{d}{dx} y(x) \right) + y(x) = 0 \quad (46)$$

$$> Sol[4] := dsolve(Ecua[4])$$

$$Sol_4 := y(x) = _C1 e^{\frac{1}{2}x} \sin\left(\frac{1}{2} \sqrt{3} x\right) + _C2 e^{\frac{1}{2}x} \cos\left(\frac{1}{2} \sqrt{3} x\right) \quad (47)$$

$$> Ecua[5] := y'' - 5 \cdot y' + 6 y = 0$$

$$Ecua_5 := \frac{d^2}{dx^2} y(x) - 5 \left(\frac{d}{dx} y(x) \right) + 6 y(x) = 0 \quad (48)$$

$$> Sol[5] := dsolve(Ecua[5])$$

$$Sol_5 := y(x) = _C1 e^{2x} + _C2 e^{3x} \quad (49)$$

$$> restart$$

$$> Ecua_5 := \frac{d^2}{dx^2} y(x) - 5 \left(\frac{d}{dx} y(x) \right) + 6 y(x) = 10 \cdot \exp(3 \cdot x)$$

$$Ecua_5 := \frac{d^2}{dx^2} y(x) - 5 \left(\frac{d}{dx} y(x) \right) + 6 y(x) = 10 e^{3x} \quad (50)$$

$$> Sol_5 := y(x) = _C1 e^{2x} + _C2 e^{3x}$$

$$Sol_5 := y(x) = _C1 e^{2x} + _C2 e^{3x} \quad (51)$$

$$> yy[1] := e^{2x}; yy[2] := e^{3x}$$

$$yy_1 := e^{2x}$$

$$yy_2 := e^{3x} \quad (52)$$

$$> Q := 10 e^{3x}$$

$$Q := 10 e^{3x} \quad (53)$$

$$> with(linalg) :$$

$$> WW := array([[yy[1], yy[2]], [diff(yy[1], x), diff(yy[2], x)]])$$

$$WW := \begin{bmatrix} e^{2x} & e^{3x} \\ 2 e^{2x} & 3 e^{3x} \end{bmatrix} \quad (54)$$

$$> BB := array([0, Q])$$

$$BB := \begin{bmatrix} 0 & 10 e^{3x} \end{bmatrix} \quad (55)$$

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> Raiz := linsolve(WW, BB)
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$$Raiz := \begin{bmatrix} -\frac{10 e^{3x}}{e^{2x}} & 10 \end{bmatrix} \quad (56)$$

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> Aprima := Raiz[1]; Bprima := Raiz[2]
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$$Aprima := -\frac{10 e^{3x}}{e^{2x}} \quad (57)$$

$$Bprima := 10$$

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> SolGral := y(x) = simplify(subs(_C2 = _C20 + 10, simplify((int(Aprima, x) + _C1) · yy[1]
+ (int(Bprima, x) + _C2) · yy[2]))))
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$$SolGral := y(x) = e^{3x} _C20 + 10 e^{3x} x + _C1 e^{2x} \quad (58)$$

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> restart
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