```
> restart
> Ecua := diff(y(t), t$2) - 2·diff(y(t), t) + 2·y(t) = (t - 5)·Heaviside(t - 5)
```

$$Ecua := \frac{d^2}{dt^2} y(t) - 2\left(\frac{d}{dt} y(t)\right) + 2\,y(t) = (t-5)\,\text{Heaviside}(t-5) \tag{1}$$

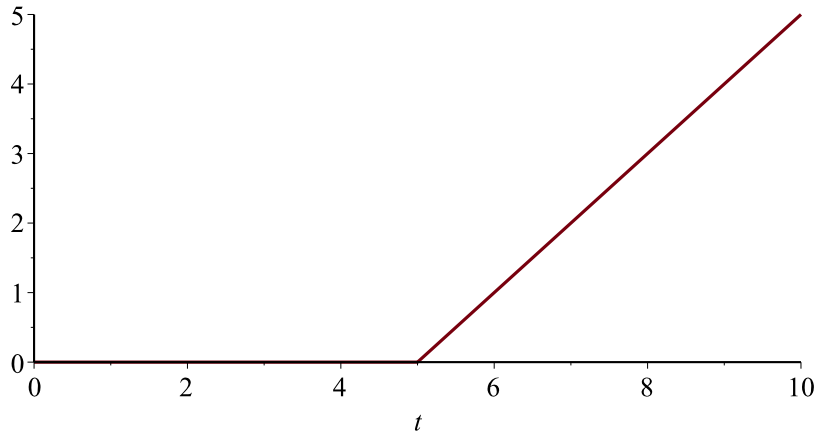
```
> Cond := y(0) = 0, D(y)(0) = 1
```

$$Cond := y(0) = 0,\ D(y)(0) = 1 \tag{2}$$

```
> Q := rhs(Ecua)
```

$$Q := (t-5)\,\text{Heaviside}(t-5) \tag{3}$$

```
> plot(Q, t = 0..10)
```




```
> with(inttrans) :
> EcuaLap := subs(Cond, laplace(Ecua, t, s))
```

$$EcuaLap := s^2\,laplace(y(t), t, s) - 1 - 2\,s\,laplace(y(t), t, s) + 2\,laplace(y(t), t, s) = \frac{e^{-5s}}{s^2} \tag{4}$$

```
> SolLap := isolate(EcuaLap, laplace(y(t), t, s))
```

$$SolLap := laplace(y(t), t, s) = \frac{\frac{e^{-5s}}{s^2} + 1}{s^2 - 2s + 2} \tag{5}$$

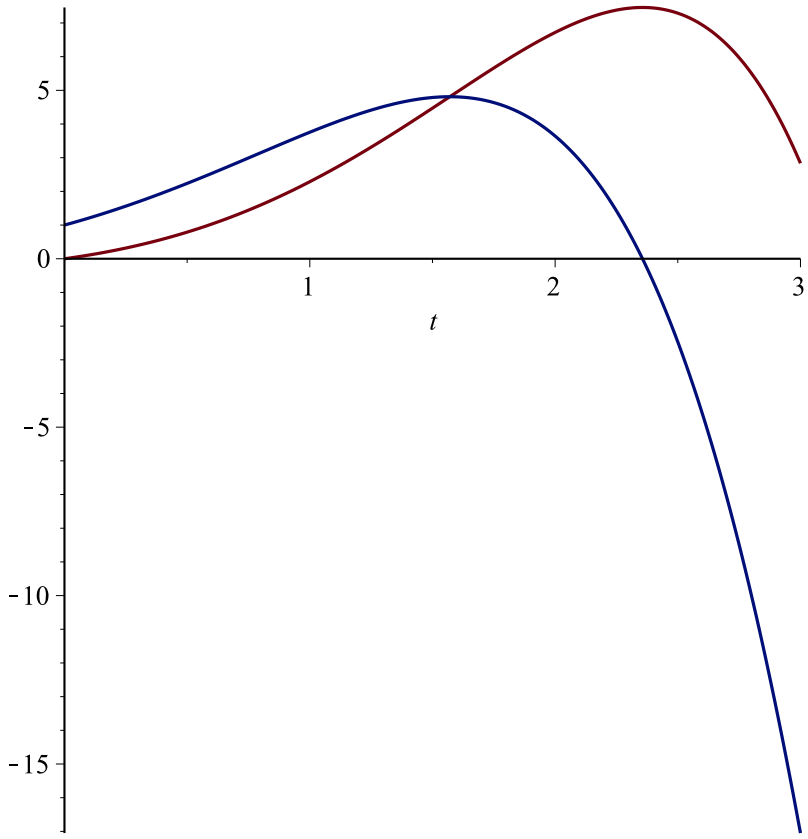
```
> SolLapFinal := simplify(SolLap)
```

$$SolLapFinal := laplace(y(t), t, s) = \frac{s^2 + e^{-5s}}{s^2\left(s^2 - 2s + 2\right)} \tag{6}$$

```
> SolPart := invlaplace(SolLap, s, t)
```

$$SolPart := y(t) = e^t \sin(t) + \frac{1}{2} e^{t-5} \cos(t-5)\,(-1 + \text{Heaviside}(-t+5)) + \frac{1}{2}\,\text{Heaviside}(t-5)\,(-4+t) \tag{7}$$

```
> plot([rhs(SolPart), rhs(diff(SolPart, t))], t = 0..3)
```




> *Ecua*

$$\frac{d^2}{dt^2} y(t) - 2\left(\frac{d}{dt} y(t)\right) + 2\,y(t) = (t-5)\,\text{Heaviside}(t-5) \tag{8}$$

> *SolPart*

$$y(t) = e^t \sin(t) + \frac{1}{2} e^{t-5} \cos(t-5)\,(-1 + \text{Heaviside}(-t+5)) + \frac{1}{2}\,\text{Heaviside}(t-5)\,(-4 + t) \tag{9}$$

> *CompUno* := *eval*(*subs*(*t* = 0, *SolPart*))

$$CompUno := y(0) = 0 \tag{10}$$

> *CompDos* := D(*y*)(0) = *eval*(*subs*(*t* = 0, *rhs*(*diff*(*SolPart*, *t*))))

$$CompDos := D(y)(0) = 1 \tag{11}$$

> *restart*

> *Sist* := *diff*(*y*[1](*t*), *t*) = 2·*y*[1](*t*) + 3·*y*[2](*t*), *diff*(*y*[2](*t*), *t*) = *y*[1](*t*) + 4·*y*[2](*t*) : *Sist*[1]; *Sist*[2]

$$\frac{d}{dt} y_1(t) = 2\,y_1(t) + 3\,y_2(t)$$

$$\frac{d}{dt} y_2(t) = y_1(t) + 4\,y_2(t) \tag{12}$$

> $Cond := y[1](0) = 1, y[2](0) = -1$

$$Cond := y_1(0) = 1, y_2(0) = -1 \tag{13}$$

> $AA := array([[2, 3], [1, 4]])$

$$AA := \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \tag{14}$$

> $Xcero := array([1, -1])$

$$Xcero := \begin{bmatrix} 1 & -1 \end{bmatrix} \tag{15}$$

> $with(linalg)$ :

> $MatExp := exponential(AA, t)$

$$MatExp := \begin{bmatrix} \frac{3}{4}\,e^{t} + \frac{1}{4}\,e^{5t} & \frac{3}{4}\,e^{5t} - \frac{3}{4}\,e^{t} \\ \frac{1}{4}\,e^{5t} - \frac{1}{4}\,e^{t} & \frac{1}{4}\,e^{t} + \frac{3}{4}\,e^{5t} \end{bmatrix} \tag{16}$$

> $SolPart := evalm(MatExp \&* Xcero) : y[1](t) = SolPart[1]; y[2](t) = SolPart[2]$

$$y_1(t) = \frac{3}{2}\,e^{t} - \frac{1}{2}\,e^{5t}$$

$$y_2(t) = -\frac{1}{2}\,e^{5t} - \frac{1}{2}\,e^{t} \tag{17}$$

> $restart$

> $Ecua := diff(z(x, y), x\$2) + 6 \cdot diff(z(x, y), y) = z(x, y)$

$$Ecua := \frac{\partial^2}{\partial x^2}\,z(x, y) + 6\left(\frac{\partial}{\partial y}\,z(x, y)\right) = z(x, y) \tag{18}$$

Con una constante de separación positiva

> $EcuaSeparable := eval(subs(z(x, y) = F(x) \cdot G(y), Ecua))$

$$EcuaSeparable := \left(\frac{d^2}{dx^2}\,F(x)\right) G(y) + 6\,F(x)\left(\frac{d}{dy}\,G(y)\right) = F(x)\,G(y) \tag{19}$$

> $EcuaSeparada := \frac{\left(lhs(EcuaSeparable) - 6\,F(x)\left(\frac{d}{dy}\,G(y)\right)\right)}{F(x) \cdot G(y)}$

$$= simplify\left(\frac{\left(rhs(EcuaSeparable) - 6\,F(x)\left(\frac{d}{dy}\,G(y)\right)\right)}{F(x) \cdot G(y)}\right)$$

$$EcuaSeparada := \frac{\frac{d^2}{dx^2}\,F(x)}{F(x)} = \frac{G(y) - 6\left(\frac{d}{dy}\,G(y)\right)}{G(y)} \tag{20}$$

> $EcuaX := lhs(EcuaSeparada) = \beta^2$

$$EcuaX := \frac{\frac{d^2}{dx^2}\,F(x)}{F(x)} = \beta^2 \tag{21}$$

> $EcuaY := rhs(EcuaSeparada) = \beta^2$

$$EcuaY := \frac{G(y) - 6\left(\frac{d}{dy}G(y)\right)}{G(y)} = \beta^2 \tag{22}$$

> $SolX := dsolve(EcuaX)$

$$SolX := F(x) = \_C1\, e^{-\beta x} + \_C2\, e^{\beta x} \tag{23}$$

> $SolY := dsolve(EcuaY)$

$$SolY := G(y) = \_C1\, e^{-\frac{1}{6}(\beta-1)(\beta+1)y} \tag{24}$$

> $SolGral := z(x,y) = rhs(SolX)\cdot subs(\_C1 = 1, rhs(SolY))$

$$SolGral := z(x,y) = \left(\_C1\, e^{-\beta x} + \_C2\, e^{\beta x}\right) e^{-\frac{1}{6}(\beta-1)(\beta+1)y} \tag{25}$$

> $Ecua$

$$\frac{\partial^2}{\partial x^2} z(x,y) + 6\left(\frac{\partial}{\partial y} z(x,y)\right) = z(x,y) \tag{26}$$

> $Comp := simplify(eval(subs(z(x,y) = rhs(SolGral), Ecua)))$

$$Comp := e^{-\beta x - \frac{1}{6}\beta^2 y + \frac{1}{6}y}\_C1 + e^{\beta x - \frac{1}{6}\beta^2 y + \frac{1}{6}y}\_C2 = \left(\_C1\, e^{-\beta x} + \_C2\, e^{\beta x}\right) e^{-\frac{1}{6}(\beta-1)(\beta+1)y} \tag{27}$$

> $ComDos := simplify(lhs(Comp) - rhs(Comp)) = 0$

$$ComDos := 0 = 0 \tag{28}$$

> $restart$

> $f := \exp(2\cdot t)$

$$f := e^{2t} \tag{29}$$

> $L := 2$

$$L := 2 \tag{30}$$

> $a[0] := \frac{1}{L}\cdot int(f, t = -L..L); C := \frac{a[0]}{2}$

$$a_0 := -\frac{1}{4} e^{-4} + \frac{1}{4} e^{4}$$

$$C := -\frac{1}{8} e^{-4} + \frac{1}{8} e^{4} \tag{31}$$

> $a[n] := subs\left(\sin(n\cdot Pi) = 0, \cos(n\cdot Pi) = (-1)^n, \frac{1}{L}\cdot int\left(f\cdot\cos\left(\frac{n\cdot Pi}{L}\cdot t\right), t = -L..L\right)\right)$

$$a_n := \frac{4 e^{4} (-1)^n - 4 e^{-4} (-1)^n}{\pi^2 n^2 + 16} \tag{32}$$

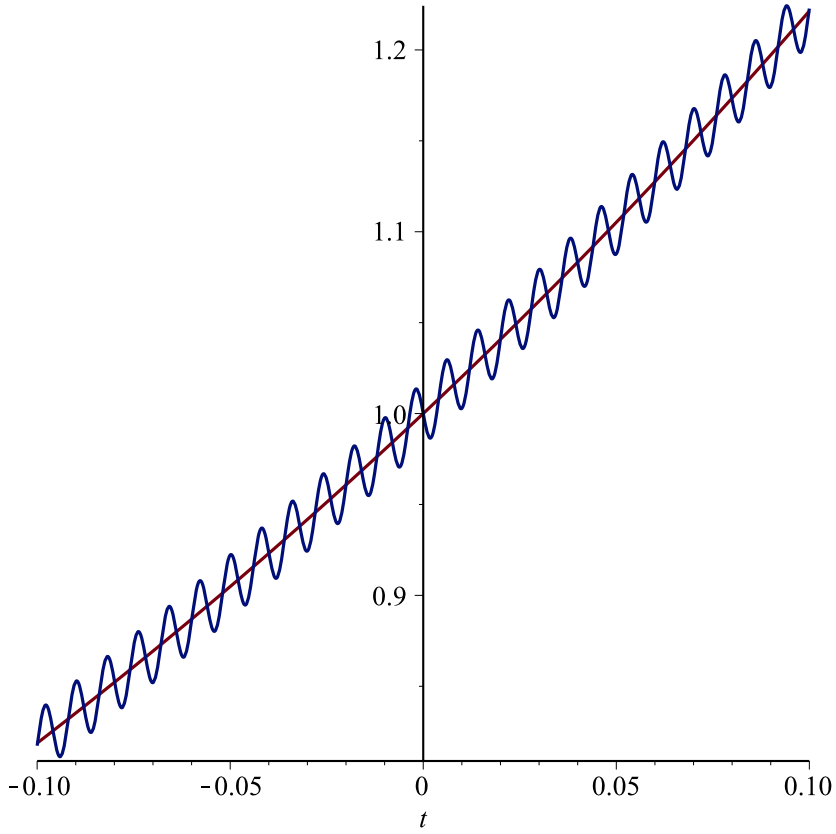
> $b[n] := subs\left(\sin(n\cdot Pi) = 0, \cos(n\cdot Pi) = (-1)^n, \frac{1}{L}\cdot int\left(f\cdot\sin\left(\frac{n\cdot Pi}{L}\cdot t\right), t = -L..L\right)\right)$

$$b_n := \frac{-e^{4} (-1)^n \pi n + e^{-4} (-1)^n \pi n}{\pi^2 n^2 + 16} \tag{33}$$

> $STFf := C + Sum\left(\left(a[n]\cdot\cos\left(\frac{n\cdot Pi}{L}\cdot t\right) + b[n]\cdot\sin\left(\frac{n\cdot Pi}{L}\cdot t\right)\right), n = 1..infinity\right)$

$$STFf := -\frac{1}{8}\,e^{-4} + \frac{1}{8}\,e^{4} + \sum_{n=1}^{\infty}\left(\frac{\left(4\,e^{4}\,(-1)^{n} - 4\,e^{-4}\,(-1)^{n}\right)\cos\left(\frac{1}{2}\,n\,\pi\,t\right)}{\pi^{2}\,n^{2}+16} + \frac{\left(-e^{4}\,(-1)^{n}\,\pi\,n + e^{-4}\,(-1)^{n}\,\pi\,n\right)\sin\left(\frac{1}{2}\,n\,\pi\,t\right)}{\pi^{2}\,n^{2}+16}\right) \quad \textbf{(34)}$$

```
> STF500 := C + sum( (a[n]·cos( (n·Pi/L)·t ) + b[n]·sin( (n·Pi/L)·t )), n = 1..500 ) :
> plot( [f, STF500], t = -0.1..0.1 )
```




```
> f
```

$$e^{2\,t} \quad \textbf{(35)}$$

```
> restart
>
```