Repaso Tema 3.- Sistemas de EDOL(1)cc
Transformada de Laplace.

$y(x)$ —— $\mathcal{L}\{y(x)\} = Y(s)$

$x \in \mathbb{R}$ $\quad y \in \mathbb{R}$ $\qquad\qquad$ $Y(s)$ $\quad s \in \mathbb{C}$ $\quad Y \in \mathbb{R}$

PROBLEMAS EDOL(n)cc con Condiciones Iniciales

$$\mathcal{L}\{a\,y(x) + b\,z(x)\} \longrightarrow a\,Y(s) + b\,Z(s)$$

$a, b \in \mathbb{R}$

Ecuación Diferencial $\qquad \mathcal{L}\{y'(x)\} \longrightarrow s\,Y(s) - f(0)$ $\qquad$ Ecuación Algebraica

$$\int y(x)\,dx \longleftarrow \frac{Y(s)}{s}$$

Propiedad superlativa:
Se puede obtener la transformada de funciones seccionalmente continuas.

$$u(t-a) = \begin{cases} 0 \; ; & t < a \\ 1 \; ; & t \geq a \end{cases}$$ escalón unitario
Heaviside $(t-a)$

$$r(t-a) = \begin{cases} 0 \; ; & t < a \\ (t-a) \; ; & t \geq a \end{cases}$$ rampa unitaria
$(t-a)\cdot$Heaviside$(t-a)$

$$\delta(t-a) = \begin{cases} 0 \; ; \; t \neq 0 \\ \int_{-\infty}^{\infty} \delta(t-a)\,dt = 1 \end{cases}$$ impulso unitario
Dirac $(t-a)$

$$y'' - 2y' + 2y = (t-5)\,u(t-5)$$

$y(0) = 0 \qquad y'(0) = 1$

$$\mathcal{L}\{y'' - 2y' + 2y\} = \mathcal{L}\{(t-5)\,u(t-5)\}$$

$$\mathcal{L}\{y''\} - 2\mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = \frac{e^{-5s}}{s^2}$$

$$\left[s^2 Y - s(0) - (1)\right] - 2\left[sY - (0)\right] + 2Y = \frac{e^{-5s}}{s^2}$$

$$(s^2-2s+2)Y - 1 = \frac{e^{-5s}}{s^2}$$

$$(s^2-2s+2)Y = \frac{e^{-5s}}{s^2} + 1$$

$$Y = \frac{\left(\frac{e^{-5s}}{s^2}+1\right)}{s^2-2s+2} \Rightarrow \frac{\left(\frac{e^{-5s}+s^2}{s^2}\right)}{s^2-2s+2}$$

$$= \frac{e^{-5s}+s^2}{s^2(s^2-2s+2)} = \frac{A}{s^2} + \frac{B}{s} + \frac{Cs+D}{s^2-2s+2}$$

$$e^{-5s}+s^2 = A(s^2-2s+2) + B(s^3-2s^2+2s) + (Cs+D)s^2$$

$$= (B+C)s^3 + (A-2B+D)s^2 + (-2A+2B)s + 2A$$

$B+C=0$ $\qquad C = -\frac{e^{-5s}}{2}$

$A-2B+D=1$ $\qquad -2B+D = 1-\frac{e^{-5s}}{2}$

$-2A+2B=0$ $\qquad A=B$

$2A = e^{-5s}$ $\longrightarrow$ $A = \frac{e^{-5s}}{2}$ $\quad B = \frac{e^{-5s}}{2}$

$$D = 1 - \frac{e^{-5s}}{2} + e^{-5s}$$

$$Y = \frac{e^{-5s}}{2}\left(\frac{1}{s^2}\right) + \frac{e^{-5s}}{2}\left(\frac{1}{s}\right) + \frac{-\frac{e^{-5s}}{2}s + \left(1+\frac{e^{-5s}}{2}\right)}{s^2-2s+2}$$

$$y(t) = \frac{1}{2}(t-5)u(t-5) + \frac{1}{2}u(t-5) + \left(-\frac{e^{-5s}}{2}s + 1 + \frac{e^{-5s}}{2}\right)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2-2s+2}\right\}$$

TEMA 3.- Sistemas de EDOL(1)cc

$$y_1'(t) = 2y_1(t) + 3y_2(t)$$
$$y_2'(t) = y_1(t) + 4y_2(t)$$

S(2)(EDOL(1)cc)H

$$y_1(0) = 1 \qquad y_2(0) = -1$$

$$\begin{cases} y_1(t) = y_2'(t) - 4y_2(t) \\ y_1'(t) = y_2''(t) - 4y_2'(t) \end{cases}$$

$$y_2''(t) - 4y_2'(t) = 2\left(y_2'(t) - 4y_2(t)\right) + 3y_2(t)$$

$$y_2''(t) - 6y_2'(t) + 5y_2(t) = 0$$

$$m^2 - 6m + 5 = 0 \qquad m_1 = 1$$
$$(m-1)(m-5) = 0 \qquad m_2 = 5$$

$$y_2(t) = C_1 e^{t} + C_2 e^{5t} \qquad -1 = C_1 + C_2$$

$$y_1(t) = (C_1 e^{t} + 5C_2 e^{5t}) - 4(C_1 e^{t} + C_2 e^{5t})$$

$$y_1(t) = -3C_1 e^{t} + C_2 e^{5t}$$

$$1 = -3C_1 + C_2$$
$$1 = -C_1 - C_2$$
$$2 = -4C_1$$

$$C_1 = -\tfrac{1}{2}$$
$$C_2 = -\tfrac{1}{2}$$

$$\boxed{\begin{aligned} y_1(t) &= \tfrac{3}{2}e^{t} - \tfrac{1}{2}e^{5t} \\ y_2(t) &= -\tfrac{1}{2}e^{t} - \tfrac{1}{2}e^{5t} \end{aligned}}$$

TEMA 4.- Ecuaciones Dif. en Der. Parciales

$$F\left(x, y, z(x,y), \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \dots\right) = 0$$

orden $\left\{ \begin{array}{l} \text{Hom.} \\ \text{Cuasi Hom} \\ \text{No Hom} \end{array} \right\}$

Método de Separación de Variables

H_0 : $z(x,y) = F(x) \cdot G(y) \longrightarrow$ # Soluciones Generales.

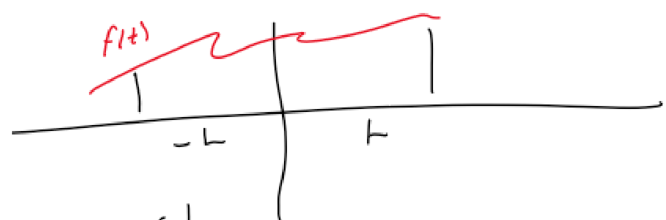
H_1 : $z(x,y) = F(x) + G(y)$

H_2 : $z(x,y) = F(x) \cdot \mathcal{L}G(y)$

H_3 : $= \mathcal{L}F(x) \cdot G_1(y)$

Serie Trigonométrica de FOURIER

$$f(t) = C + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi}{L} t\right) + b_n \operatorname{sen}\left(\frac{n\pi}{L} t\right) \right)$$



$$a_0 = \frac{1}{L} \int_{-L}^{L} f(t)\,dt \qquad C = \frac{a_0}{2}$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(t) \cos\left(\frac{n\pi}{L} t\right) dt$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(t) \operatorname{sen}\left(\frac{n\pi}{L} t\right) dt$$