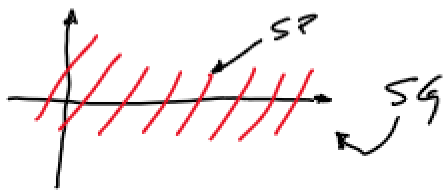
Clase 3: Ecuaciones Diferenciales

$y' = f(x,y)$ EDO(1) NL

Solución general única
soluciones particulares (∞)
soluciones singulares (#). (NL)

} TODAS EDO.



Teorema Existencia y unicidad sol. part. en un punto (x_0, y_0)

$y' = f(x,y)$ (x_0, y_0) cond. inicial

a) $f(x,y)$ x_0, y_0 &
b) $\frac{\partial f}{\partial y}$ x_0, y_0

entonces la solución particular existe y única en x_0, y_0

$y = C_1 x$ SG $y' = C_1$

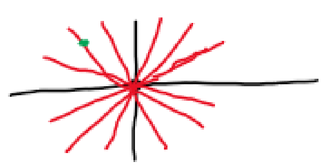
$C_1 = \frac{y}{x}$ $\quad y' = \frac{y}{x}$ EDO(1) L. CVH.

$f(x,y) = \frac{y}{x}$ $\quad x_0 = 0$

$\frac{\partial f}{\partial y} = \frac{1}{x} \longrightarrow x_0 = 0$

} NO PODEMOS GARANTIZAR QUE EXISTA SP, O SEA ÚNICA.

$x = -1$
$y = 1$



$$2y(y'+2)-x(y')^2=0 \quad \text{EDO(1) NL}$$

$$c_1 y-(c_1-x)^2=0 \quad \underline{\text{SG.}}$$

$$y_g=\frac{(c_1-x)^2}{c_1} \qquad y'=\frac{-2(c_1-x)}{c_1}$$

$c_1=1$

$$y_p=\frac{(1-x)^2}{1}$$

$c_1=2$

$$y_p=\frac{(2-x)^2}{2}$$

$c_1=-\sqrt{2}$

$$y_p=\frac{(-\sqrt{2}-x)^2}{-\sqrt{2}}$$

$$y_s=-4x \qquad y_s=0 \quad \underline{\text{SINGULARES}}$$

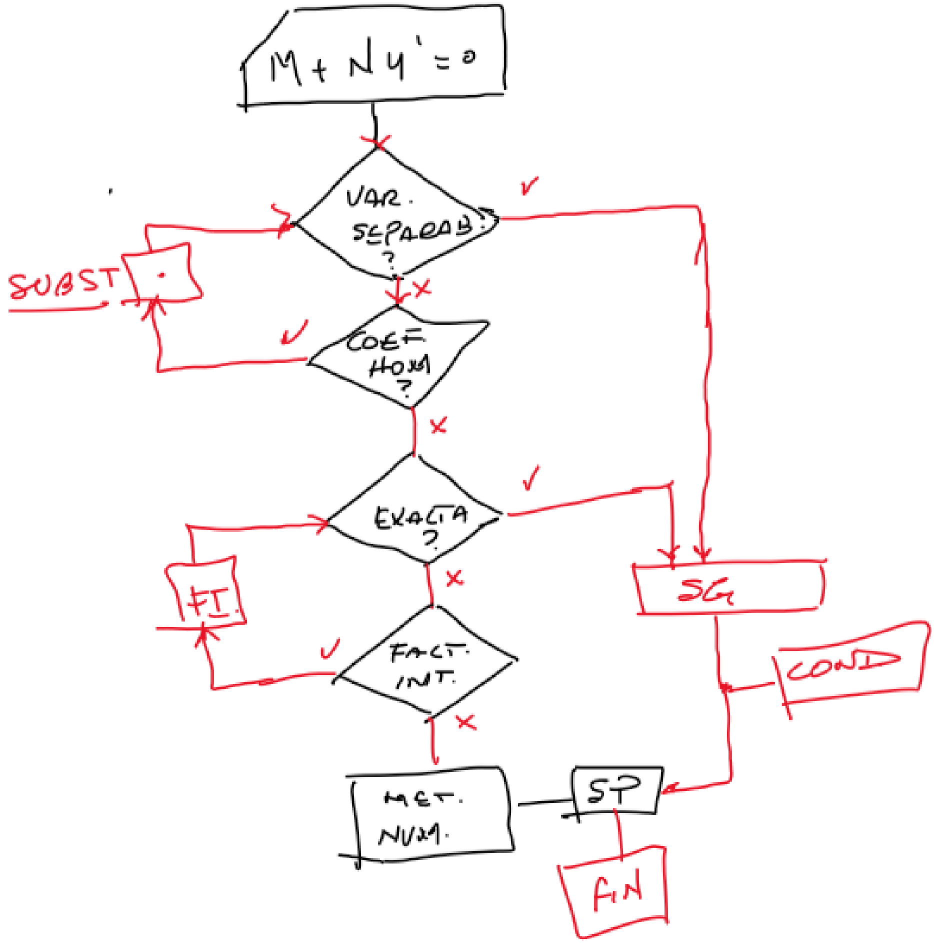
EDO(n)L — $\begin{cases} \text{SOL. GENERAL} \\ \text{SOL. PARTICULAR} \end{cases}$

$\underline{\text{EDO(n)NL}}$ — $\begin{cases} \text{EDO(n) L} \begin{cases} \text{SG} \\ \text{SP.} \end{cases} \\ \begin{cases} \text{SOL. GRAL.} \\ \text{SOL. PART.} \\ \text{SOL. SINGULARES} \end{cases} \end{cases}$

ECUACIONES PRIMER ORDEN.

→ NO LINEALES DE PRIMER ORDEN

$$M(x,y) + N(x,y)\,y' = 0 \quad \left\{ \text{forma general.} \right.$$



Método Variables Separables

$$M(x,y) + N(x,y)\,y' = 0$$

si $\quad P(x)Q(y) + R(x)S(y)\,y' = 0$

$$\text{Sol.} \Rightarrow \int \frac{P(x)}{R(x)}\,dx + \int \frac{S(y)}{Q(y)}\,dy = C_1$$

$$(y^2 + xy^2)\,y' = yx^2 - x^2$$

$$\underset{N}{} \qquad\qquad -M$$

$$(x^2 - x^2y) + (y^2 + xy^2)\,y' = 0$$

$$x^2(1-y) + (1+x)\,y^2 \cdot y' = 0$$

$$P(x) = x^2$$
$$Q(y) = 1-y$$
$$R(x) = 1+x$$
$$S(y) = y^2$$

$$\int \frac{x^2}{1+x}\,dx + \int \frac{y^2}{1-y}\,dy = C_1$$

$$SolGral := -\frac{1}{2}x^2 + x - \ln(x+1) + \frac{1}{2}y^2 + y + \ln(y-1) = \_C1$$

$$\underset{M}{y\,Ly} + \underset{N}{x}\,\frac{dy}{dx} = 0$$

$P(x) = 1$
$Q(y) = y\,Ly$
$R(x) = x$
$S(y) = 1$

Sq $$\int \frac{1}{x}\,dx + \int \frac{1}{y\,Ly}\,dy = C_1$$

$$L(x) + \int \left(\frac{\frac{dy}{y}}{Ly} \right) = C_1 \qquad \begin{cases} u = Ly \\ du = \frac{dy}{y} \end{cases}$$

$$L(x) + \int \frac{du}{u} = C_1$$

$$L(x) + L(u) = C_1$$

$$L(x) + L(Ly) = C_1$$

$$L(x\,Ly) = C_1$$

$$x\,Ly = e^{C_1}$$

$$x\,Ly = C_2$$

$$Ly = \frac{C_2}{x}$$

$$y = e^{\frac{C_2}{x}}$$