

$$\begin{aligned}
& \text{restart} \\
& \text{Ecua} := \text{sqrt}(x^2 + y^2) + y - x \cdot y' = 0 \\
& \text{Ecua} := \sqrt{x^2 + y(x)^2} + y(x) - x \left(\frac{d}{dx} y(x) \right) = 0 \quad (1) \\
& M := \sqrt{x^2 + y^2} + y; N := -x \\
& M := \sqrt{x^2 + y^2} + y \\
& N := -x \quad (2) \\
& M\text{lambda} := \text{factor}(\text{subs}(x = \text{lambda} \cdot x, y = \text{lambda} \cdot y, M)) \\
& M\text{lambda} := \sqrt{\lambda^2 (x^2 + y^2)} + \lambda y \quad (3) \\
& M\text{lambda} := \text{lambda} \cdot (\text{sqrt}(x^2 + y^2) + y) \\
& M\text{lambda} := \lambda (\sqrt{x^2 + y^2} + y) \quad (4) \\
& n := 1 \\
& n := 1 \quad (5) \\
& N\text{lambda} := \text{lambda}(-x) \\
& N\text{lambda} := \lambda(-x) \quad (6) \\
& m := 1 \\
& m := 1 \quad (7) \\
& \text{comprobar} := m - n = 0 \\
& \text{comprobar} := 0 = 0 \quad (8) \\
& \text{with(DEtools)} : \\
& \text{odeadvisor}(\text{Ecua}) \\
& [[_homogeneous, class A], _rational, _dAlembert] \quad (9) \\
& \text{Ecua} \\
& \sqrt{x^2 + y(x)^2} + y(x) - x \left(\frac{d}{dx} y(x) \right) = 0 \quad (10) \\
& \text{EcuaDos} := \text{simplify}(\text{isolate}(\text{eval}(\text{subs}(y(x) = u(x) \cdot x, \text{Ecua})), \text{diff}(u(x), x))) \\
& \text{EcuaDos} := \frac{d}{dx} u(x) = \frac{\sqrt{x^2 (u(x)^2 + 1)}}{x^2} \quad (11) \\
& \text{EcuaTres} := \text{diff}(u(x), x) = \frac{x \cdot \text{sqrt}(u(x)^2 + 1)}{x^2} \\
& \text{EcuaTres} := \frac{d}{dx} u(x) = \frac{\sqrt{u(x)^2 + 1}}{x} \quad (12) \\
& \text{odeadvisor}(\text{EcuaTres}) \\
& [_separable] \quad (13) \\
& \text{SolGral} := \text{Int}\left(-\frac{1}{x}, x\right) + \text{Int}\left(\frac{1}{\text{sqrt}(u^2 + 1)}, u\right) = _C1 \\
& \text{SolGral} := \int \left(-\frac{1}{x}\right) dx + \int \frac{1}{\sqrt{u^2 + 1}} du = _C1 \quad (14)
\end{aligned}$$

$$\begin{aligned} &> \text{SolGralUno} := \text{int}\left(-\frac{1}{x}, x\right) + \text{int}\left(\frac{1}{\text{sqrt}(u^2 + 1)}, u\right) = _CI \\ &\quad \text{SolGralUno} := -\ln(x) + \text{arcsinh}(u) = _CI \end{aligned} \quad (15)$$

$$\begin{aligned} &> \text{SolGralDos} := \text{isolate}\left(\text{subs}\left(u = \frac{y}{x}, \text{SolGralUno}\right), y\right) \\ &\quad \text{SolGralDos} := y = \sinh(_CI + \ln(x)) \, x \end{aligned} \quad (16)$$

$$\begin{aligned} &> \text{SolGralFinal} := y(x) = \sinh(_CI + \ln(x)) \, x \\ &\quad \text{SolGralFinal} := y(x) = \sinh(_CI + \ln(x)) \, x \end{aligned} \quad (17)$$

$$\begin{aligned} &> \text{DerSolGral} := \text{diff}(\text{SolGralFinal}, x) \\ &\quad \text{DerSolGral} := \frac{d}{dx} y(x) = \cosh(_CI + \ln(x)) + \sinh(_CI + \ln(x)) \end{aligned} \quad (18)$$

$$\begin{aligned} &> \text{Ecua} \\ &\quad \sqrt{x^2 + y(x)^2} + y(x) - x \left(\frac{d}{dx} y(x) \right) = 0 \end{aligned} \quad (19)$$

$$\begin{aligned} &> \text{Comprobacion} := \text{simplify}(\text{eval}(\text{subs}(y(x) = \text{rhs}(\text{SolGralFinal}), \text{Ecua}))) \\ &\quad \text{Comprobacion} := -x \cosh(_CI + \ln(x)) + \sqrt{x^2 \cosh(_CI + \ln(x))^2} = 0 \end{aligned} \quad (20)$$

$$\begin{aligned} &> \text{SolucionGeneral} := \log\left(\frac{1}{x}\right) + \cos^{-1}\left(\frac{y(x)}{x}\right) = _CI \\ &\quad \text{SolucionGeneral} := \ln\left(\frac{1}{x}\right) + \arccos\left(\frac{y(x)}{x}\right) = _CI \end{aligned} \quad (21)$$

$$\begin{aligned} &> \text{DerSolGeneral} := \text{isolate}(\text{diff}(\text{SolucionGeneral}, x), \text{diff}(y(x), x)) \\ &\quad \text{DerSolGeneral} := \frac{d}{dx} y(x) = \left(-\frac{\sqrt{1 - \frac{y(x)^2}{x^2}}}{x} + \frac{y(x)}{x^2} \right) x \end{aligned} \quad (22)$$

$$\begin{aligned} &> \text{DerEcua} := \text{isolate}(\text{Ecua}, \text{diff}(y(x), x)) \\ &\quad \text{DerEcua} := \frac{d}{dx} y(x) = -\frac{-\sqrt{x^2 + y(x)^2} - y(x)}{x} \end{aligned} \quad (23)$$

$$\begin{aligned} &> \text{ComprobacionDos} := \text{simplify}(\text{rhs}(\text{DerEcua}) - \text{rhs}(\text{DerSolGeneral})) = 0 \\ &\quad \text{ComprobacionDos} := \frac{\sqrt{-\frac{y(x)^2 - x^2}{x^2}} \, x + \sqrt{x^2 + y(x)^2}}{x} = 0 \end{aligned} \quad (24)$$

> restart

$$\begin{aligned} &> \text{Ecua} := y' = \frac{2 \cdot x \cdot y}{(3 \cdot x^2 - y^2)} \\ &\quad \text{Ecua} := \frac{d}{dx} y(x) = \frac{2 \, x \, y(x)}{3 \, x^2 - y(x)^2} \end{aligned} \quad (25)$$

> with(DEtools) :

$$\begin{aligned} &> \text{odeadvisor}(\text{Ecua}) \\ &\quad [[_homogeneous, class A], _rational, _dAlembert] \end{aligned} \quad (26)$$

$$\begin{aligned} &> \text{EcuaDos} := \text{simplify}(\text{isolate}(\text{eval}(\text{subs}(y(x) = u(x) \cdot x, \text{Ecua})), \text{diff}(u(x), x))) \\ &\quad \text{EcuaDos} := \text{simplify}(\text{isolate}(\text{eval}(\text{subs}(y(x) = u(x) \cdot x, \text{Ecua})), \text{diff}(u(x), x))) \end{aligned} \quad (27)$$

$$EcuaDos := \frac{d}{dx} u(x) = - \frac{u(x) (u(x)^2 - 1)}{x (u(x)^2 - 3)} \quad (27)$$

> with(DEtools) :

$$\text{odeadvisor}(EcuaDos) \quad [_{separable}] \quad (28)$$

$$Q := \frac{u \cdot (u^2 - 1)}{(u^2 - 3)} \quad Q := \frac{u (u^2 - 1)}{u^2 - 3} \quad (29)$$

$$R := x \quad R := x \quad (30)$$

$$\text{SolGral} := \text{Int}\left(\frac{1}{R}, x\right) + \text{Int}\left(\frac{1}{Q}, u\right) = _C1$$

$$\text{SolGral} := \int \frac{1}{x} dx + \int \frac{u^2 - 3}{u (u^2 - 1)} du = _C1 \quad (31)$$

$$\text{SolGralDos} := \text{int}\left(\frac{1}{R}, x\right) + \text{int}\left(\frac{1}{Q}, u\right) = _C1$$

$$\text{SolGralDos} := \ln(x) + 3 \ln(u) - \ln(u - 1) - \ln(u + 1) = _C1 \quad (32)$$

$$\text{SolGralFinal} := \text{subs}\left(u = \frac{y}{x}, \text{SolGralDos}\right)$$

$$\text{SolGralFinal} := \ln(x) + 3 \ln\left(\frac{y}{x}\right) - \ln\left(\frac{y}{x} - 1\right) - \ln\left(\frac{y}{x} + 1\right) = _C1 \quad (33)$$

$$\text{SolGralFinalDos} := \text{simplify}(\exp(\text{lhs}(\text{SolGralFinal}))) = _C2$$

$$\text{SolGralFinalDos} := - \frac{y^3}{x^2 - y^2} = _C2 \quad (34)$$

$$\text{SolGralFinalTres} := - \frac{y(x)^3}{x^2 - y(x)^2} = _C2$$

$$\text{SolGralFinalTres} := - \frac{y(x)^3}{x^2 - y(x)^2} = _C2 \quad (35)$$

$$\text{DerSolGral} := \text{isolate}(\text{diff}(\text{lhs}(\text{SolGralFinalTres}), x) = 0, \text{diff}(y(x), x))$$

$$\text{DerSolGral} := \frac{d}{dx} y(x) = - \frac{2 y(x) x}{y(x)^2 - 3 x^2} \quad (36)$$

$$Ecua \quad \frac{d}{dx} y(x) = \frac{2 x y(x)}{3 x^2 - y(x)^2} \quad (37)$$

$$\text{Comprobacion} := \text{simplify}(\text{rhs}(Ecua) - \text{rhs}(\text{DerSolGral})) = 0$$

$$\text{Comprobacion} := 0 = 0 \quad (38)$$

> restart

$$\text{IntegralU} := \text{int}\left(\left(\frac{3 - u^2}{u - u^2}\right), u\right)$$

$$\text{IntegralU} := u - 2 \ln(u - 1) + 3 \ln(u) \quad (39)$$

$\left[\begin{array}{l} \textcolor{red}{>} \\ \text{=} \\ \textcolor{red}{>} \end{array} \right]$

$$SolGral := int\left(\frac{1}{x}, x\right) + subs\left(u = \frac{y}{x}, IntegralU\right) = _CI$$

$$\textcolor{blue}{SolGral := \ln(x) + \frac{y}{x} - 2 \ln\left(\frac{y}{x} - 1\right) + 3 \ln\left(\frac{y}{x}\right) = _CI}$$

(40)