

## CLASE 04 - LA ECUACIÓN NO LINEAL PRIMER ORDEN (CONTINUÁ)

$$\text{EDO(1)NL} \quad M(x, y) + N(x, y) y' = 0$$

$$y' = - \frac{M(x, y)}{N(x, y)}$$

Método: ¿Variables separables?

$$\text{si } \left\{ \begin{array}{l} M(x, y) \Rightarrow P(x) \cdot Q(y) \\ N(x, y) \Rightarrow R(x) \cdot S(y) \end{array} \right\} \text{ entonces.}$$

$$\text{SOL. GENERAL} \Rightarrow \int \frac{P(x)}{R(x)} dx + \int \frac{S(y)}{Q(y)} dy = C_1$$

Método: COEFICIENTES HOMOGÉNEOS

$$M(x, y) + N(x, y) y' = 0$$

¿Sies de Coef. Hom?

$$\text{si } \begin{array}{l} x \rightarrow \lambda x \\ y \rightarrow \lambda y \end{array} \quad \begin{array}{l} M(\lambda x, \lambda y) = \lambda^m M(x, y) \\ N(\lambda x, \lambda y) = \lambda^n N(x, y) \end{array} \quad \& \quad m=n \text{ entonces.}$$

se afirma que EDO(1)NL es de C. H.

$$\left[ u = \frac{y}{x} \rightarrow y = u \cdot x \quad y' = u'x + u \right]$$

$$\left[ M(x, ux) + N(x, ux)(u'x + u) = 0 \right]$$

Mágicamente se vuelve de V.S.

$$\sqrt{x^2+y^2}+y - x y' = 0 \quad \text{Ecuación}$$

$$M(\lambda x, \lambda y) = \sqrt{(\lambda x)^2 + (\lambda y)^2} + (\lambda y) \Rightarrow \sqrt{\lambda^2(x^2+y^2)} + \lambda y$$

coef.  
hom.

$$= \sqrt{\lambda^2} \cdot \sqrt{x^2+y^2} + \lambda y$$

$$= \lambda \sqrt{x^2+y^2} + \lambda y \Rightarrow \lambda (\sqrt{x^2+y^2} + y) \quad m=1$$

$$N(\lambda x, \lambda y) = -(\lambda x) \Rightarrow \lambda(-x); \quad n=1; \quad m=n;$$

$$y = m \cdot x \rightarrow y' = m'x + m$$

$$\sqrt{x^2+(mx)^2} + (mx) - x(m'x+m) = 0$$

$$\sqrt{x^2+u^2x^2} + ux - u'x^2 - ux = 0$$

$$\sqrt{x^2(u^2+1)} - u'x^2 = 0$$

$$\sqrt{x^2} \cdot \sqrt{u^2+1} - u'x^2 = 0$$

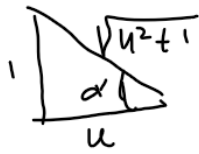
$$x \sqrt{u^2+1} - u'x^2 = 0$$

$$-\frac{1}{x} + \frac{u'}{\sqrt{u^2+1}} = 0$$

$$-\frac{1}{x} dx + \frac{1}{\sqrt{u^2+1}} du = 0$$

$$-\int \frac{dx}{x} + \int \frac{du}{\sqrt{u^2+1}} = C_1$$

$$- \ln x + \int \frac{du}{\sqrt{u^2+1}} = C_1$$



$$\frac{u}{1} = \cos(\alpha) \quad du = -\sin(\alpha) d\alpha$$

$$\frac{1}{\sqrt{u^2+1}} = \sin(\alpha)$$

$$\int \frac{-\sin(\alpha) d\alpha}{\sin(\alpha)} = \int -d\alpha \Rightarrow -\alpha$$

$$\alpha = \arccos(u)$$

$$-\ln x + \arccos\left(\frac{y}{x}\right) = C_1$$

$$u = \frac{y}{x}$$

SOL  
G.EAL

$$\ln\left(\frac{1}{x}\right) + \arccos\left(\frac{y}{x}\right) = C_1$$

$$\text{EDO(1)NL} \quad y' = \frac{2xy}{3x^2 - y^2}$$

$$-2xy + (3x^2 - y^2) y' = 0$$

$$M(x, y) = 2(xy) \Rightarrow \lambda^2(2xy) \quad m=2$$

$$N(x, y) = 3(x)^2 - (y)^2 \Rightarrow \lambda^2(3x^2 - y^2) \quad n=2 \quad m=n$$

$$y = ux$$

$$y' = u'x + u$$

CH.

$$\begin{aligned}
 -2x(ux) + (3x^2 - (ux)^2)(u'x + u) &= 0 \\
 -2ux^2 + (3x^2 - u^2x^2)u'x + u(3x^2 - u^2x^2) &= 0 \\
 -2ux^2 + 3x^2u - u^2x^2 + (x^2(3 - u^2))u'x &= 0 \\
 x^2u - u^2x^2 + x^3(3 - u^2)u' &= 0 \\
 x^2(-u^2 + u) + x^3(3 - u^2)u' &= 0
 \end{aligned}$$

$$\boxed{\frac{1}{x} + \frac{3 - u^2}{-u^2 + u} \frac{du}{dx} = 0} \quad \text{VS.}$$

$$\frac{dx}{x} + \frac{3 - u^2}{u - u^2} du = 0$$

$$\begin{aligned}
 \text{SOL GRAL} \quad \int \frac{dx}{x} + \int \frac{3 - u^2}{u - u^2} du &= C_1 & \text{sust}^1 \\
 & & u = \frac{y}{x}
 \end{aligned}$$

$$\begin{array}{l} \text{SOL GRAL} \\ \hline \text{BONITA} \end{array} \rightarrow \boxed{\frac{y^3}{y^2 - x^2} = C_1}$$

$$\int \frac{3 - u^2}{u - u^2} du = u + 3\ln u - 2\ln(u-1)$$

$$\text{sust. } u = \frac{y}{x}$$

$$\text{SOL GRAL} \rightarrow \text{FEA.} \quad \ln(x) + \frac{y}{x} + 3\ln\left(\frac{y}{x}\right) - 2\ln\left(\frac{y}{x} - 1\right) = C_1$$