Clase 05 EDO(1)NL "EXACTA."

SOLUCIÓN GENERAL

$$\boxed{x^2 + x^3y^2 + 6x^2y^3 - 5y^3 = C_1}$$

$$x^2 + x^3y^2 + 6x^2y^3 - 5y^3$$

$\int M\,dx$ $\qquad$ $\int N\,dy$

$\int M\,dx \cap \int N\,dy$

$$\begin{cases} \dfrac{d}{dx}F(x,y) = \dfrac{d}{dx}(C_1) \\[2mm] \dfrac{\partial F}{\partial x} + \dfrac{\partial F}{\partial y} \cdot \dfrac{dy}{dx} = 0 \end{cases}$$

$$(2x + 3x^2y^2 + 12xy^3 + (0)) + ((0) + 2x^3y + 18x^2y^2 - 15y^2)\frac{dy}{dx} = 0$$

$$\underbrace{(2x + 3x^2y^2 + 12xy^3)}_{M(x,y)} + \underbrace{(2x^3y + 18x^2y^2 - 15y^2)}_{N(x,y)}\frac{dy}{dx} = 0$$

$$\frac{\partial^2 F}{\partial x \partial y} \equiv \frac{\partial^2 F}{\partial y \partial x}$$

$$\frac{\partial M}{\partial y} = (0) + 6x^2y + 36xy^2 \qquad \frac{\partial N}{\partial x} = 6x^2y + 36xy^2 + (0)$$

Esta EDO(1) NL Es <u>EXACTA</u>

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$SG \quad \left[\int M(x,y)\,dx\right] \cup \left[\int N(x,y)\,dy\right] = C_1$$

$$\int M\,dx = \int (2x + 3x^2y^2 + 12xy^3)\,dx$$
$$= 2\int x\,dx + 3y^2\int x^2\,dx + 12y^3\int x\,dx$$
$$= x^2 + x^3y^2 + 6x^2y^3$$

$$\int N\,dy = \int (2x^3y + 18x^2y^2 - 15y^2)\,dy$$
$$= 2x^3\int y\,dy + 18x^2\int y^2\,dy - 15\int y^2\,dy$$
$$= x^3y^2 + 6x^2y^3 - 5y^3$$

$$SG \Rightarrow \boxed{x^3y^2 + 6x^2y^3 + x^2 - 5y^3 = C_1}$$

$$M + N\frac{dy}{dx} = 0 \qquad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \qquad \text{EXACTA.}$$

$$SG \Rightarrow \quad \int M\,dx + \int\left[N - \frac{\partial}{\partial y}\int M\,dx\right]dy = C_1$$

$$SG \Rightarrow \quad \int N\,dy + \int\left[M - \frac{\partial}{\partial x}\int N\,dy\right]dx = C_1$$

$$\left(\frac{x}{\sqrt{x^2+y^2}}+\frac{1}{x}+\frac{1}{y}\right)+\left(\frac{y}{\sqrt{x^2+y^2}}+\frac{1}{y}-\frac{x}{y^2}\right)\frac{dy}{dx}=0$$

EDO(1)NL

$$M=\frac{x}{\sqrt{x^2+y^2}}+\frac{1}{x}+\frac{1}{y} \qquad \frac{\partial M}{\partial y}=x\,\frac{\partial}{\partial y}\left(\frac{1}{\sqrt{x^2+y^2}}\right)+(0)+\frac{\partial}{\partial y}\left(\frac{1}{y}\right)$$

$$=x\left(\frac{-y}{(x^2+y^2)^{3/2}}\right)-\frac{1}{y^2}$$

$$N=\frac{y}{\sqrt{x^2+y^2}}+\frac{1}{y}-\frac{x}{y^2} \qquad \frac{\partial N}{\partial x}=y\left(\frac{-x}{(x^2+y^2)^{3/2}}\right)+(0)-\frac{1}{y^2}$$

EDO(1)NL es EXACTA.

$$\int M\,dx=\int\frac{x}{\sqrt{x^2+y^2}}\,dx+\int\frac{dx}{x}+\frac{1}{y}\int dx$$

$$=\sqrt{x^2+y^2}+Lx+\frac{x}{y}$$

$$\left[\int M\,dx\right]\cup\left[\int N\,dy\right]=C_1$$

$$\int N\,dy=\int\frac{y}{\sqrt{x^2+y^2}}+\int\frac{dy}{y}-x\int\frac{dy}{y^2}$$

$$=\sqrt{x^2+y^2}+Ly+\frac{x}{y}$$

$$SG \Rightarrow \boxed{\sqrt{x^2+y^2}+\frac{x}{y}+Lx+Ly=C_1}$$

Método del FACTOR INTEGRANTE

$$x^2y + x^3y^2 + x^4y^3 = C_1$$ SOLUCIÓN GENERAL

$F(x,y) = C_1$

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} \cdot \frac{dy}{dx} = 0$$ EDO(1) NL.

EXACTA. $$\underbrace{(2xy + 3x^2y^2 + 4x^3y^3)}_{M} + \underbrace{(x^2 + 2x^3y + 3x^4y^2)}_{N}\frac{dy}{dx} = 0$$

$$\frac{\partial M}{\partial y} = 2x + 6x^2y + 12x^3y^2$$

$$\frac{\partial N}{\partial x} = 2x + 6x^2y + 12x^3y^2$$

EXACTA $$x(2y + 3xy^2 + 4x^2y^3) + x(x + 2x^2y + 3x^3y^2)\frac{dy}{dx} = 0$$

$\downarrow$

NO EXACTA. $$\underbrace{2y + 3xy^2 + 4x^2y^3}_{MM} + \underbrace{(x + 2x^2y + 3x^3y^2)}_{NN}\frac{dy}{dx} = 0$$

$$\frac{\partial MM}{\partial y} = 2 + 6xy + 12x^2y^2$$

$$\frac{\partial MM}{\partial y} \neq \frac{\partial NN}{\partial x} \qquad \frac{\partial NN}{\partial x} = 1 + 4xy + 9x^2y^2$$

$$M + N\frac{dy}{dx} = 0 \qquad \text{EDO(1)NL}$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \qquad \text{NO EXACTA.}$$

$$\mu(x,y)M(x,y) + \mu(x,y)N(x,y)\frac{dy}{dx} = 0$$

FACTOR INTEGRANTE.

EXACTA.

$$\frac{\partial}{\partial y}(\mu M) = \frac{\partial}{\partial x}(\mu N)$$

$$\frac{\partial \mu}{\partial y}M + \mu\frac{\partial M}{\partial y} = \frac{\partial \mu}{\partial x}N + \mu\frac{\partial N}{\partial x}$$

$\mu(x,y) \leftarrow$ incógnita

datos conocidos: M, N, $\frac{\partial M}{\partial y}$, $\frac{\partial N}{\partial x}$

ED en DP que CAPITULO IV

$\mu(x) = x \qquad \mu(y)$