Clase 06 EDO(1)NL Factor Integrante

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

NO EXACTA

existe una función $\mu(x,y)$ conocida como "factor integrante" tal que si lo multiplico por toda la EDO(1)NL se vuelve EXACTA.

$$\mu(x,y)M(x,y) + \mu(x,y)N(x,y)\frac{dy}{dx} = 0$$

EXACTA.

¿Como encontramos F.I.?

EDP. $$\frac{\partial}{\partial y}(\mu M) = \frac{\partial}{\partial x}(\mu N)$$

Cap. IV.

1) si $\mu(x)$ sólo de "x"

$$\frac{\partial \mu}{\partial y}M + \mu\frac{\partial M}{\partial y} = \frac{\partial \mu}{\partial x}N + \mu\frac{\partial N}{\partial x}$$

$$(0) + \mu\frac{\partial M}{\partial y} - \mu\frac{\partial N}{\partial x} = \frac{d\mu}{dx}N$$

$$\mu\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)dx = d\mu\, N$$

$$\frac{d\mu}{\mu} = \left(\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}\right)dx.$$

Sea $f(x)$

$$(x+y^2) - 2xy\frac{dy}{dx} = 0$$

$$M = x + y^2 \quad N = -2xy$$

$$\frac{\partial M}{\partial y} = +2y \quad \frac{\partial N}{\partial x} = -2y$$ NO ES EXACTA.

$$f(x) = \frac{2y-(-2y)}{-2xy} \Rightarrow \frac{4y}{-2xy} = -\frac{2}{x}$$

$$\int \frac{d\mu}{\mu} = \int f(x)dx$$

$$\ln \mu = \int \frac{-2}{x} dx$$

$$\ln \mu = -2\ln x$$

$$\ln \mu = \ln\left(\frac{1}{x^2}\right)$$

$$\mu(x) = \frac{1}{x^2}$$ FACTOR INTEGRANTE.

$$\frac{1}{x^2}(x+y^2) - \frac{1}{x^2}(2xy)\frac{dy}{dx} = 0$$

$$\left(\frac{1}{x} + \frac{y^2}{x^2}\right) - \frac{2y}{x}\frac{dy}{dx} = 0$$

$$\quad MM \qquad\qquad NN$$

$$\frac{\partial MM}{\partial y} = \frac{2y}{x^2} \qquad \frac{\partial NN}{\partial x} = -2y\frac{d}{dx}\left(x^{-1}\right)$$

$$= -2y\left(-1 \cdot x^{-2}\right)$$

ES EXACTA.

$$= \frac{2y}{x^2}$$

$$\left[\int MM\,dx\right] \cup \left[\int NN\,dy\right] = C_1$$

$$\int MM\,dx = \int \frac{dx}{x} + y^2\int\frac{dx}{x^2}$$

$$= \ln x + y^2\left(-\frac{1}{x}\right)$$

$$= \ln x - \frac{y^2}{x}$$

$$\int NN\,dy = -\frac{2}{x}\int y\,dy$$

$$= -\frac{y^2}{x}$$

SOLUCIÓN GENERAL $\quad \ln x - \frac{y^2}{x} = C_1$

Si $\mu(y)$

$$\frac{\partial \mu}{\partial y}M + \mu\frac{\partial M}{\partial y} = \frac{\partial \mu}{\partial x}N + \mu\frac{\partial N}{\partial x}$$

$$\frac{d\mu}{dy}M + \mu\frac{\partial M}{\partial y} = (0) + \mu\frac{\partial N}{\partial x}$$

$$\frac{d\mu}{dy}M = \mu\frac{\partial N}{\partial x} - \mu\frac{\partial M}{\partial y}$$

$$\frac{d\mu}{\mu} = \left(\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} \right) dy$$

$g(y)$

$$\int \frac{d\mu}{\mu} = \int g(y)\,dy$$

$$\underbrace{(2xy^2 - 3y^3)}_{M} + \underbrace{(7 - 3xy^2)}_{N} \frac{dy}{dx} = 0$$

$$\frac{\partial M}{\partial y} = 4xy - 9y^2 \qquad \frac{\partial N}{\partial x} = (0) - 3y^2$$

NO EXACTA.

$$g(y) = \left(\frac{-3y^2 - 4xy + 9y^2}{2xy^2 - 3y^3} \right)$$

$$= \frac{-4xy + 6y^2}{2xy^2 - 3y^3} \Rightarrow \frac{2\cancel{(-2xy + 3y^2)}}{-y\cancel{(-2xy + 3y^2)}}$$

$$g(y) = -\frac{2}{y} \checkmark$$

$$\int \frac{d\mu}{\mu} = -2\int \frac{dy}{y}$$

$$L\mu = -2Ly$$

$$L\mu = L\left(\frac{1}{y^2}\right)$$

$$\mu(y) = \frac{1}{y^2}$$

$$\frac{1}{y^2}\left(2xy^2-3y^3\right)+\frac{1}{y^2}\left(7-3xy^2\right)\frac{dy}{dx}=0$$

$$\underbrace{(2x-3y)}_{MM}+\underbrace{\left(\frac{7}{y^2}-3x\right)}_{NN}\frac{dy}{dx}=0$$

$$\frac{\partial MM}{\partial y}=(0)-3 \qquad \frac{\partial NN}{\partial x}=(0)-3$$

EXACTA.

$$\int MM\,dx = 2\int x\,dx - 3y\int dx$$

$$= x^2-3xy$$

$$\int NN\,dy = 7\int\frac{dy}{y^2}-3x\int dy$$

$$=-\frac{7}{y}-3xy$$

SOL GRAL $\boxed{x^2-3xy-\frac{7}{y}=C_1}$

Cerramos EDO(1) NL.

EDO(1)L cv. H.

$$\frac{dy}{dx} + p(x)y = 0$$

$$\frac{dy}{y} = -p(x)dx$$

$$\int \frac{dy}{y} = \int -p(x)dx$$

$$Ly = \int -p(x)dx + C_1$$

$$y = e^{-\int p(x)dx + C_1}$$

$$y = e^{C_1} e^{-\int p(x)dx}$$

$$y = C_2 e^{-\int p(x)dx.}$$

$$\frac{dy}{dx} - \frac{y}{x} = 0$$

$$\frac{dy}{dx} = \frac{y}{x}$$

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$Ly = Lx + LC_1$$

$$y = C_1 x$$

$$\frac{dy}{dx} = C_1$$

$$\frac{dy}{dx} - \frac{y}{x} = 0 \qquad p(x) = -\frac{1}{x}$$

$$y = C_2 e^{-\int p(x)dx}$$

$$= C_2 e^{+\int \frac{dx}{x}}$$

$$= C_2 e^{\ln x}$$

$$y = C_2 x$$

$$-\frac{y}{x} + \frac{dy}{dx} = 0$$

$M \quad N$

$$\frac{\partial M}{\partial y} = -\frac{1}{x} \qquad \frac{\partial N}{\partial x} = 0$$ NO ES EXACTA.

$$f(x) = \left(\frac{-\frac{1}{x} - 0}{1}\right) \Rightarrow -\frac{1}{x}$$

$$\frac{d\mu}{\mu} = f(x)dx$$

$$\int \frac{d\mu}{\mu} = -\int \frac{dx}{x}$$

$$\ln \mu = -\ln x$$

$$\ln \mu = \ln\left(\frac{1}{x}\right)$$

$$\mu = \frac{1}{x}$$

$$\frac{1}{x}\cdot\left(-\frac{y}{x}\right) + \frac{1}{x}\frac{dy}{dx} = 0$$

$$-\frac{y}{x^2} + \frac{1}{x}\frac{dy}{dx} = 0$$

$$\frac{\partial MM}{\partial y} = -\frac{1}{x^2} \qquad \frac{\partial NN}{\partial x} = -\frac{1}{x^2}$$

$$\int MM\,dx = -y\int x^{-2}\,dx$$

$$= y(x^{-1})$$

$$= \frac{y}{x}$$

$$\int NN\,dy = \frac{1}{x}\int dy$$

$$= \frac{y}{x}$$

$$\frac{y}{x} = C_1 \qquad \boxed{y = C_1 x}$$

$$p(x)y + \frac{dy}{dx} = 0$$

$$\frac{\partial M}{\partial y} = p(x) \qquad \frac{\partial N}{\partial x} = 0$$

$$f(x) = \left(\frac{p(x) - 0}{1}\right) \Rightarrow p(x)$$

$$\int \frac{d\mu}{\mu} = \int p(x)\,dx$$

$$\ln \mu = \int p(x)\,dx$$

$$\mu = e^{\int p(x)\,dx}$$

$$e^{\int p(x)dx} p(x) y + e^{\int p(x)dx} \frac{dy}{dx} = 0$$

$$\frac{d}{dx}\left(y e^{\int p(x)dx}\right) = 0$$

$$y e^{\int p(x)dx} = C_1$$

$$\boxed{y = C_1 e^{-\int p(x)dx}}$$

EDO(1)L.c.v.NH.

$$(1)\ \frac{dy}{dx}+p(x)y=q(x).$$

factor integrante.

$$e^{\int p(x)dx}\frac{dy}{dx}+e^{\int p(x)dx}p(x)y=e^{\int p(x)dx}q(x)$$

$$\frac{d}{dx}\left(ye^{\int p(x)dx}\right)=e^{\int p(x)dx}q(x).$$

$$\int d\left(ye^{\int p(x)dx}\right)=\int e^{\int p(x)dx}q(x)dx$$

$$ye^{\int p(x)dx}=C_1+\int e^{\int p(x)dx}q(x)dx$$

$$y=C_1e^{-\int p(x)dx}+e^{-\int p(x)dx}\int e^{\int p(x)dx}q(x)dx$$

$$\frac{dy}{dx}+p(x)y=q(x).$$

$$\boxed{2x\frac{dy}{dx}-y=3x^2}$$

$$\frac{dy}{dx}+\left(\frac{-1}{2x}\right)y=\frac{3x^2}{2x}$$

$$\frac{dy}{dx}+\left(-\frac{1}{2x}\right)y=\frac{3x}{2}$$

$$p(x)=-\frac{1}{2x}$$

$$q(x)=\frac{3x}{2}$$

$$\int p(x)dx=-\frac{1}{2}\int\frac{dx}{x}$$

$$=-\frac{1}{2}\ln x$$

$$=\ln\left(\frac{1}{\sqrt{x}}\right)$$

$$e^{\int p(x)dx}=e^{\ln\left(\frac{1}{\sqrt{x}}\right)}$$

$$=\frac{1}{\sqrt{x}}$$

$$-\int p(x)\,dx = \frac{1}{2}\int \frac{dx}{x}$$

$$= \frac{1}{2} L x$$

$$= L\sqrt{x}$$

$$e^{-\int p(x)dx} = e^{L\sqrt{x}}$$

$$= \sqrt{x}$$

$$y = C_1\sqrt{x} + \sqrt{x}\int \frac{1}{\sqrt{x}}\left(\frac{3x}{2}\right)dx$$

$$\frac{3}{2}\int \frac{x}{\sqrt{x}}\,dx = \frac{3}{2}\int \sqrt{x}\,dx$$

$$= \frac{3}{2}\left[-2\frac{1}{\sqrt{x}}\right]$$

$$= 3\left(-\frac{1}{\sqrt{x}}\right)$$

$$\boxed{y = C_1\sqrt{x} - 3}$$