Clase 8 - Repaso Tema 1 : La EDO(1) $\begin{cases} NL \\ L. \end{cases}$

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

NL

$$\frac{dy}{dx} = -\frac{M(x,y)}{N(x,y)}$$

$$\frac{dy}{dx} + p(x)y = q(x)$$

L.

$$\frac{dy}{dx} = -p(x)y + q(x).$$

Métodos de Solución

1- Variables separables
2- Coeficientes homogéneos
3- Exacta
4- Factor integrante

Fórmula única.

EDO(1)NL

$$M + Ny' = 0 \rightarrow VS \rightarrow P(x)Q(y) + R(x)S(y)\,y' = 0$$

sol. gral $\int \frac{P}{R}\,dx + \int \frac{S}{Q}\,dy = C_1$

(1) $-e^{y} + (1+e^{x})\,y\,y' = 0$

$P = -1$
$Q = e^{y}$
$R = 1+e^{x}$
$S = y$

$$-\int \frac{1}{(1+e^{x})}\,dx + \int \frac{y}{e^{y}}\,dy = C_1$$

$$\int \frac{-e^{-x}}{(e^{-x}+1)}\,dx + \int e^{-y}y\,dy = C_1$$

$u = e^{-x}+1 \qquad \int \frac{du}{u} - ye^{-y} + \int -e^{-y}dy = c_1 \quad v = y \quad dv = dy$

$du = -e^{-x}dx \qquad\qquad dw = e^{-y}dy$

$\qquad w = -e^{-y}$

$\ln u - ye^{-y} + e^{-y} = c_1$

$$\ln(e^{-x}+1) - ye^{-y} - e^{-y} = C_1$$

$$\frac{d}{dx}\left(\ln(e^{-x}+1)\right) + \frac{d}{dx}\left(-ye^{-y} - e^{-y}\right) = 0$$

$$\frac{1}{e^{-x}+1}\cdot\frac{d}{dx}(e^{-x}+1) + \Big[\underbrace{\frac{d}{dy}(-ye^{-y} - e^{-y})}_{y}\Big]\cdot\frac{dy}{dx} = 0$$

$$\frac{-e^{-x}}{e^{-x}+1} - (-ye^{-y} + e^{-y} - e^{-y})\frac{dy}{dx} = 0$$

$$-\frac{1}{(e^{-x}+1)e^{x}} + ye^{-y}\frac{dy}{dx} = 0$$

$$-\frac{1}{(1+e^{x})} + ye^{-y}\frac{dy}{dx} = 0$$

$$+\frac{y}{e^{y}}\frac{dy}{dx} = \frac{1}{(1+e^{x})}$$

$$\frac{dy}{dx} = \frac{1}{(1+e^{x})}\cdot\left(+\frac{e^{y}}{y}\right)$$

$$\frac{dy}{dx} = +\frac{e^{y}}{y(1+e^{x})}$$

ECUA $\quad (1+e^{x})y\frac{dy}{dx} = e^{y}$

$$\frac{dy}{dx} = \frac{e^{y}}{y(1+e^{x})}$$

$=$

COEFICIENTES HOMOGÊNEOS

$$M(x,y)+N(x,y)\,y'=0 \qquad \lambda \Rightarrow CH.$$

$$M(\lambda x,\lambda y)=\lambda^{m}\cdot M(x,y)$$
$$N(\lambda x,\lambda y)=\lambda^{n}N(x,y) \qquad \text{si } m=n \quad CH$$

$$2x\,y'(x^2+y^2)=y(y^2+2x^2)$$
$$M=-y(y^2+2x^2)$$
$$N=2x(x^2+y^2)$$

$$M(\lambda x,\lambda y)=-\lambda y\left((\lambda y)^2+2(\lambda x)^2\right)$$
$$=-\lambda y\left(\lambda^2y^2+2\lambda^2x^2\right)$$
$$=\lambda^3\left(-y(y^2+2x^2)\right) \qquad m=3$$
$$N(\lambda x,\lambda y)=2(\lambda x)\left((\lambda x)^2+(\lambda y)^2\right) \qquad m=n$$
$$=\lambda 2x\left(\lambda^2x^2+\lambda^2y^2\right)$$
$$=\lambda^3\left(2x(x^2+y^2)\right) \qquad n=3$$

$$-y(y^2+2x^2)+2x(x^2+y^2)\,y'=0$$
$$u=\frac{y}{x} \qquad \boxed{y=u\cdot x \quad y'=xu'+u}$$
$$-(u\cdot x)\left((u\cdot x)^2+2x^2\right)+2x\left(x^2+(ux)^2\right)\cdot(xu'+u)=0$$
$$-(u\cdot x)(u^2x^2+2x^2)+(2x^3+2u^2x^3)\cdot(xu'+u)=0$$
$$-(u^3x^3+2ux^3)+(2x^4u'+2u^2x^4u'+2x^3u+2u^3x^3)=0$$
$$-u^3x^3-\cancel{2ux^3}+\cancel{2x^3u}+2u^3x^3+(2x^4u'+2u^2x^4u')=0$$
$$-u^3x^3+2u^3x^3+(2x^4+2u^2x^4)u'=0$$
$$u^3x^3+(2+2u^2)x^4u'=0$$
$$\frac{1}{x}+\frac{2+2u^2}{u^3}u'=0$$

SOL GERAL $\displaystyle\int\frac{dx}{x}+\int\frac{2+2u^2}{u^3}\,du=C_1$

$$Lx + 2\int \frac{1+u^2}{u^3}\,du = C_1$$

$$Lx + 2\int \frac{du}{u^3} + 2\int \frac{du}{u} = C_1$$

$$Lx + 2\left(\frac{u^{-2}}{-2}\right) + 2Lu = C_1$$

$$Lx - \frac{1}{u^2} + Lu^2 = C_1$$

$u = \frac{y}{x}$

$$Lx - \frac{x^2}{y^2} + L\left(\frac{y}{x}\right)^2 = C_1$$

SOL GRAL

EXACTA.

$$M + N y' = 0 \qquad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \text{EXACTA.}$$

$$\int M\,dx \quad \cup \quad \int N\,dy = C_1$$

$$\int M\,dx + \int \left(N - \frac{\partial}{\partial y}\int M\,dx\right) dy = C_1$$

$$\int N\,dy + \int \left(M - \frac{\partial}{\partial x}\int N\,dy\right) dx = C_1$$

ECUA

$$\underbrace{(x^3 + xy^2)}_{M} + \underbrace{(x^2y + y^3)}_{N}\, y' = 0$$

$$\frac{\partial M}{\partial y} = 2xy \qquad \frac{\partial N}{\partial x} = 2xy \qquad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

EXACTA.

$$\int M\,dx = \int x^3 dx + y^2 \int x\,dx$$
$$= \frac{x^4}{4} + \frac{x^2y^2}{2}$$
$$\int N\,dy = x^2 \int y\,dy + \int y^3 dy$$
$$= \frac{x^2y^2}{2} + \frac{y^4}{4}$$

SOL GEAL $\quad \frac{x^4}{4} + \frac{x^2y^2}{2} + \frac{y^4}{4} = C_1$