$$y'' + a_1 y' + a_2 y = 0 \quad \text{EDO(2)LCCH.}$$

$$H = y_p = e^{mx}$$

$$m^2 + a_1 m + a_2 = 0 \quad \text{E(A) CARACTERÍSTICA}$$

CASO I.- $m_1, m_2 \in \mathbb{R} \quad m_1 \neq m_2$

$$y_g = C_1 e^{m_1 x} + C_2 e^{m_2 x} \qquad \left.\begin{matrix} e^{m_1 x} \\ e^{m_2 x} \end{matrix}\right\} \text{SPF.}$$

$$W = \begin{vmatrix} e^{m_1 x} & e^{m_2 x} \\ m_1 e^{m_1 x} & m_2 e^{m_2 x} \end{vmatrix} \neq 0.$$

$$(m_2 - m_1) e^{m_1 x} e^{m_2 x} \neq 0$$

$$m_2 - m_1 \neq 0$$

$$m_2 \neq m_1$$

CASO II.- Raíces E(A)C

$$m_1, m_2 \in \mathbb{R} \quad m_1 = m_2$$

$$e^{mx} \xrightarrow{m=m_1} e^{m_1x}$$

$$\frac{d}{dm}$$

$$xe^{mx} \xrightarrow{m=m_1} xe^{m_1x}$$

$$y_g = C_1e^{m_1x} + C_2xe^{m_1x}$$

CASO II $\to (m-a)^4 = 0 \qquad m_1 = m_2 = m_3 = m_4$

$$e^{mx} \xrightarrow{m=a} e^{ax}$$

$$\frac{d}{dm} \quad xe^{mx} \xrightarrow{m=a} xe^{ax}$$

$$\frac{d}{dm} \quad x^2e^{mx} \xrightarrow{m=a} x^2e^{ax}$$

$$\frac{d}{dm} \quad x^3e^{mx} \xrightarrow{m=a} x^3e^{ax}$$

$$y_g = C_1e^{ax} + C_2xe^{ax} + C_3x^2e^{ax} + C_4x^3e^{ax}.$$

$$\frac{d^4y}{dx^4} = 0 \qquad m^4 = 0 \qquad m_1 = m_2 = m_3 = m_4 = 0.$$

$$y_g = C_1 + C_2x + C_3x^2 + C_4x^3.$$

EDO(2)LCCH.

$$y'' + a_1 y' + a_2 y = 0$$

$$m^2 + a_1 m + a_2 = 0$$

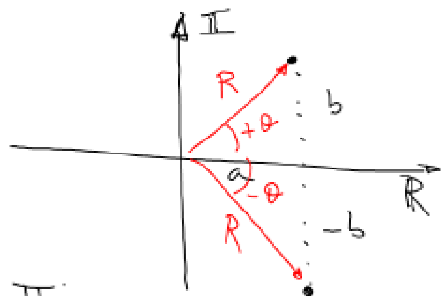
CASO III: $m_1 m_2 \in \mathbb{C}$ $\quad m_1 = a + bi \quad i = \sqrt{-1}$

$\quad m_2 = a - bi$

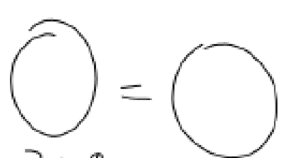
$$y_g = c_1 e^{(a+bi)x} + c_2 e^{(a-bi)x} \quad m_1 \neq m_2$$

$$y_g = c_1 e^{ax} e^{bxi} + c_2 e^{ax} e^{-bxi}$$

$$y_g = e^{ax}\left(c_1 e^{bxi} + c_2 e^{-bxi}\right) \quad \begin{matrix} x \in \mathbb{R} \\ y \in \mathbb{R} \end{matrix}$$



Euler $e^{\pi i} = -1$



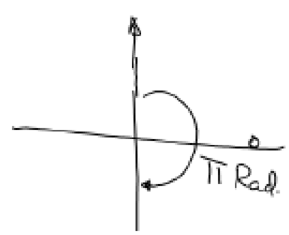
360° $\quad$ 2π Rad.

180° $\quad$ π Rad.

$$\begin{cases} Re^{\theta i} = R\cos\theta + iR\,\mathrm{sen}\,\theta \\ Re^{-\theta i} = R\cos(\theta) - iR\,\mathrm{sen}\,\theta \end{cases}$$

$$e^{\theta i} = \cos(\theta) + i\,\mathrm{sen}(\theta)$$

$$e^{-\theta i} = \cos(\theta) - i\,\mathrm{sen}(\theta).$$



$$y_g = e^{ax}\left(c_1 e^{bxi} + c_2 e^{-bxi}\right)$$

$$y_g = e^{ax}\left(c_1\left[\cos(bx) + i\,\mathrm{sen}(bx)\right] + c_2\left[\cos(bx) - i\,\mathrm{sen}(bx)\right]\right)$$

$$y_g = e^{ax}\left(\left[c_1+c_2\right]\cos(bx) + \left[c_1 i - c_2 i\right]\mathrm{sen}(bx)\right)$$

$$= e^{ax}\left(c_{10}\cos(bx) + c_{20}\,\mathrm{sen}(bx)\right)$$

$$y_g = c_{10}e^{ax}\cos(bx) + c_{20}e^{ax}\,\mathrm{sen}(bx)$$

$$\begin{cases} m_1 = a + bi & a \in \mathbb{R} \\ m_2 = a - bi & b \in \mathbb{R}^+ \end{cases}$$

$$y'' + 2y' + 2y = 0$$

$$m^2 + 2m + 2 = 0.$$

$$m_{1,2} = \frac{-2 \pm \sqrt{4 - 4(2)}}{2}$$

$$= \frac{-2 \pm \sqrt{-4}}{2}$$

$$= \frac{-2 \pm 2i}{2}$$

$$= -1 \pm i \qquad \begin{matrix} a = -1 \\ b = 1 \end{matrix}$$

$$y_g = c_1 e^{-x}\cos(x) + c_2 e^{-x}\,\mathrm{sen}(x)$$

$$y = C_1 e^{3x} + C_2 e^{3x}\cos 2x + C_3 e^{3x}\operatorname{sen} 2x$$

$$(m-3)(m-(3+2i))(m-(3-2i)) = 0$$

$$(m-3)((m-3)+2i)((m-3)-2i)) = 0$$

$$(m^2 - a^2) = (m-a)(m+a)$$

$$= m^2 - \cancel{a}m + \cancel{a}m - a^2$$

$$(m-3)((m-3)^2 - (2i)^2) = 0$$

$$(m-3)((m^2-6m+9)+4) = 0$$

$$(m-3)(m^2-6m+13) = 0$$

$$m^3 - 6m^2 + 13m - 3m^2 + 18m - 39 = 0$$

$$m^3 - 9m^2 + 31m - 39 = 0$$

$$y''' - 9y'' + 31y' - 39y = 0$$

$$y = C_1 e^{2x}\cos 3x + C_2 e^{2x}\operatorname{sen}(3x) +$$
$$+ C_3 x e^{2x}\cos(3x) + C_4 x e^{2x}\operatorname{sen}(3x)$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$$((m-2)^2 - (3i)^2)^2 = 0$$

$$(m-2)^4 - 2(m-2)^2\cdot(3i)^2 + (3i)^4 = 0$$

$$m^4 - 8m^3 + 24m^2 - 32m + 16 + 18(m-2)^2 + 81 = 0$$

$$+18m^2 - 72m + 72 + 81 = 0$$

$$m^4 - 8m^3 + 42m^2 - 104m + 169 = 0$$

$$\frac{d^4y}{dx^4} - 8\frac{d^3y}{dx^3} + 42\frac{d^2y}{dx^2} - 104\frac{dy}{dx} + 169y = 0$$

$$((m+a)+bi)^2\cdot((m+a)-bi)^2 = 0$$

EDO(2)LCC $\underline{\underline{NH}}$ $\quad y'' + a_1 y' + a_2 y = Q(x)$