TEMA 2.- EDO(n)L CC $\left\{\begin{array}{l} H. \\ NH. \end{array}\right.$

$$y_g = C_1 y_1 + C_2 y_2 + \ldots + C_n y_n$$

$$EDO(n)L \left\{\begin{array}{l} CC \\ CV \end{array}\right\} H.$$

$$y_g = C_1 y_1 + C_2 y_2 + \ldots + C_n y_n + F(x)$$

$$EDO(n)L \left\{\begin{array}{l} CC \\ CV \end{array}\right\} \underline{\underline{NH}} \quad y_{p/Q.}$$

$$y_g = C_1 e^{2x} + C_2 x e^{2x} + x^2 e^{2x}$$

EDO(2)L CC NH.

$$y_{g/H_A} = C_1 e^{2x} + C_2 x e^{2x} \qquad y_{p/Q} = x^2 e^{2x}$$

$$y_{g/NH} = y_{g/H_A} + y_{p/Q.}$$

$$y'' + a_1 y' + a_2 y = Q(x)$$

$$m^2 + a_1 m + a_2 = 0 \quad E(A)C.$$

$$(m-2)^2 = 0$$

$$m^2 - 4m + 4 = 0$$

$$y'' - 4y' + 4y = 0 \quad \text{EDO(2)LCCH}_A$$

$$y = e^{2x} \quad y' = 2e^{2x} \quad y'' = 4e^{2x}$$

$$(4e^{2x}) - 4(2e^{2x}) + 4(e^{2x}) = 0$$

$$(8-8)e^{2x} = 0$$

$$(0)e^{2x} = 0$$

$$0 \equiv 0$$

$$y'' - 4y' + 4y = 0$$

$$y_{P/F} = xe^{2x}$$

$$y' = e^{2x} + 2xe^{2x}$$

$$y'' = 2e^{2x} + 2e^{2x} + 4xe^{2x}$$

$$= 4e^{2x} + 4xe^{2x}$$

$$[4e^{2x} + 4xe^{2x}] - 4[e^{2x} + 2xe^{2x}] + 4[xe^{2x}] = 0$$

$$[4 - 8 + 4]xe^{2x} + [4 - 4]e^{2x} = 0$$

$$(0)xe^{2x} + (0)e^{2x} = 0$$

$$0 \equiv 0$$

$$y_{p/Q} = x^2 e^{2x}$$

$$y' = 2x^2 e^{2x} + 2x e^{2x}$$

$$y'' = 2\left(2x^2 e^{2x} + 2x e^{2x}\right) + 2\left(2x e^{2x} + e^{2x}\right)$$

$$= 4x^2 e^{2x} + 8x e^{2x} + 2e^{2x}$$

$$\left[4x^2 e^{2x} + 8x e^{2x} + 2e^{2x}\right] - 4\left[2x^2 e^{2x} + 2x e^{2x}\right] + 4\left[x^2 e^{2x}\right] = Q.$$

$$(0)x^2 e^{2x} + (0)x e^{2x} + (2e^{2x}) = Q.$$

$$y_g = C_1 e^{2x} + C_2 x e^{2x} + x^2 e^{2x}$$

$$y'' - 4y' + 4y = 2e^{2x}$$

EDO(2) LCC NH.

$$y^{(n)} + a_1 y^{(n-1)} + \ldots + a_{n-1} y' + a_n y = Q.$$

$$y_{g/NH} = y_{g/A} + y_{p/Q}.$$

$$y_{g/NH} = \overbrace{C_1 y_1 + C_2 y_2 + \ldots + C_n y_n}^{} + F(x)$$

$$y_{g/NH} = C_1 e^{4x} + C_2 \cos(2x) + C_3 \operatorname{sen}(2x) + 5e^{3x} + 4x^2 + 2\cos(5x)$$

$$y''' + a_1 y'' + a_2 y' + a_3 y = Q(x).$$

$$(m-4)(m-2i)(m+2i)=0$$

$$(m-4)(m^2-(2i)^2)=0$$

$$(m-4)(m^2+4)=0$$

$$m^3-4m^2+4m-16=0$$

$$y'''-4y''+4y'-16y=Q(x)$$

EDO(3)LccNH.

$$y''+a_1y'+a_2y=Q(x)$$

$M_1 \Rightarrow$ Operador Diferencial

$M_2 \Rightarrow$ Parámetros Variables.

1- Resolver la EDO(2)LccH$_A$.

$$y'' \quad \frac{d^2y}{dx^2} \quad \ddot{y} \quad D_x^2 y \quad D^2 y$$

$$y''+a_1y'+a_2y=0$$

$$D^2y+a_1Dy+a_2y=0$$

$$(D^2+a_1D+a_2)y=0$$

$$m^2+a_1m+a_2=0$$

$$(m - m_1)(m - m_2) = 0 \qquad m_1, m_2 \in \mathbb{R}$$
$$m_1 \neq m_2$$

$$(D - m_1)(D - m_2)\, y = 0$$

$$y'' - 5y' + 6y = 0$$

$$(D^2 - 5D + 6)\, y = 0$$

$$(D - 2)(D - 3)\, y = 0$$

$$y = c_1 e^{2x} + c_2 e^{3x}$$

$$(D - 2)(D - 3)\left[c_1 e^{2x} + c_2 e^{3x}\right] = 0$$

$$(D - 2)\left[2c_1 e^{2x} + \cancel{3c_2 e^{3x}} - 3c_1 e^{2x} - \cancel{3c_2 e^{3x}}\right] = 0$$

$$(D - 2)\left[-c_1 e^{2x}\right] = 0$$

$$-2c_1 e^{2x} + 2c_1 e^{2x} = 0$$

$$(-2 + 2)\, c_1 e^{2x} = 0$$

$$(0)\, c_1 e^{2x} = 0$$

$$0 = 0$$

$$D\left(D^{n}\right)y=0$$

$$D^{n+1}y=0$$

$$D\left(a_1 f+a_2 g\right)=0$$

$$a_1 Df+a_2 Dg=0$$

$$D\left(D^{-1}\right)y=0$$

$$D^{0}\quad y=0$$

$$\int f\,dx=D^{-1}f+C_1$$

$$\int_a^b g\,dx=D^{-1}g\Big|_a^b$$

TABLA OPERADOR ANIQUILADOR

$P(D)$	y
$(D-a)$	e^{ax}
$(D-a)^2$	xe^{ax}
$(D-a)^3$	x^2e^{ax}
$\vdots$	$\vdots$
$(D-a)^{n+1}$	x^ne^{ax}
D	1
D^2	x
$\vdots$	$\vdots$
D^{n+1}	x^n
(D^2+b^2)	$\begin{cases}\cos(bx)\\ \operatorname{sen}(bx)\end{cases}$
$((D-a)^2+b^2)$	$\begin{cases}e^{ax}\cos(bx)\\ e^{ax}\operatorname{sen}(bx)\end{cases}$
$(D^2+b^2)^2$	$\begin{cases}x\cos(bx)\\ x\operatorname{sen}(bx)\end{cases}$
$((D-a)^2+b^2)^2$	$\begin{cases}xe^{ax}\cos(bx)\\ xe^{ax}\operatorname{sen}(bx)\end{cases}$

$$(D-a)\left[e^{ax}\right]=0$$

$$0=0$$

$$(D-a)(D-a)xe^{ax}=0$$

$$(D-a)\left(e^{ax}+axe^{ax}-axe^{ax}\right)=0$$

$$(D-a)\left[e^{ax}\right]=0$$

$$0=0$$

$$y'' - 6y' + 9y = 2e^{4x} + 3x^2$$

$$y'' - 6y' + 9y = 0$$

$$(D^2 - 6D + 9)y = 0$$

$$(D-3)^2 y = 0$$

$$y = c_1 e^{3x} + c_2 x e^{3x}$$

EDO(2) LccNH. $(D-3)^2 y = 2e^{4x} + 3x^2$

$$(D-3)^2 (D-4) y = (D-4)\left[2e^{4x} + 3x^2\right]$$

$$= 8e^{4x} - 8e^{4x} + 6x - 12x^2$$

$$(D-3)^2 (D-4) y = 6x - 12x^2$$

$$(D-3)^2 (D-4) D^3 y = D^3 \left[6x - 12x^2\right]$$

$$= D^2 (6 - 24x)$$

$$= D(0 - 24)$$

EDO(6) LccH. $(D-3)^2 (D-4) D^3 y = (0)$

$$y = c_1 e^{3x} + c_2 x e^{3x} + c_3 e^{4x} + c_4 x^2 + c_5 x + c_6$$

$$y_{original} = c_1 e^{3x} + c_2 x e^{3x}$$

$$y_{p/a} = Ae^{4x} + Bx^2 + Dx + E$$

$$y' = 4Ae^{4x} + 2Bx + D + (0)$$

$$y'' = 16Ae^{4x} + 2B + (0)$$

$$(16Ae^{4x} + 2B) - 6(4Ae^{4x} + 2Bx + D) +$$

$$+ 9(Ae^{4x} + Bx^2 + Dx + E) = 0$$

$$(16A - 24A + 9A)e^{4x} + (9Bx^2) + (9D - 12B)x +$$

$$+ (2B - 6D + 9E) = 2e^{4x} + 3x^2$$

$$Ae^{4x} + 9Bx^2 + (9D - 12B)x + (2B - 6D + 9E) = 2e^{4x} + 3x^2$$

$A = 2$

$B = \frac{1}{3}$

$D = \frac{4}{9}$

$E = \frac{1}{9}\left(-\frac{2}{3} + \frac{24}{9}\right) = \frac{18}{81}$