

```

> restart
> Ecua := y'' + 4 y' + 3 y = 9 exp(-3 x)

```

$$Ecua := \frac{d^2}{dx^2} y(x) + 4 \left(\frac{d}{dx} y(x) \right) + 3 y(x) = 9 e^{-3x} \quad (1)$$

```

> Q := rhs(Ecua)

```

$$Q := 9 e^{-3x} \quad (2)$$

```

> SolHom := lhs(Ecua) = 0

```

$$SolHom := \frac{d^2}{dx^2} y(x) + 4 \left(\frac{d}{dx} y(x) \right) + 3 y(x) = 0 \quad (3)$$

```

> EcuaCarac := m^2 + 4 m + 3 = 0

```

$$EcuaCarac := m^2 + 4 m + 3 = 0 \quad (4)$$

```

> Raiz := solve(EcuaCarac)

```

$$Raiz := -1, -3 \quad (5)$$

```

> yy[1] := exp(Raiz[1]·x)

```

$$yy_1 := e^{-x} \quad (6)$$

```

> yy[2] := exp(Raiz[2]·x)

```

$$yy_2 := e^{-3x} \quad (7)$$

```

> SolGralHom := y(x) = _C1·yy[1] + _C2·yy[2]

```

$$SolGralHom := y(x) = _C1 e^{-x} + _C2 e^{-3x} \quad (8)$$

```

> SolGralNoHom := y(x) = A(x)·yy[1] + B(x)·yy[2]

```

$$SolGralNoHom := y(x) = A(x) e^{-x} + B(x) e^{-3x} \quad (9)$$

```

> with(linalg) :
> WW := wronskian([yy[1], yy[2]], x)

```

$$WW := \begin{bmatrix} e^{-x} & e^{-3x} \\ -e^{-x} & -3 e^{-3x} \end{bmatrix} \quad (10)$$

```

> BB := array([0, Q])

```

$$BB := \begin{bmatrix} 0 & 9 e^{-3x} \end{bmatrix} \quad (11)$$

```

> InvWW := inverse(WW)

```

$$InvWW := \begin{bmatrix} \frac{3}{2 e^{-x}} & \frac{1}{2 e^{-x}} \\ -\frac{1}{2 e^{-3x}} & -\frac{1}{2 e^{-3x}} \end{bmatrix} \quad (12)$$

```

> Probar := evalm(WW &* InvWW)

```

$$Probar := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (13)$$

```

> SolUno := evalm(InvWW &* BB)

```

(14)

$$SolUno := \begin{bmatrix} \frac{9}{2} \frac{e^{-3x}}{e^{-x}} & -\frac{9}{2} \end{bmatrix} \quad (14)$$

> Aprima := simplify(SolUno[1])

$$Aprima := \frac{9}{2} e^{-2x} \quad (15)$$

> Bprima := SolUno[2]

$$Bprima := -\frac{9}{2} \quad (16)$$

> Para := linsolve(WW, BB)

$$Para := \begin{bmatrix} \frac{9}{2} \frac{e^{-3x}}{e^{-x}} & -\frac{9}{2} \end{bmatrix} \quad (17)$$

> A(x) := int(Aprima, x) + _C1

$$A(x) := -\frac{9}{4} e^{-2x} + _C1 \quad (18)$$

> B(x) := int(Bprima, x) + _C2

$$B(x) := -\frac{9}{2} x + _C2 \quad (19)$$

> SolGralFinal := simplify(SolGralNoHom)

$$SolGralFinal := y(x) = -\frac{9}{4} e^{-3x} + _C1 e^{-x} - \frac{9}{2} e^{-3x} x + _C2 e^{-3x} \quad (20)$$

> SolGralDobleFinal := simplify(subs(_C1 = _C10, _C2 = _C20 + \frac{9}{4}, SolGralFinal))

$$SolGralDobleFinal := y(x) = _C10 e^{-x} - \frac{9}{2} e^{-3x} x + e^{-3x} _C20 \quad (21)$$

> Comprobar := eval(subs(y(x) = rhs(SolGralDobleFinal), lhs(Ecua) - rhs(Ecua) = 0))

$$Comprobar := 0 = 0 \quad (22)$$

> Comprobar := eval(subs(y(x) = rhs(SolGralFinal), lhs(Ecua) - rhs(Ecua) = 0))

$$Comprobar := 0 = 0 \quad (23)$$

> restart

> Ecua := diff(y(t), t\$4) + 5·diff(y(t), t\$2) - 4·y(t) = 5·exp(-3 t)·cos(2 t)

$$Ecua := \frac{d^4}{dt^4} y(t) + 5 \left(\frac{d^2}{dt^2} y(t) \right) - 4 y(t) = 5 e^{-3t} \cos(2 t) \quad (24)$$

> EcuaHom := lhs(Ecua) = 0

$$EcuaHom := \frac{d^4}{dt^4} y(t) + 5 \left(\frac{d^2}{dt^2} y(t) \right) - 4 y(t) = 0 \quad (25)$$

> Q := rhs(Ecua)

$$Q := 5 e^{-3t} \cos(2 t) \quad (26)$$

> EcuaCarac := m^4 + 5 m^2 - 4 = 0

$$EcuaCarac := m^4 + 5 m^2 - 4 = 0 \quad (27)$$

> Raiz := solve(EcuaCarac); evalf(%)

$$\begin{aligned} \text{Raiz} := & \frac{1}{2} I \sqrt{2 \sqrt{41} + 10}, -\frac{1}{2} I \sqrt{2 \sqrt{41} + 10}, \frac{1}{2} \sqrt{-10 + 2 \sqrt{41}}, \\ & -\frac{1}{2} \sqrt{-10 + 2 \sqrt{41}} \\ & 2.387794404 I, -2.387794404 I, 0.8375930500, -0.8375930500 \end{aligned} \quad (28)$$

$$\begin{aligned} > \text{yy}[1] := \cos(\text{Im}(\text{Raiz}[1]) \cdot t) \\ & \text{yy}_1 := \cos\left(\frac{1}{2} \sqrt{2 \sqrt{41} + 10} t\right) \end{aligned} \quad (29)$$

$$\begin{aligned} > \text{yy}[2] := \sin(\text{Im}(\text{Raiz}[1]) \cdot t) \\ & \text{yy}_2 := \sin\left(\frac{1}{2} \sqrt{2 \sqrt{41} + 10} t\right) \end{aligned} \quad (30)$$

$$\begin{aligned} > \text{yy}[3] := \exp(\text{Raiz}[3] \cdot t) \\ & \text{yy}_3 := e^{\frac{1}{2} \sqrt{-10 + 2 \sqrt{41}} t} \end{aligned} \quad (31)$$

$$\begin{aligned} > \text{yy}[4] := \exp(\text{Raiz}[4] \cdot t) \\ & \text{yy}_4 := e^{-\frac{1}{2} \sqrt{-10 + 2 \sqrt{41}} t} \end{aligned} \quad (32)$$

$$\begin{aligned} > \text{with}(\text{linalg}) : \\ > \text{WW} := \text{wronskian}([\text{yy}[1], \text{yy}[2], \text{yy}[3], \text{yy}[4]], t) : \text{evalf}(\%, 3) \\ & \begin{bmatrix} \cos(2.38 t) & \sin(2.38 t) & e^{0.835 t} & e^{-0.835 t} \\ -2.38 \sin(2.38 t) & 2.38 \cos(2.38 t) & 0.835 e^{0.835 t} & -0.835 e^{-0.835 t} \\ -5.70 \cos(2.38 t) & -5.70 \sin(2.38 t) & 0.700 e^{0.835 t} & 0.700 e^{-0.835 t} \\ 13.6 \sin(2.38 t) & -13.6 \cos(2.38 t) & 0.582 e^{0.835 t} & -0.582 e^{-0.835 t} \end{bmatrix} \end{aligned} \quad (33)$$

$$\begin{aligned} > \text{BB} := \text{array}([0, 0, 0, Q]) \\ & \text{BB} := \begin{bmatrix} 0 & 0 & 0 & 5 e^{-3 t} \cos(2 t) \end{bmatrix} \end{aligned} \quad (34)$$

$$\begin{aligned} > \text{Para} := \text{linsolve}(\text{WW}, \text{BB}) : \text{evalf}(\%, 3) \\ & \begin{bmatrix} \frac{0.327 \sin(2.38 t) e^{-3 t} \cos(2 t)}{\cos(2.38 t)^2 + \sin(2.38 t)^2}, -\frac{0.327 \cos(2.38 t) e^{-3 t} \cos(2 t)}{\cos(2.38 t)^2 + \sin(2.38 t)^2}, \frac{0.467 e^{-3 t} \cos(2 t)}{e^{0.835 t}}, \\ -\frac{0.467 e^{-3 t} \cos(2 t)}{e^{-0.835 t}} \end{bmatrix} \end{aligned} \quad (35)$$

$$\begin{aligned} > \text{Aprima} := \text{simplify}(\text{Para}[1]) : \text{evalf}(\%, 3) \\ & 0.327 \sin(2.38 t) e^{-3 t} \cos(2 t) \end{aligned} \quad (36)$$

$$\begin{aligned} > \text{Bprima} := \text{simplify}(\text{Para}[2]) : \text{evalf}(\%, 3) \\ & -0.327 \cos(2.38 t) e^{-3 t} \cos(2 t) \end{aligned} \quad (37)$$

$$\begin{aligned} > \text{Dprima} := \text{simplify}(\text{Para}[3]) : \text{evalf}(\%, 3) \\ & 0.467 e^{-3.84 t} \cos(2 t) \end{aligned} \quad (38)$$

$$\begin{aligned} > \text{Eprima} := \text{simplify}(\text{Para}[4]) : \text{evalf}(\%, 3) \\ & -0.467 e^{-2.16 t} \cos(2 t) \end{aligned} \quad (39)$$

$$\begin{aligned} &> \text{SolHom} := y(t) = _C1 \cdot yy[1] + _C2 \cdot yy[2] + _C3 \cdot yy[3] + _C4 \cdot yy[4] : \text{evalf}(\%, 3) \\ &\quad y(t) = _C1 \cos(2.38 t) + _C2 \sin(2.38 t) + _C3 e^{0.835 t} + _C4 e^{-0.835 t} \end{aligned} \quad (40)$$

$$\begin{aligned} &> \text{SolNoHom} := y(t) = A(t) \cdot yy[1] + B(t) \cdot yy[2] + D(t) \cdot yy[3] + E(t) \cdot yy[4] : \text{evalf}(\%, 3) \\ &\quad y(t) = A(t) \cos(2.38 t) + B(t) \sin(2.38 t) + D(t) e^{0.835 t} + E(t) e^{-0.835 t} \end{aligned} \quad (41)$$

$$\begin{aligned} &> A(t) := \text{int}(Aprima, t) + _C1 : \text{evalf}(\%, 3) \\ &\quad -0.0254 e^{-3 \cdot t} \cos(4.38 t) - 0.0174 e^{-3 \cdot t} \sin(4.38 t) - 0.00681 e^{-3 \cdot t} \cos(0.38 t) \\ &\quad - 0.0532 e^{-3 \cdot t} \sin(0.38 t) + _C1 \end{aligned} \quad (42)$$

$$\begin{aligned} &> B(t) := \text{int}(Bprima, t) + _C2 : \text{evalf}(\%, 3) \\ &\quad 0.0532 e^{-3 \cdot t} \cos(0.38 t) - 0.00681 e^{-3 \cdot t} \sin(0.38 t) + 0.0174 e^{-3 \cdot t} \cos(4.38 t) \\ &\quad - 0.0254 e^{-3 \cdot t} \sin(4.38 t) + _C2 \end{aligned} \quad (43)$$

$$\begin{aligned} &> D(t) := \text{int}(Dprima, t) + _C3 : \text{evalf}(\%, 3) \\ &\quad -0.0958 e^{-3.84 t} \cos(2. t) + 0.0500 e^{-3.84 t} \sin(2. t) + _C3 \end{aligned} \quad (44)$$

$$\begin{aligned} &> E(t) := \text{int}(Eprima, t) + _C4 : \text{evalf}(\%, 3) \\ &\quad 0.116 e^{-2.16 t} \cos(2. t) - 0.107 e^{-2.16 t} \sin(2. t) + _C4 \end{aligned} \quad (45)$$

$$\begin{aligned} &> \text{SolFinal} := \text{simplify}(\text{SolNoHom}) : \text{evalf}(\%, 3) \\ &\quad y(t) = 0.995 \cos(2.38 t) _C1 + 0.995 \sin(2.38 t) _C2 + 0.995 e^{0.835 t} _C3 + 0.995 e^{-0.835 t} _C4 \\ &\quad - 0.0574 \sin(2. t) e^{-3 \cdot t} + 0.0204 e^{-3 \cdot t} \cos(2. t) - 0.0253 \cos(4.38 t) e^{-3 \cdot t} \cos(2.38 t) \\ &\quad + 0.0174 \cos(4.38 t) e^{-3 \cdot t} \sin(2.38 t) - 0.0175 \sin(4.38 t) e^{-3 \cdot t} \cos(2.38 t) \\ &\quad - 0.0253 \sin(4.38 t) e^{-3 \cdot t} \sin(2.38 t) - 0.00686 \cos(0.385 t) e^{-3 \cdot t} \cos(2.38 t) \\ &\quad + 0.0534 \cos(0.385 t) e^{-3 \cdot t} \sin(2.38 t) - 0.0534 \sin(0.385 t) e^{-3 \cdot t} \cos(2.38 t) \\ &\quad - 0.00686 \sin(0.385 t) e^{-3 \cdot t} \sin(2.38 t) \end{aligned} \quad (46)$$

$$\begin{aligned} &> \text{CondIni} := y(0) = -2, D(y)(0) = 0, D(D(y))(0) = 7, D(D(D(y)))(0) = -5 \\ &\quad \text{CondIni} := y(0) = -2, D(y)(0) = 0, D^{(2)}(y)(0) = 7, D^{(3)}(y)(0) = -5 \end{aligned} \quad (47)$$

$$\begin{aligned} &> \text{SolUno} := \text{simplify}(\text{subs}(t=0, \text{rhs}(\text{SolFinal})) = -2) : \text{evalf}(\%, 3) \\ &\quad -0.0117 + _C3 + _C4 + _C1 = -2. \end{aligned} \quad (48)$$

$$\begin{aligned} &> \text{SolDos} := \text{simplify}(\text{subs}(t=0, \text{rhs}(\text{diff}(\text{SolFinal}, t))) = 0) : \text{evalf}(\%, 3) \\ &\quad -0.00786 + 0.83 _C3 - 0.83 _C4 + 2.38 _C2 = 0. \end{aligned} \quad (49)$$

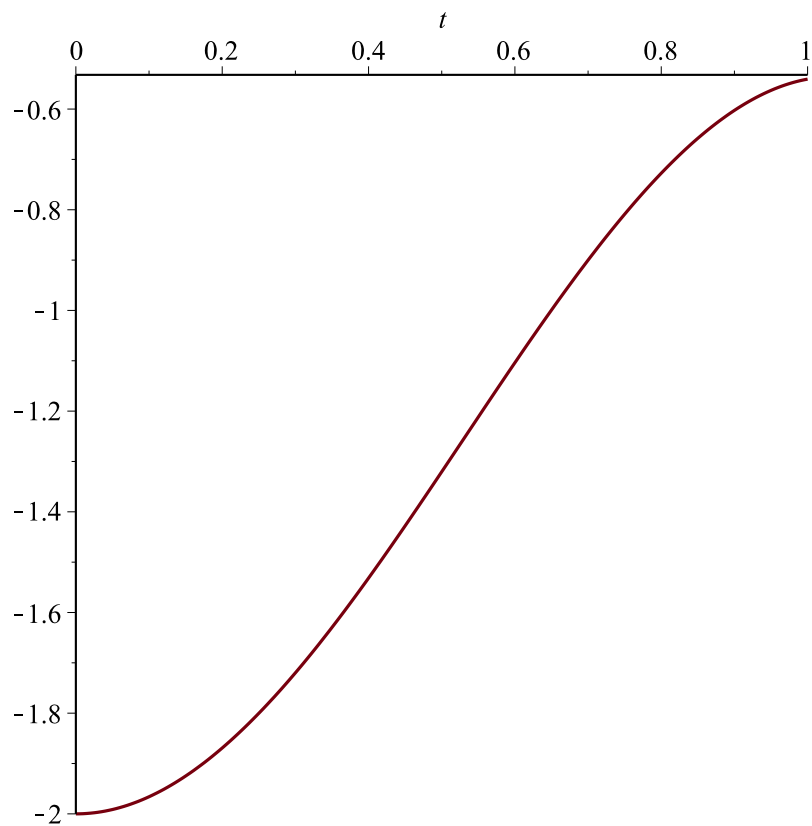
$$\begin{aligned} &> \text{SolTres} := \text{simplify}(\text{subs}(t=0, \text{rhs}(\text{diff}(\text{SolFinal}, t\$2))) = 7) : \text{evalf}(\%, 3) \\ &\quad 0.199 + 0.70 _C3 + 0.70 _C4 - 5.70 _C1 = 7. \end{aligned} \quad (50)$$

$$\begin{aligned} &> \text{SolCuatro} := \text{simplify}(\text{subs}(t=0, \text{rhs}(\text{diff}(\text{SolFinal}, t\$3))) = -5) : \text{evalf}(\%, 3) \\ &\quad -1.09 + 0.6 _C3 - 0.6 _C4 - 13.6 _C2 = -5. \end{aligned} \quad (51)$$

$$\begin{aligned} &> \text{Parametros} := \text{solve}([\text{SolUno}, \text{SolDos}, \text{SolTres}, \text{SolCuatro}]) : \text{evalf}(\%, 3) \\ &\quad \{ _C1 = -1.28, _C2 = 0.258, _C3 = -0.708, _C4 = 0.00858 \} \end{aligned} \quad (52)$$

$$\begin{aligned} &> \text{SolPart} := \text{subs}(\text{Parametros}, \text{SolFinal}) : \text{evalf}(\%, 3) \\ &\quad y(t) = -0.0574 \sin(2. t) e^{-3 \cdot t} + 0.0204 e^{-3 \cdot t} \cos(2. t) - 0.00686 \sin(0.385 t) e^{-3 \cdot t} \sin(2.38 t) \\ &\quad + 0.0534 \cos(0.385 t) e^{-3 \cdot t} \sin(2.38 t) - 0.0253 \sin(4.38 t) e^{-3 \cdot t} \sin(2.38 t) \\ &\quad - 0.0534 \sin(0.385 t) e^{-3 \cdot t} \cos(2.38 t) + 0.0175 \cos(4.38 t) e^{-3 \cdot t} \sin(2.38 t) \\ &\quad - 0.0175 \sin(4.38 t) e^{-3 \cdot t} \cos(2.38 t) - 0.00686 \cos(0.385 t) e^{-3 \cdot t} \cos(2.38 t) \\ &\quad - 0.0253 \cos(4.38 t) e^{-3 \cdot t} \cos(2.38 t) - 0.704 e^{0.835 t} + 0.00853 e^{-0.835 t} \\ &\quad + 0.258 \sin(2.38 t) - 1.29 \cos(2.38 t) \end{aligned} \quad (53)$$

```
> plot(rhs(SolPart), t=0..1)
```



```
> plot([rhs(SolPart), rhs(diff(SolPart, t)), rhs(diff(SolPart, t$2)), rhs(diff(SolPart, t$3))], t  
=0..1)
```

