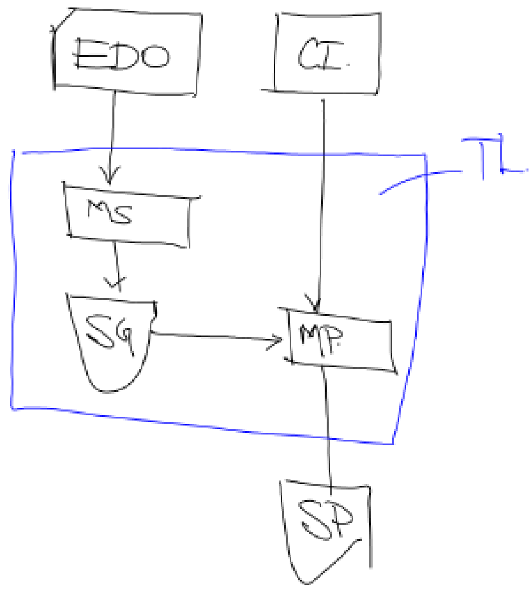
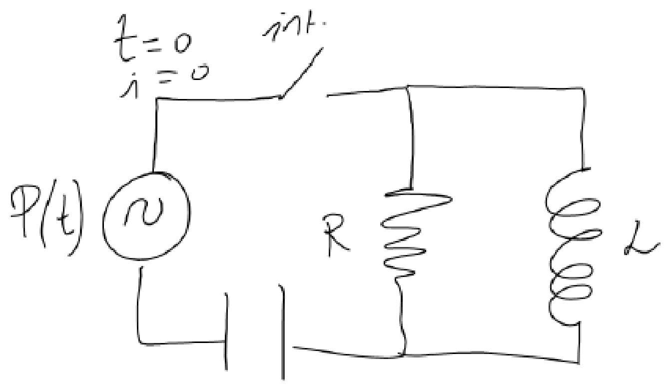
¡TEMA 3.- Transformada de Laplace y Sistemas de Ecs. Dfs. Ord.

TL → es muy útil para resolver problemas de EDO con cond. iniciales.

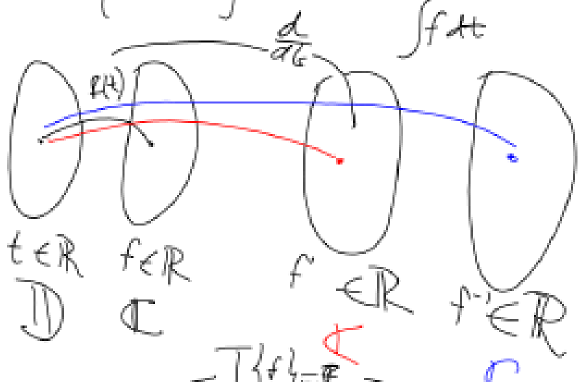




$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = P(t)$$

$$L\frac{d^2i}{dt} + R\frac{di}{dt} + \frac{1}{C}i = \frac{d}{dt}P(t)$$

$T\{f(t)\} = F(s)$



$af + bg \xrightarrow{T\{\}} aF + bG$

$f' \xrightarrow{\text{fácil}} sF - f(0)$

$\int f \longleftarrow \frac{F}{s}$

fácil

difícil

$$T\{f(t)\} = \int_{-\infty}^{\infty} f(t)\cdot N(s,t)\,dt$$

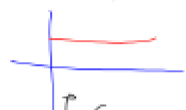
Laplace

$$N(s,t) = \begin{cases} 0 & ; t<0 \\ e^{-st} & ; t \geq 0 \end{cases}$$

$$\mathcal{L}\{f(t)\} = \int_{0}^{\infty} f(t)\,e^{-st}\,dt$$

$$\mathcal{L}\{f(t)\} = \left[\int f(t)\,e^{-st}\,dt\right]_0^{\infty}$$

$f(t) = 1$



$$\mathcal{L}\{1\} = \left[\int (1)\,e^{-st}\,dt\right]_0^{\infty}$$

$$\mathcal{L}\{1\} = \left[\frac{e^{-st}}{-s}\right]_0^{\infty}$$

$$= -\frac{1}{s}\left[\lim_{t\to\infty} e^{-st} - 1\right]$$

$$\lim_{t\to\infty} e^{-st} = \lim_{t\to\infty} \frac{1}{e^{st}} \Rightarrow \lim_{b\to\infty} \frac{1}{b} = 0$$

$$\mathcal{L}\{1\} = -\frac{1}{s}\left[(0) - 1\right]$$

$$\boxed{\mathcal{L}\{1\} = \frac{1}{s}} \quad s \in \mathbb{C}$$

$$\mathcal{L}\{\sqrt{3}\} = \sqrt{3}\,\mathcal{L}\{1\}$$

$$= \frac{\sqrt{3}}{s}$$

① propiedad lineal

$$\mathcal{L}\{a f(t) + b\, g(t)\} = a F(s) + b\, G(s)$$

$a, b \in \mathbb{R}$

② propiedad semejanza

$$\mathcal{L}\{f(\alpha t)\} = \frac{1}{\alpha} F\left(\frac{s}{\alpha}\right) \qquad \mathcal{L}\{f(t)\} = F(p)$$

$\alpha > 0$

$$\mathcal{L}\{t\} = \left[\int t e^{-st} dt\right]_0^\infty$$

$$\int t e^{-st} dt = -\frac{t e^{-st}}{s} + \frac{1}{s}\int e^{-st} dt$$

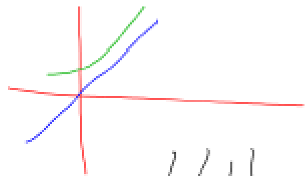
$u = t \quad du = dt$

$dV = e^{-st} dt \quad V = \frac{e^{-st}}{-s}$

$$\int t e^{-st} dt = \left[-\frac{t e^{-st}}{s} - \frac{e^{-st}}{s^2}\right]_0^\infty$$

$$= -\frac{1}{s}\left[\lim_{t\to\infty} \overset{0}{e^{-st}} \lim_{t\to\infty} \overset{\infty}{t} - (0)\right] -$$

$$= -\frac{1}{s^2}\left[\lim_{t\to\infty} \overset{0}{e^{-st}} - 1\right]$$



$$\mathcal{L}\{1\} = \frac{1}{s}$$

$$\mathcal{L}\{t\} = \frac{1}{s^2}$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$
$$n \in \mathbb{Z}^+$$

$$\mathcal{L}\{t^3\} = \frac{6}{s^4}$$

$$\mathcal{L}\{e^{at}\} = \left[\int e^{at} e^{-st} dt\right]_0^\infty$$
$$a \in \mathbb{R}$$

$$= \left[\int e^{-(s-a)t} dt\right]_0^\infty$$

$$= \left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^\infty$$

$$= -\frac{1}{s-a}\left[\lim_{t\to\infty} e^{-(s-a)t} - 1\right]$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a} \qquad \mathcal{L}\{1\} = \frac{1}{s}$$
$$a \in \mathbb{R}$$

$$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$$

$$\mathcal{L}\{t^2 e^{at}\} = \frac{2}{(s-a)^3} \qquad \mathcal{L}\{t^2\} = \frac{2!}{s^3}$$

$$\mathcal{L}\{\cos(bt)\} = \frac{s}{s^2+b^2}$$

$$\mathcal{L}\{\text{sen}(bt)\} = \frac{b}{s^2+b^2}$$

$$\mathcal{L}\{e^t\} = \frac{1}{s-1}$$

$$\mathcal{L}\{e^{2t}\} = \frac{1}{2} \cdot \frac{1}{\frac{s}{2}-1}$$

$$= \frac{1}{2}\left(\frac{1}{\frac{s-2}{2}}\right)$$

$$= \frac{1}{2}\left(\frac{2}{s-2}\right)$$

$$\mathcal{L}\{e^{2t}\} = \frac{1}{s-2}$$

$$\mathcal{L}\{e^{-3t}\cos(5t)\} = \frac{(s+3)}{(s+3)^2+25}$$

$$\mathcal{L}\{\cos(5t)\} = \frac{s}{s^2+25}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+25}\right\} = \cos(5t)$$

$f(t)$ $\qquad F(s)=$

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

$$\mathcal{L}\{f'''(t)\} = s^3F(s) - s^2f(0) - sf'(0) - f''(0)$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^nF(s) - \sum_{i=0}^{n-1} s^{(n-1-i)} f^{(i)}(0)$$

$$\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{F(s)}{s}$$

$$y'' - 5y' + 6y = 0 \qquad \begin{array}{l} y(0) = 2 \\ y'(0) = -2 \end{array}$$

$$\left[s^2F(s) - s\cdot 2 - (-2)\right] - 5\left[sF(s) - 2\right] + 6F(s) = 0$$

$$\left(s^2 - 5s + 6\right)F(s) - 2s + 12 = 0$$

$$F(s) = \frac{2s - 12}{s^2 - 5s + 6} \Rightarrow \frac{2s-12}{(s-2)(s-3)}$$

$$F(s) = \frac{A}{s-2} + \frac{B}{s-3}$$

$$\frac{2s-12}{(s-2)(s-3)} = \frac{A}{s-2} + \frac{B}{s-3}$$

$$2s - 12 = A(s-3) + B(s-2)$$

$$= (A+B)s - (3A+2B)$$

$$\begin{array}{ll} A+B = 2 & 8+B = 2 \\ 3A+2B = 12 & B = -6 \\ -2A-2B = -4 & \\ \hline A = 8 & \end{array}$$

$$F(s) = \frac{8}{s-2} - \frac{6}{s-3}$$

$$\mathcal{L}^{-1}\{F(s)\} = y(t) = 8\,\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} - 6\,\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\}$$

$$y(t) = 8e^{2t} - 6e^{3t}$$