

$$\mathcal{L} \left\{ f(t) \right\} = \int_{-\infty}^{\infty} f(t) e^{-st} dt \Rightarrow F(s)$$

$$\mathcal{L}^{-1} \left\{ F(s) \right\} = \int_{-\infty}^{\infty} e^{st} F(s) ds \Rightarrow f(t)$$

$$\mathcal{L} \left\{ f'(t) \right\} = sF(s) - f(0)$$

$$\mathcal{L} \left\{ \int_0^t f(\tau) d\tau \right\} = \frac{F(s)}{s}$$

$$\mathcal{L}^{-1} \left\{ \int_s^t f(\tau) d\tau \right\} = \frac{f(t)}{t}$$

$$\mathcal{L} \left\{ e^{\alpha t} f(t) \right\} = F(s-\alpha)$$

$$\mathcal{L}^{-1} \left\{ e^{-\alpha s} F(s) \right\} = f(t-\alpha)$$

$$\mathcal{L}^{-1} \left\{ F(s) G(s) \right\} = f(t) * g(t)$$

convolución

$$f(t) * g(t) = \int_0^t f(z) \cdot g(t-z) dz$$

$$\begin{aligned}
& \mathcal{L}^{-1} \left\{ \frac{\alpha s}{(s^2 + \alpha^2)^2} \right\} \\
& \mathcal{L}^{-1} \left\{ \frac{s}{(s^2 + \alpha^2)} \cdot \frac{\alpha}{(s^2 + \alpha^2)} \right\} = \cos(\alpha t) * \sin(\alpha t) \\
& \cos(\alpha t) * \sin(\alpha t) = \int_0^t \cos(\alpha z) \sin(\alpha(t-z)) dz \\
& = \left[ \int \cos(\alpha z) [\sin(\alpha t) \cos(\alpha z) - \cos(\alpha t) \sin(\alpha z)] dz \right]_0^t \\
& = \left[ \sin(\alpha t) \left[ (\cos^2 \alpha z) dz - \cos(\alpha t) \int \cos(\alpha z) \sin(\alpha z) dz \right] \right]_0^t \\
& = \left[ \sin(\alpha t) \left[ \left( \frac{1}{2} + \frac{1}{2} \cos(2\alpha z) \right) dz - \right. \right. \\
& \quad \left. \left. - (\sin(\alpha t)) \int \sin(\alpha z) \cos(\alpha z) dz \right] \right]_0^t \\
& = \left[ \frac{\sin(\alpha t)}{2} \int dz + \frac{\sin(\alpha t)}{4\alpha} \int \cos(2\alpha z) 2\alpha dz - \right. \\
& \quad \left. - \cos(\alpha t) \frac{\sin^2(\alpha z)}{\alpha} \right]_0^t \\
& = \left[ \frac{\sin(\alpha t)}{2} + \frac{\sin(\alpha t) \sin(2\alpha t)}{4\alpha} - \right. \\
& \quad \left. - \frac{\cos(\alpha t)}{2\alpha} \sin^2(\alpha t) \right] \\
& = \frac{t \sin(\alpha t)}{2} + \frac{\sin(\alpha t) \sin(2\alpha t)}{4\alpha} - \\
& \quad - \frac{\cos(\alpha t)}{2\alpha} \sin^2(\alpha t) \\
& = \frac{t \sin(\alpha t)}{2} + \frac{\sin(\alpha t) (\sin(\alpha t) \cos(\alpha t))}{2\alpha} - \\
& \quad - \frac{\cos(\alpha t)}{2\alpha} \sin^2(\alpha t) \\
& = \frac{t \sin(\alpha t)}{2}
\end{aligned}$$

$$L^{-1} \left\{ \frac{s-1}{s^2+25} \right\} =$$

$$L^{-1} \left\{ \frac{s}{s^2+25} - \frac{1}{s^2+25} \right\} =$$

$$L^{-1} \left\{ \frac{s}{s^2+25} \right\} - \frac{1}{5} L^{-1} \left\{ \frac{5}{s^2+25} \right\} =$$

$$\cos(5t) - \frac{1}{5} \operatorname{sech}(5t)$$

