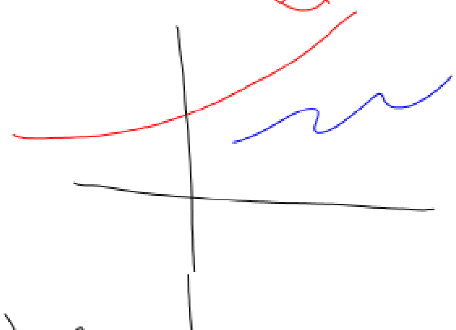
# Teorema de existencia y unicidad de la transf. de Laplace

Dada una función $f(t)$ que cumpla con:

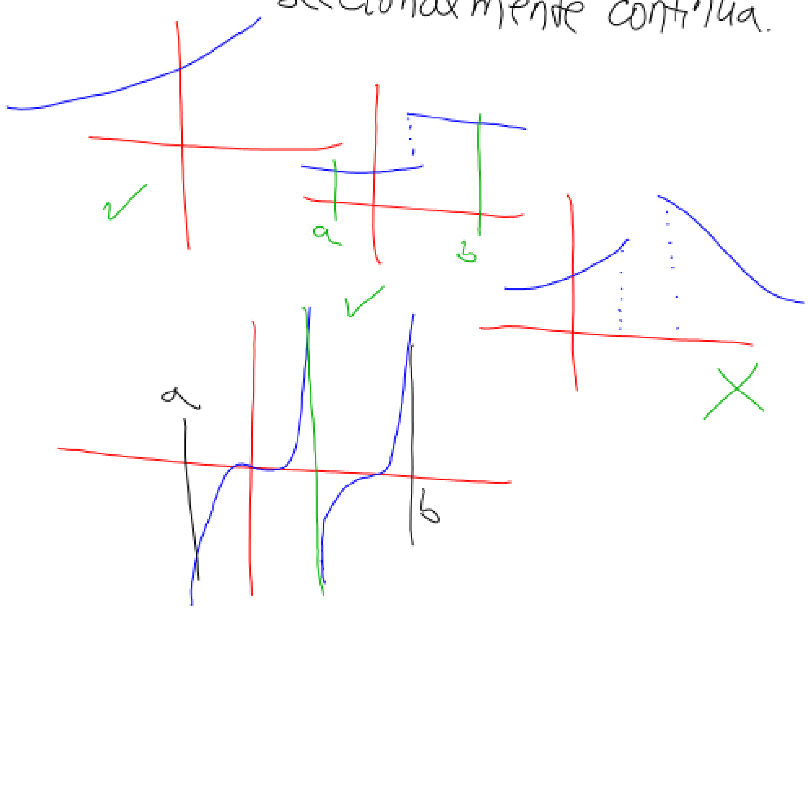
a) ser una función de orden exponencial

$$|f(t)| \le Me^{at} \quad M, a \in \mathbb{R}$$

$$e^{5t} \longrightarrow e^{5t^2} \quad e^{5t^3} \quad e^{5t^n} \quad n>1$$

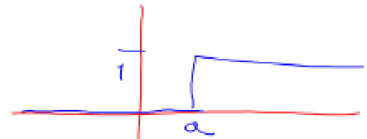


b) $f(t)$ debe ser continua ó seccionalmente continua.



$$u(t-a) = \begin{cases} 0 \; ; & t < a \\ 1 \; ; & t \geq a \end{cases}$$

Escalón unitario




$$\mathcal{L}\left(\text{Heaviside}(t-a)\right) = \frac{e^{-as}}{s}$$

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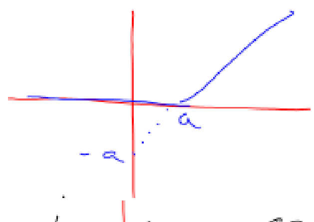
$$r(t-a) = \begin{cases} 0 \; ; & t < a \\ (t-a) \; ; & t \geq a \end{cases}$$

rampa unitaria




$$\mathcal{L}\left(u(t-a)\right) = \frac{e^{-as}}{s}$$

$$\mathcal{L}\left(r(t-a)\right) = \frac{e^{-as}}{s^2}$$

$$\mathcal{L}\left\{ f'(t) \right\} = sF(s) - f(0)$$

$$= s\left(\frac{e^{-as}}{s^2}\right)$$

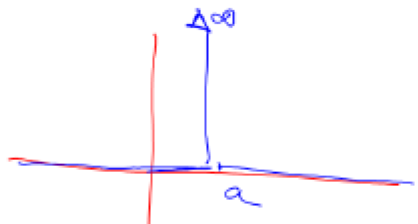
$$\mathcal{L}\left\{ u(t-a) \right\} = \frac{e^{-as}}{s}$$

$$\mathcal{L}\left\{ \frac{d}{dt} r(t-a) \right\} = \mathcal{L}\left\{ u(t-a) \right.$$

$$\frac{d}{dt} r(t-a) = u(t-a)$$

$$\delta(t-a) = \begin{cases} 0 \; ; \; t \neq a \\ \displaystyle\int_{-\infty}^{\infty} \delta(t-a)\,dt = 1. \end{cases}$$

impulso unitario




Dirac(t - a)

$$\mathcal{L}\{u(t-5)\} = \frac{e^{-5s}}{s}$$

$$\mathcal{L}\{\delta(t-5)\} = e^{-5s}$$

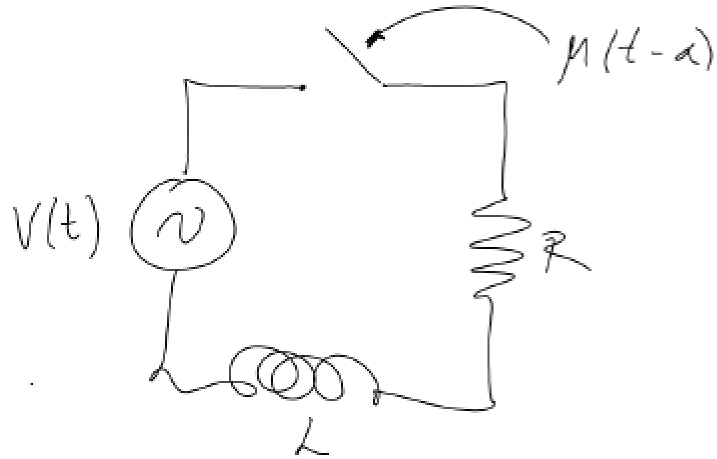
$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$= s\left(\frac{e^{-5s}}{s}\right)$$

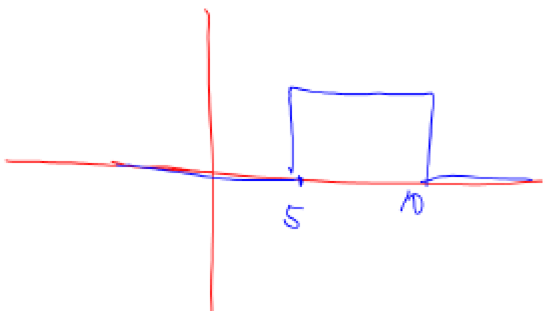
$$\mathcal{L}\{u'(t-a)\} = \mathcal{L}\{\delta(t-a)\}$$

$$u'(t-a) = \delta(t-a).$$

$$r'(t-a) = u(t-a)$$




$u(t-5) - u(t-10)$




TAREA: Utilizando TL.

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 3y = u(t-4)$$

$y(0) = 1 \quad y'(0) = -2$

lunes 25 antes de 23:59.