TEMA 3b. SISTEMAS EDO(1)LCC $\begin{cases} H. \\ NH. \end{cases}$

$$\frac{dx_1(t)}{dt} = a_{11}x_1(t) + a_{12}x_2(t)$$

$$\frac{dx_2(t)}{dt} = a_{21}x_1(t) + a_{22}x_2(t)$$

$$a_{21}x_1(t) = \frac{dx_2(t)}{dt} - a_{22}x_2(t)$$

$$x_1(t) = \frac{1}{a_{21}}\frac{dx_2(t)}{dt} - \frac{a_{22}}{a_{21}}x_2(t)$$

$\frac{d}{dt}$

$$\frac{dx_1(t)}{dt} = \frac{1}{a_{21}}\frac{d^2x_2(t)}{dt^2} - \frac{a_{22}}{a_{21}}\frac{dx_2}{dt}$$

$$\left[\frac{1}{a_{21}}\frac{d^2x_2(t)}{dt} - \frac{a_{22}}{a_{21}}\frac{dx_2}{dt}\right] = a_{11}\left[\frac{1}{a_{21}}\frac{dx_2}{dt} - \frac{a_{22}}{a_{21}}x_2(t)\right] + a_{12}x_2(t)$$

$$\frac{d^2x_2(t)}{dt^2} - a_{22}\frac{dx_2(t)}{dt} = a_{11}\frac{dx_2}{dt} - a_{22}x_2(t) + a_{21}a_{12}x_2(t)$$

$$\frac{d^2x_2(t)}{dt^2} - (a_{22}+a_{11})\frac{dx_2}{dt} + (a_{22} - a_{21}a_{12})x_2(t) = 0.$$

EDO(2)LCCH. ⟶ S(2)EDO(1)LCCH.

$$x_2(t) = C_1 x x_1(t) + C_2 x x_2(t)$$

$$\frac{dx_2(t)}{dt} = C_1\frac{d}{dt}x x_1(t) + C_2\frac{d}{dt}x x_2(t).$$

(I) $X_1'(t) = 2X_1(t) + 3X_2(t)$ $\quad X_1(0) = 4$

(II) $X_2'(t) = X_1(t) + 4X_2(t)$ $\quad X_2(0) = -3$

De (II) despejamos $X_1(t)$

$\rightarrow X_1(t) = X_2'(t) - 4X_2(t)$

$X_1'(t) = X_2''(t) - 4X_2'(t)$

$X_2''(t) - 4X_2'(t) = 2\left(X_2'(t) - 4X_2(t)\right) + 3X_2(t)$

$X_2''(t) - 6X_2'(t) + 5X_2(t) = 0$. EDO(2) Lcch

$(D^2 - 6D + 5)X_2(t) = 0$

$(D-1)(D-5)X_2(t) = 0$

$\rightarrow X_2(t) = C_1 e^t + C_2 e^{5t}$

$X_2'(t) = C_1 e^t + 5C_2 e^{5t}$

$X_1(t) = \left(C_1 e^t + 5C_2 e^{5t}\right) - 4\left(C_1 e^t + C_2 e^{5t}\right)$

$X_1(t) = -3C_1 e^t + C_2 e^{5t}$

$X_2(t) = C_1 e^t + C_2 e^{5t}$

$-3C_1 = C_{10}$

$C_2 = C_{20}$

$C_1 = \frac{C_{10}}{-3}$

$X_1(0) \Rightarrow 4 = -3C_1 + C_2$

$X_2(t) \Rightarrow -3 = C_1 + C_2$

$7 = -4C_1$ $\quad C_1 = -\frac{7}{4}$

$-3 = -\frac{7}{4} + C_2$ $\quad C_2 = -\frac{5}{4}$

$-3 + \frac{7}{4} = C_2$

$-\frac{5}{4} = C_2$

$X_1(t) = \frac{21}{4}e^t - \frac{5}{4}e^{5t}$

$X_2(t) = -\frac{7}{4}e^t - \frac{5}{4}e^{5t}$

$$\bar{X} = \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} \qquad \frac{d}{dt}\bar{X} = \begin{bmatrix} X_1'(t) \\ X_2'(t) \end{bmatrix}$$

$$\begin{bmatrix} X_1'(t) \\ X_2'(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}}_{A} \cdot \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix}$$

$$\frac{d}{dt}\bar{X} = A \cdot \bar{X} \qquad \bar{X}(0) = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

$$\bar{X} = e^{At} \cdot \bar{X}(0)$$

Matriz exponencial

$$e^{At}\Big|_{t=0} = I$$

$$\frac{d}{dt}e^{At} = Ae^{At}$$

$$\frac{d^3y}{dt^3} - 4\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - 6y = 0$$

$y(0) = 1$
$y'(0) = -2$
$y''(0) = 3$

EDO (3) Lech.

$$\frac{dy(t)}{dt} = y_2(t)$$

$$\frac{dy_2(t)}{dt} = y_3(t)$$

$$\frac{dy_3(t)}{dt} = 6y_1(t) - 3y_2(t) + 4y_3(t)$$

$$y_1(0) = 1$$
$$y_2(0) = -2$$
$$y_3(0) = 3$$

$$\frac{d}{dt}\begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -3 & 4 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix}$$