Matriz Exponencial.

$$e^{At}\Big|_{t=0} = I.$$

$$\frac{d}{dt} e^{At} = A e^{At}$$

$$\left[e^{At}\right]^{-1} = e^{At}\Big|_{t=-t}$$

$$MatExp := \begin{bmatrix} \frac{3}{4}e^{t} + \frac{1}{4}e^{5t} & \frac{3}{4}e^{5t} - \frac{3}{4}e^{t} \\ \frac{1}{4}e^{5t} - \frac{1}{4}e^{t} & \frac{1}{4}e^{t} + \frac{3}{4}e^{5t} \end{bmatrix}$$

$$e^{At} = \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} e^{t} + \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} e^{5t}$$

$$e^{At}\Big|_{t=0} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$e^{t}=1+t+\frac{t^2}{2!}+\frac{t^3}{3!}+\ldots+\frac{t^n}{n!}+\ldots$$

$$\longrightarrow e^{at}=1+at+\frac{a^2}{2!}t^2+\frac{a^3}{3!}t^3+\ldots+\frac{a^n}{n!}t^n+\ldots$$

$$e=1+1+\frac{1}{2}+\frac{1}{6}+\ldots+\frac{1}{n!}+\ldots$$

$$\doteq 2{,}72\ldots$$

$$\longrightarrow e^{At}=I+At+\frac{A^2}{2!}t^2+\frac{A^3}{3!}t^3+\ldots+\frac{A^n}{n!}t^n+\ldots$$

$A_{n\times n}$ Teorema Hamilton-Cayle

"Toda matriz A satisface su propia ecuación característica"

$$A=\begin{bmatrix}2 & 3\\ 1 & 4\end{bmatrix}$$

$$\det(A-\lambda I)=0$$

$$\begin{vmatrix}2-\lambda & 3\\ 1 & 4-\lambda\end{vmatrix}=0$$

$$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$(2-\lambda)(4-\lambda)-(3)(1)=0$$

$$\lambda^2-6\lambda+8-3=0$$

$$\lambda^2-6\lambda+5=0$$

$$= \begin{bmatrix} 4+3 & 6+12 \\ 2+4 & 3+16 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 18 \\ 6 & 19 \end{bmatrix}$$

$$A^2-6A+5I=[0]$$

$$\begin{bmatrix} 7 & 18 \\ 6 & 19 \end{bmatrix} - \begin{bmatrix} 12 & 18 \\ 6 & 24 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$e^{At} = I + At + \frac{A^2}{2!}t^2 + \ldots\ldots + \frac{A^{n-1}}{(n-1)!}t^{n-1} + \frac{A^n}{n!}t^n + \ldots\ldots$$

$$\det(A-\lambda I)=0$$

$$A^n + a_1A^{n-1} + a_2A^{n-2} + \ldots\ldots + a_nI = 0$$

$$A^n = \underbrace{-a_nI - a_{n-1}A - \ldots\ldots - a_2A^{n-2} - a_1A^{n-1}}_{\text{"n" términos}}$$

$$A^{n+1} = -a_nA - a_{n-1}A^2 - \ldots\ldots - a_2A^{n-1} - a_1A^n$$

$$A^{n+1} = b_nI + b_{n-1}A + b_{n-2}A^2 + \ldots\ldots + b_1A^{n-1}$$

$$A^{n+2} = c_nI + c_{n-1}A + c_{n-2}A^2 + \ldots\ldots + c_nA^{n-1}$$

$$\underline{e^{At}} = B_0(t)I + B_1(t)A + B_2(t)A^2 + \ldots\ldots + B_{n-1}(t)A^{n-1}$$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \qquad \overline{e^{At}}$$

$$\lambda^2 - 6\lambda + 5 = 0$$

$$(\lambda - 1)(\lambda - 5) = 0 \qquad \lambda_1 = 1, \quad \lambda_2 = 5$$

$$\boxed{e^{At} = B_0(t)\, I + B_1(t)\, A.}$$

$$\begin{cases} e^{t} = B_0 + B_1 \\ e^{5t} = B_0 + 5B_1 \end{cases}$$

$$-e^{t} = -B_0 - B_1$$

$$e^{5t} - e^{t} = 4B_1$$

$$B_1(t) = \frac{1}{4}\left(e^{5t} - e^{t}\right)$$

$$B_0(t) = e^{t} - \frac{1}{4}\left(e^{5t} - e^{t}\right)$$

$$B_0(t) = \frac{1}{4}\left(5e^{t} - e^{5t}\right)$$

$$e^{At} = \frac{1}{4}\left(5e^{t} - e^{5t}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{1}{4}\left(e^{5t} - e^{t}\right)\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$e^{At} = \frac{1}{4}\begin{bmatrix} 5-2 & -3 \\ -1 & 5-4 \end{bmatrix} e^{t} + \frac{1}{4}\begin{bmatrix} -1+2 & 3 \\ 1 & -1+4 \end{bmatrix} e^{5t}$$

$$e^{At} = \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} e^{t} + \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} e^{5t}$$

$$\frac{dx_1(t)}{dt} = a_{11}x_1(t) + a_{12}x_2(t) + b_1(t)$$

$$\frac{dx_2(t)}{dt} = a_{21}x_1(t) + a_{22}x_2(t) + b_2(t)$$

$S(z) \in$ DOL(1)cc NH.

$$\frac{d}{dt}\bar{X}(t) = A\bar{x}(t) + \bar{b}(t)$$

$$\frac{d}{dt}\bar{x}(t) = A\bar{x}(t)$$

$$\bar{X}_H(t) = e^{At}\bar{x}(0)$$

$$\bar{X}(t) = e^{At}\bar{x}(0) + \int_0^t e^{A(t-z)}\bar{b}(z)\,dz$$

$$\frac{d}{dt}\bar{X}(t) = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\bar{x}(t) + \begin{bmatrix} t^2 \\ 5e^t \end{bmatrix} \qquad \bar{X}(0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$e^{At} = \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}e^t + \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}e^{5t}$$

$$\bar{X}_H(t) = \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix}e^t + \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix}e^{5t}$$

$$= \frac{1}{4}\begin{bmatrix} -6 \\ 2 \end{bmatrix}e^t + \frac{1}{4}\begin{bmatrix} 2 \\ 2 \end{bmatrix}e^{5t}$$

$$\int_0^t e^{A(t-z)}\bar{b}(z)\,dz = \int_0^t\left(\frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} z^2 \\ 5e^z \end{bmatrix}e^{(t-z)} + \right.$$

$$\left. + \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} z^2 \\ 5e^z \end{bmatrix}e^{5(t-z)}\right)dz.$$