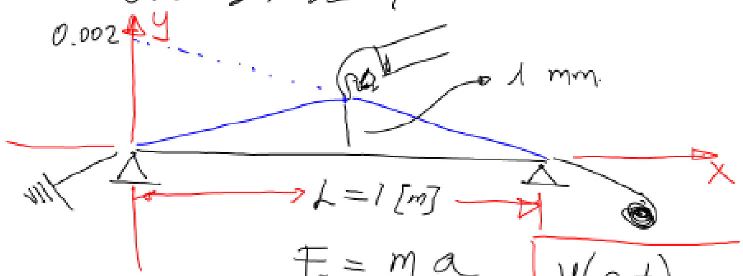
CUERDA DE GUITARRA.



$$F_r = m\,a$$

$$a = \frac{\partial^2 y}{\partial t^2}$$

$$F_r = k^2 \frac{\partial^2 y}{\partial t^2}$$

$$\boxed{\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}}$$

$$\left.\begin{aligned} y(0,t) &= 0 \\ y(l,t) &= 0 \end{aligned}\right\} \text{CF.}$$

$$y(x,0) = \begin{cases} \frac{0.001}{0.50}x \ ; & 0 \leq x \leq 0.50 \\ 0.002 - \frac{0.001}{0.5}x \ ; & 0.5 < x < 1 \end{cases}$$

$$\left.\frac{\partial y}{\partial t}\right|_{t=0} = 0$$

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \qquad c^2 \in \mathbb{R}^+$$

H_0 $\quad y(x,t) = F(x)\cdot G(t)$

$$\frac{\partial y}{\partial t} = F(x)\cdot G'(t) \qquad \frac{\partial y}{\partial x} = F'(x)\cdot G(t)$$

$$\frac{\partial^2 y}{\partial t^2} = F(x)\cdot G''(t) \qquad \frac{\partial^2 y}{\partial x^2} = F''(x)\cdot G(t)$$

$$F\cdot G'' = c^2 F''\cdot G$$

$$\frac{G''}{G} = c^2 \frac{F''}{F}$$

$$c^2 \frac{F''}{F} = \alpha \qquad\qquad \frac{G''}{G} = \alpha$$

para $\alpha = 0$

$$c^2 \frac{F''}{F} = 0 \quad F''(x) = 0 \quad F'(x) = k_1$$

$$\boxed{F(x) = k_1 x + k_2}$$

$y(0,t) = 0$

$F(0) \cdot G(t) = 0$

$F(0) = 0$

$y(\ell, t) = 0$

$F(\ell) \cdot G(t) = 0.$

$F(\ell) = 0$

$k_1(0) + k_2 = 0 \quad - ①$

$k_1(\ell) + k_2 = 0 \quad - ②$

De ① $\quad k_2 = 0$

$k_1(\ell) = 0 \quad k_1 = 0$

para $\alpha > 0 \quad \alpha = \beta^2 \quad \forall \beta \neq 0 \in \mathbb{R}$

$$c^2 \frac{F''(x)}{F(x)} = \beta^2 \quad F''(x) = \frac{\beta^2}{c^2} F(x)$$

$$F''(x) - \frac{\beta^2}{c^2} F(x) = 0$$

$$\left(D^2 - \frac{\beta^2}{c^2}\right) F(x) = 0 \quad F(x) = k_1 e^{\frac{\beta}{c}x} + k_2 e^{-\frac{\beta}{c}x}$$

$F(0) = 0$

$F(\ell) = 0$

$$k_1 e^{\frac{\beta}{c}(0)} + \frac{k_2}{e^{\frac{\beta}{c}(0)}} = 0$$

$$K_1 + k_2 = 0$$

$$k_1 e^{\frac{\beta}{c}\ell} + \frac{k_2}{e^{\frac{\beta}{c}\ell}} = 0$$

$$-k_2 e^{\frac{\beta}{c}\ell} + \frac{k_2}{e^{\frac{\beta}{c}\ell}} = 0$$

$$k_2 e^{\frac{\beta}{c}\ell} = \frac{k_2}{e^{\frac{\beta}{c}\ell}}$$

$$\left(e^{\frac{\beta}{c}\ell}\right)^2 = 1 \quad \times$$

para $\alpha < 0$ $\quad \alpha = -\beta^2 \quad \forall \beta \neq 0 \in \mathbb{R}$.

$$c^2 \frac{F''(x)}{F(x)} = -\beta^2 \qquad F''(x) = -\frac{\beta^2}{c^2} F(x)$$

$$F''(x) + \frac{\beta^2}{c^2} F(x) = 0$$

$$\left(D^2 + \frac{\beta^2}{c^2}\right) F(x) = 0$$

$$m^2 + \frac{\beta^2}{c^2} = 0 \qquad m_1 = \frac{\beta}{c} i$$

$$m_2 = -\frac{\beta}{c} i$$

$$F(x) = k_1 \cos\left(\frac{\beta}{c} x\right) + k_2 \operatorname{sen}\left(\frac{\beta}{c} x\right)$$

$F(0) = 0$

$F(\ell) = 0 \qquad k_1(1) + k_2(0) = 0 \qquad k_1 = 0$

$$k_2 \operatorname{sen}\left(\frac{\beta}{c} \ell\right) = 0 \qquad \operatorname{sen}\left(\frac{\beta}{c} \ell\right) = \operatorname{sen}(n\pi) \quad n \in \mathbb{Z}$$

$$n\pi = \frac{\beta}{c} x$$

$$\beta = n\pi c$$

$$F(x) = k_2 \operatorname{sen}(n\pi x) \qquad k_2 \neq 0.$$

$$\frac{G''(t)}{G(t)} = -n^2 \pi^2 c^2$$

$$G''(t) = -n^2 \pi^2 c^2 G(t)$$

$$G''(t) + n^2 \pi^2 c^2 G(t) = 0$$

$$G(t) = k_{10} \cos(n\pi c t) + k_{20} \operatorname{sen}(n\pi c t)$$

$$y(x,t) = \left(k_1 \cos(n\pi c t) + k_2 \operatorname{sen}(n\pi c t)\right) \operatorname{sen}(n\pi x).$$

$$t\left(\frac{\partial u}{\partial t}\right)=x\left(\frac{\partial u}{\partial x}\right) \qquad \alpha=1$$

$$u(x,t)=F(x)\,G(t)$$

$$\frac{\partial u}{\partial t}=F\cdot G' \qquad \frac{\partial u}{\partial x}=F'G$$

$$tF\cdot G'=xF'G$$

$$\frac{tG'}{G}=\frac{xF'}{F} \qquad \frac{tG'}{G}=1 \qquad \frac{xF'}{F}=1$$