```
> restart
> AA := array([[2, 3], [1, 4]])
```

$$AA := \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \tag{1}$$

```
> with(linalg) :
> MatExp := exponential(AA, t)
```

$$MatExp := \begin{bmatrix} \frac{3}{4}\,e^{t} + \frac{1}{4}\,e^{5t} & \frac{3}{4}\,e^{5t} - \frac{3}{4}\,e^{t} \\ \frac{1}{4}\,e^{5t} - \frac{1}{4}\,e^{t} & \frac{1}{4}\,e^{t} + \frac{3}{4}\,e^{5t} \end{bmatrix} \tag{2}$$

```
> DerMatExp := map(diff, MatExp, t)
```

$$DerMatExp := \begin{bmatrix} \frac{3}{4}\,e^{t} + \frac{5}{4}\,e^{5t} & \frac{15}{4}\,e^{5t} - \frac{3}{4}\,e^{t} \\ \frac{5}{4}\,e^{5t} - \frac{1}{4}\,e^{t} & \frac{1}{4}\,e^{t} + \frac{15}{4}\,e^{5t} \end{bmatrix} \tag{3}$$

```
> InvMatExp := map(rcurry(eval, t='-t'), MatExp)
```

$$InvMatExp := \begin{bmatrix} \frac{3}{4}\,e^{-t} + \frac{1}{4}\,e^{-5t} & \frac{3}{4}\,e^{-5t} - \frac{3}{4}\,e^{-t} \\ \frac{1}{4}\,e^{-5t} - \frac{1}{4}\,e^{-t} & \frac{1}{4}\,e^{-t} + \frac{3}{4}\,e^{-5t} \end{bmatrix} \tag{4}$$

```
> AAA := simplify(evalm(DerMatExp &* InvMatExp))
```

$$AAA := \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \tag{5}$$

```
> restart
> Ecua := diff(z(x, y), x$2) + 4·diff(z(x, y), y) = z(x, y)
```

$$Ecua := \frac{\partial^2}{\partial x^2}\,z(x,y) + 4\left(\frac{\partial}{\partial y}\,z(x,y)\right) = z(x,y) \tag{6}$$

```
> with(PDEtools)
```

[*CanonicalCoordinates*, *ChangeSymmetry*, *CharacteristicQ*, *CharacteristicQInvariants*, *ConservedCurrentTest*, *ConservedCurrents*, *ConsistencyTest*, *D_Dx*, *DeterminingPDE*, *Eta_k*, *Euler*, *FromJet*, *FunctionFieldSolutions*, *InfinitesimalGenerator*, *Infinitesimals*, *IntegratingFactorTest*, *IntegratingFactors*, *InvariantEquation*, *InvariantSolutions*, *InvariantTransformation*, *Invariants*, *Laplace*, *Library*, *PDEplot*, *PolynomialSolutions*, *ReducedForm*, *SimilaritySolutions*, *SimilarityTransformation*, *Solve*, *SymmetryCommutator*, *SymmetryGauge*, *SymmetrySolutions*, *SymmetryTest*, *SymmetryTransformation*, *TWSolutions*, *ToJet*, *build*, *casesplit*, *charstrip*, *dchange*, *dcoeffs*, *declare*, *diff_table*, *difforder*, *dpolyform*, *dsubs*, *mapde*, *separability*, *splitstrip*, *splitsys*, *undeclare*] (7)

```
> SolGral := build(pdsolve(Ecua))
```

(8)

$$SolGral := z(x,y) = e^{\sqrt{-c_1}\,x}\,\_C3\,e^{-\frac{1}{4}y-c_1}\,e^{\frac{1}{4}y}\,\_C1 + \frac{\_C3\,e^{-\frac{1}{4}y-c_1}\,e^{\frac{1}{4}y}\,\_C2}{e^{\sqrt{-c_1}\,x}} \tag{8}$$

> $Comprobar := simplify(eval(subs(z(x,y) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \tag{9}$$

> $SolGralDos := z(x,y) = \_C1 \cdot \exp(-x) + \_C2 \cdot \exp(x)$

$$SolGralDos := z(x,y) = \_C1\,e^{-x} + \_C2\,e^{x} \tag{10}$$

> $ComprobarDos := simplify(eval(subs(z(x,y) = rhs(SolGralDos), lhs(Ecua) - rhs(Ecua) = 0)))$

$$ComprobarDos := 0 = 0 \tag{11}$$

> $SolGralTres := z(x,y) = \left(\_C1 \cdot \exp\left(\sqrt{1+\beta^2}\cdot x\right) + \_C2 \cdot \exp\left(-\sqrt{1+\beta^2}\cdot x\right)\right) \cdot \exp\left(-\frac{\beta^2}{4}\cdot y\right)$

$$SolGralTres := z(x,y) = \left(\_C1\,e^{\sqrt{\beta^2+1}\,x} + \_C2\,e^{-\sqrt{\beta^2+1}\,x}\right) e^{-\frac{1}{4}\beta^2 y} \tag{12}$$

> $ComprobarTres := simplify(eval(subs(z(x,y) = rhs(SolGralTres), lhs(Ecua) - rhs(Ecua) = 0)))$

$$ComprobarTres := 0 = 0 \tag{13}$$

> $SolGralCuatro := z(x,y) = \left(\_C1 \cdot \cos\left(\sqrt{\beta^2-1}\cdot x\right) + \_C2 \cdot \sin\left(\sqrt{\beta^2-1}\cdot x\right)\right) \cdot \exp\left(\frac{\beta^2}{4}\cdot y\right)$

$$SolGralCuatro := z(x,y) = \left(\_C1\cos\left(\sqrt{\beta^2-1}\,x\right) + \_C2\sin\left(\sqrt{\beta^2-1}\,x\right)\right) e^{\frac{1}{4}\beta^2 y} \tag{14}$$

> $ComprobarCuatro := simplify(eval(subs(z(x,y) = rhs(SolGralCuatro), lhs(Ecua) - rhs(Ecua) = 0)))$

$$ComprobarCuatro := 0 = 0 \tag{15}$$

> $SolGralCinco := z(x,y) = \left(\_C1 \cdot \exp\left(\sqrt{1-\beta^2}\cdot x\right) + \_C2 \cdot \exp\left(-\sqrt{1-\beta^2}\cdot x\right)\right) \cdot e^{\frac{1}{4}\beta^2 y}$

$$SolGralCinco := z(x,y) = \left(\_C1\,e^{\sqrt{-\beta^2+1}\,x} + \_C2\,e^{-\sqrt{-\beta^2+1}\,x}\right) e^{\frac{1}{4}\beta^2 y} \tag{16}$$

> $ComprobarCinco := simplify(eval(subs(z(x,y) = rhs(SolGralCinco), lhs(Ecua) - rhs(Ecua) = 0)))$

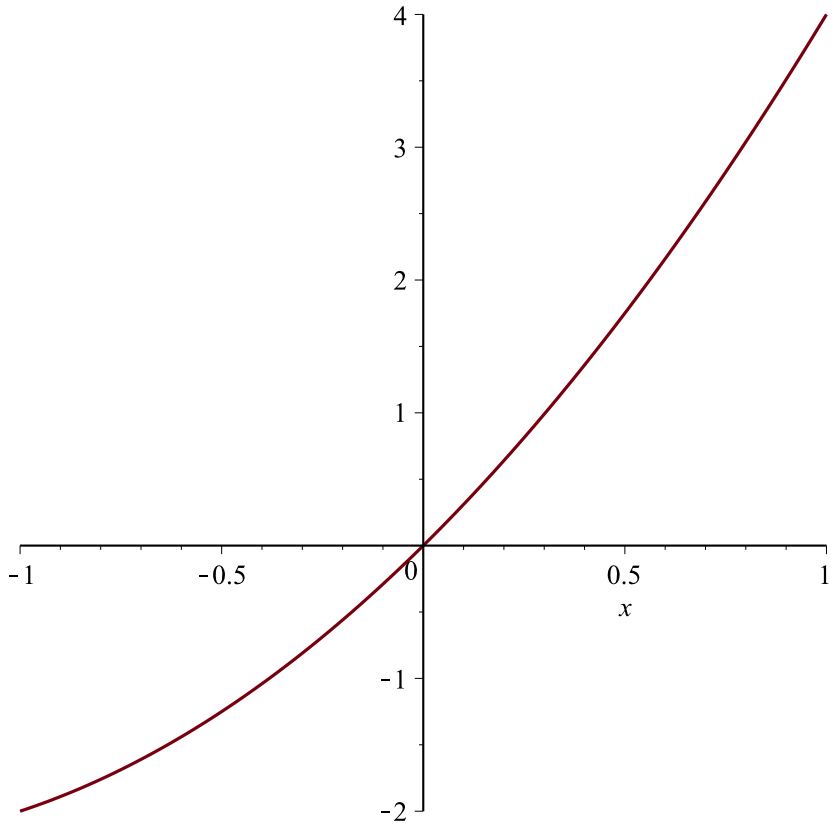
$$ComprobarCinco := 0 = 0 \tag{17}$$

> $restart$

> $f := x^2 + 3x$

$$f := x^2 + 3x \tag{18}$$

> $plot(f, x = -1..1)$



> $L := 1$

$$L := 1 \tag{19}$$

> $a[0] := \frac{1}{L} \cdot int(f, x = -L..L)$

$$a_0 := \frac{2}{3} \tag{20}$$

> $a[n] := \frac{1}{L} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), x = -L..L\right)$

$$a_n := \frac{2\left(\sin(n\pi)\,\pi^2 n^2 + 2\,n\pi\cos(n\pi) - 2\sin(n\pi)\right)}{n^3\,\pi^3} \tag{21}$$

> $b[n] := \frac{1}{L} \cdot int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), x = -L..L\right)$

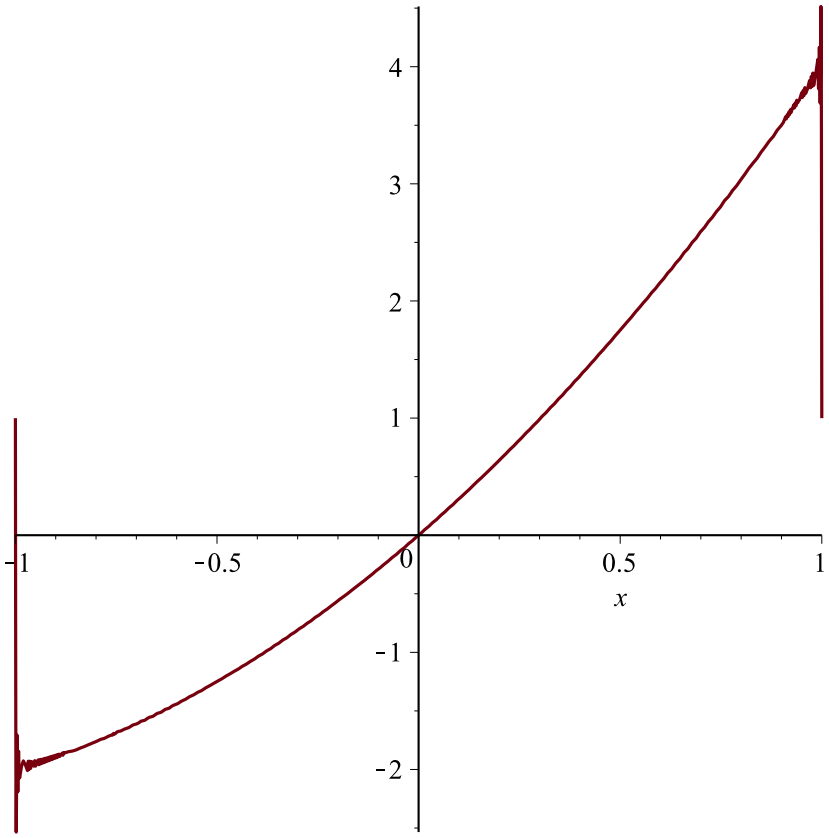
$$b_n := -\frac{6\left(n\pi\cos(n\pi) - \sin(n\pi)\right)}{n^2\,\pi^2} \tag{22}$$

> $STF := \frac{a[0]}{2} + Sum\left(a[n] \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right) + b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), n = 1..infinity\right)$

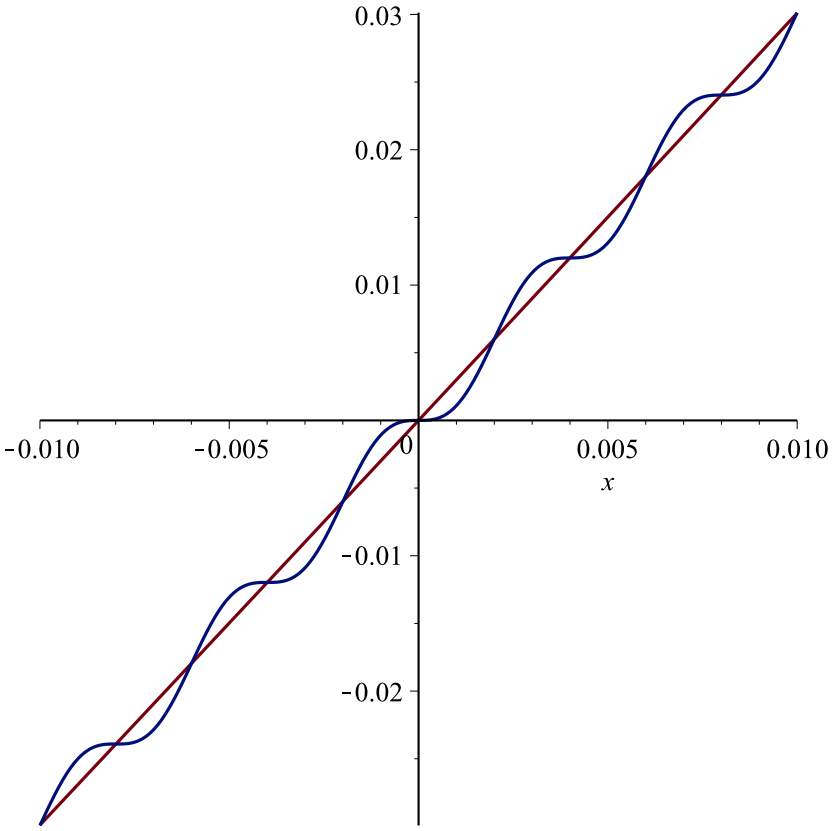
(23)

$$STF := \frac{1}{3} + \sum_{n=1}^{\infty} \left(\frac{2\left(\sin(n\pi)\,\pi^2\,n^2 + 2\,n\,\pi\cos(n\pi) - 2\sin(n\pi)\right)\cos(n\pi x)}{n^3\,\pi^3} - \frac{6\left(n\,\pi\cos(n\pi) - \sin(n\pi)\right)\sin(n\pi x)}{n^2\,\pi^2} \right) \tag{23}$$

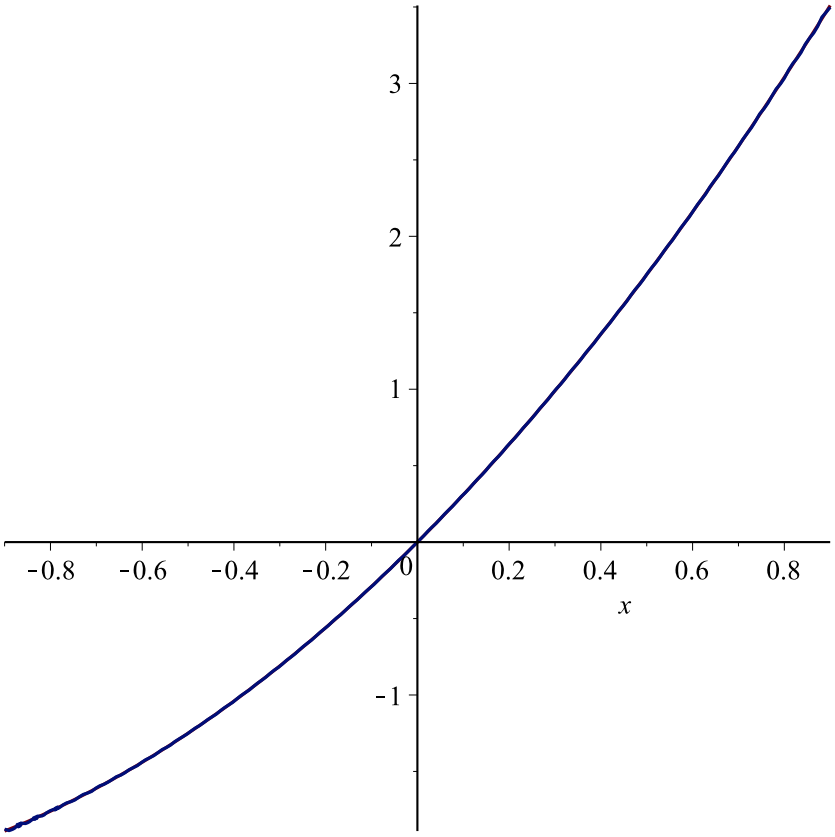
```
> STF500 := a[0]/2 + sum(a[n]*cos(n*Pi*x/L) + b[n]*sin(n*Pi*x/L), n = 1..500):
> plot(STF500, x = -1..1)
```




```
> plot([f, STF500], x = -0.01..0.01)
```

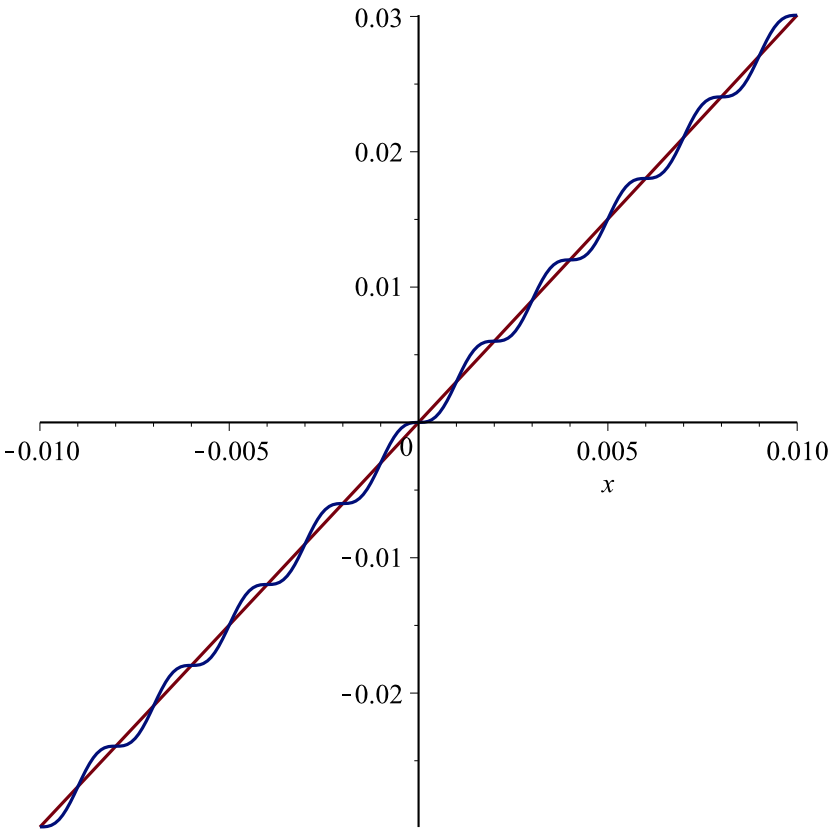



```
> plot( [ f, STF500 ], x =-0.9 ..0.9 )
```



> $STF1000 := \frac{a[0]}{2} + sum\left(a[n]\cdot\cos\left(\frac{n\cdot\mathrm{Pi}\cdot x}{L}\right) + b[n]\cdot\sin\left(\frac{n\cdot\mathrm{Pi}\cdot x}{L}\right), n = 1..1000\right):$

> $plot([f, STF1000], x = -0.01..0.01)$


0.03
0.02
0.01
-0.010
-0.005
0
0.005
0.010
x
-0.01
-0.02