

A

$$e^{At} \frac{d}{dt} e^{At} = A e^{At}$$

$$\frac{d}{dt} e^{At} \times \left[e^{At} \right]' = A$$

Método de Separación Variables
para EDP.

$$\frac{\partial z}{\partial x^2} + 4 \frac{\partial z}{\partial y} = z$$

$$\text{H: } z(x, y) = F(x) \cdot G(y) \quad \begin{cases} z = F(x) + G(y) \\ z = F(x) \cdot L_y \\ z = L_x \cdot G(y) \end{cases}$$
$$\frac{\partial z}{\partial x} = F' \cdot G \quad \frac{\partial z}{\partial x^2} = F'' \cdot G$$
$$\frac{\partial z}{\partial y} = F \cdot G'$$

$$F''G + 4FG' = FG'$$

$$\begin{aligned} F''G &= -4FG' + FG' & F''G - FG' &= -4FG' \\ F''G &= (-4G' + G)F & (F'' - F)G' &= -4G'F \\ \frac{F''}{F} &= \frac{-4G' + G}{G} & \frac{F'' - F}{F} &= \frac{-4G'}{G} \end{aligned}$$

$$\frac{F'' - F}{F} = \frac{-4G'}{g}$$

$$\frac{F'' - F}{F} = \alpha \quad -\frac{4G'}{g} = \alpha$$

$\alpha = 0, \alpha > 0, \alpha < 0$

$\therefore \alpha = 0$

$$\begin{aligned} F'' - F &= 0 & -4G' &= 0 \\ (D^2 - 1)F &= 0 & \frac{-4G'}{g} &= 0 \\ (D+1)(D-1)F &= 0 & -4G' &= 0 \\ F(x) &= C_1 e^{-x} + C_2 e^x & G(y) &= C_3 \\ Z(x,y)_{\alpha=0} &= C_3 (C_1 e^{-x} + C_2 e^x) \\ \boxed{Z(x,y)_{\alpha=0} = C_{10} e^{-x} + C_{20} e^x} \end{aligned}$$

para $\alpha > 0 \quad \alpha = \beta^2 \quad \forall \beta \neq 0 \in \mathbb{R}$

$$\begin{aligned} \frac{F'' - F}{F} &= \beta^2 & -\frac{4G'}{g} &= \beta^2 \\ F'' - \beta^2 F &= 0 & -4G' &= \beta^2 G \\ F'' - (1+\beta^2)F &= 0 & G' &= -\frac{\beta^2}{4} G \\ (D^2 - (1+\beta^2))F &= 0 & G' + \frac{\beta^2}{4} G &= 0 \\ (D - \sqrt{1+\beta^2})(D + \sqrt{1+\beta^2})F &= 0 & G(y) &= C_3 e^{-\frac{\beta^2}{4}y} \\ F(x) &= C_1 e^{\sqrt{1+\beta^2}x} + C_2 e^{-\sqrt{1+\beta^2}x} \\ Z(x,y)_{\alpha>0} &= C_3 \left(C_1 e^{\sqrt{1+\beta^2}x} + C_2 e^{-\sqrt{1+\beta^2}x} \right) e^{-\frac{\beta^2}{4}y} \\ \boxed{Z(x,y)_{\alpha>0} = \left(C_{10} e^{\sqrt{1+\beta^2}x} + C_{20} e^{-\sqrt{1+\beta^2}x} \right) e^{-\frac{\beta^2}{4}y}} \end{aligned}$$

para $\alpha < 0 \quad \alpha = -\beta^2 \quad \forall \beta \neq 0 \in \mathbb{R}$

$$\frac{F'' - F}{F} = -\beta^2 \quad -\frac{4G'}{g} = -\beta^2$$

$$F'' - F = -\beta^2 F \quad G' = \frac{\beta^2}{4} g$$

$$F'' + (-1 + \beta^2)F = 0 \quad G' - \frac{\beta^2}{4}g = 0$$

$$(D + (\beta^2 - 1))F = 0 \quad (D - \frac{\beta^2}{4})g = 0$$

$$(D + (\sqrt{\beta^2 - 1})^2)F = 0 \quad G(y) = C_2 e^{\frac{\beta^2}{4}y}$$

$$f(x) = C_1 \cos(\sqrt{\beta^2 - 1}x) + C_2 \sin(\sqrt{\beta^2 - 1}x)$$

$$\beta^2 - 1 > 0$$

$$\beta^2 > 1$$

$$f(x)_{\substack{\alpha < 0 \\ \beta^2 > 1}} = C_1 \cos(\sqrt{\beta^2 - 1}x) + C_2 \sin(\sqrt{\beta^2 - 1}x)$$

$$z(x, y)_{\substack{\alpha < 0 \\ \beta^2 > 1}} = C_1 \left(C_1 \cos(\sqrt{\beta^2 - 1}x) + C_2 \sin(\sqrt{\beta^2 - 1}x) \right) e^{\frac{\beta^2}{4}y}$$

$$z(x, y)_{\substack{\alpha < 0 \\ \beta^2 > 1}} = (C_{10} \cos(\sqrt{\beta^2 - 1}x) + C_{20} \sin(\sqrt{\beta^2 - 1}x)) e^{\frac{\beta^2}{4}y}$$

$$\underline{z(x, y)}_{\substack{\alpha < 0 \\ \beta^2 < 1}} = (C_{10} e^{\sqrt{1-\beta^2}x} + C_{20} e^{-\sqrt{1-\beta^2}x}) e^{\frac{\beta^2}{4}y}$$

$$(D + \beta^2 - 1)F(x)$$

$$M^2 + \beta^2 - 1 = 0$$

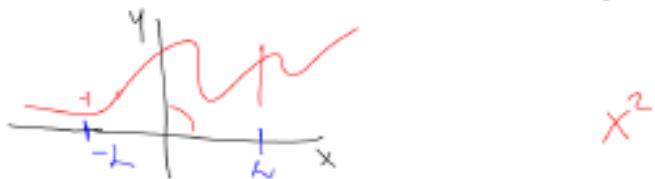
$$M^2 = -(\beta^2 - 1)$$

$$M = \pm \sqrt{\beta^2 - 1} i$$

$$\mathcal{P}(D)z(x,y) = 0$$

$$\text{SOL(GRA)} \quad z(x,y) = f_1(x,y) + f_2(x,y) + \dots + f_n(x,y)$$

SÉRIE TRIGONOMÉTRICA FOURIER



$$\cos(ax) \perp \operatorname{Sen}(ax)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \operatorname{Sen}\left(\frac{n\pi x}{L}\right) \right).$$

$$a_0 = \frac{1}{L} \int_{-L}^{L} f(x) dx$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \operatorname{Sen}\left(\frac{n\pi x}{L}\right) dx$$