

$$\begin{aligned}
& \text{restart} \\
& \text{Ecua} := -(\text{sqrt}(x^2 - y(x)^2) + y(x)) + x \cdot \text{diff}(y(x), x) = 0 \\
& \text{Ecua} := -\sqrt{x^2 - y(x)^2} - y(x) + x \left(\frac{d}{dx} y(x) \right) = 0 \quad (1) \\
& \text{with(DEtools)} : \\
& \text{odeadvisor}(\text{Ecua}) \\
& \quad [[_{\text{homogeneous}}, \text{class } A], _{\text{rational}}, _{\text{dAlembert}}] \quad (2) \\
& \text{EcuaDos} := \text{eval}(\text{subs}(y(x) = x \cdot u(x), \text{Ecua})) \\
& \text{EcuaDos} := -\sqrt{x^2 - x^2 u(x)^2} - x u(x) + x \left(u(x) + x \left(\frac{d}{dx} u(x) \right) \right) = 0 \quad (3) \\
& \text{EcuaTres} := \text{simplify}(\text{isolate}(\text{EcuaDos}, \text{diff}(u(x), x))) \\
& \text{EcuaTres} := \frac{d}{dx} u(x) = \frac{\sqrt{-x^2 (u(x)^2 - 1)}}{x^2} \quad (4) \\
& \text{EcuaTresCud} := \text{lhs}(\text{EcuaTres})^2 = \text{rhs}(\text{EcuaTres})^2 \\
& \text{EcuaTresCud} := \left(\frac{d}{dx} u(x) \right)^2 = -\frac{u(x)^2 - 1}{x^2} \quad (5) \\
& \text{EcuaCuatro} := x^2 \cdot \text{lhs}(\text{EcuaTresCud}) = x^2 \cdot \text{rhs}(\text{EcuaTresCud}) \\
& \text{EcuaCuatro} := x^2 \left(\frac{d}{dx} u(x) \right)^2 = -u(x)^2 + 1 \quad (6) \\
& \text{EcuaCuatroRaiz} := x \cdot \text{diff}(u(x), x) - \text{sqrt}(\text{rhs}(\text{EcuaCuatro})) = 0 \\
& \text{EcuaCuatroRaiz} := x \left(\frac{d}{dx} u(x) \right) - \sqrt{-u(x)^2 + 1} = 0 \quad (7) \\
& P := 1 \\
& P := 1 \quad (8) \\
& Q := -\sqrt{-u^2 + 1} \\
& Q := -\sqrt{-u^2 + 1} \quad (9) \\
& R := x \\
& R := x \quad (10) \\
& S := 1 \\
& S := 1 \quad (11) \\
& \text{SolGral} := \text{int}\left(\frac{P}{R}, x\right) + \text{int}\left(\frac{S}{Q}, u\right) = _CI \\
& \text{SolGral} := \ln(x) - \arcsin(u) = _CI \quad (12) \\
& \text{SolGralFinal} := \text{subs}\left(u = \frac{y}{x}, \text{SolGral}\right) \\
& \text{SolGralFinal} := \ln(x) - \arcsin\left(\frac{y}{x}\right) = _CI \quad (13) \\
& \text{SolGralDos} := \ln(x) - \arcsin\left(\frac{y(x)}{x}\right) = _CI \\
& \text{SolGralDos} := \ln(x) - \arcsin\left(\frac{y(x)}{x}\right) = _CI \quad (14)
\end{aligned}$$

> $DerSol := expand(isolate(simplify(diff(SolGralDos, x)), diff(y(x), x)))$

$$DerSol := \frac{d}{dx} y(x) = \sqrt{1 - \frac{y(x)^2}{x^2}} + \frac{y(x)}{x} \quad (15)$$

> $DerSolUno := lhs(DerSol) - \frac{y(x)}{x} = rhs(DerSol) - \frac{y(x)}{x}$

$$DerSolUno := \frac{d}{dx} y(x) - \frac{y(x)}{x} = \sqrt{1 - \frac{y(x)^2}{x^2}} \quad (16)$$

> $DerSolDos := simplify(lhs(DerSolUno)^2 = rhs(DerSolUno)^2)$

$$DerSolDos := \frac{\left(-x \left(\frac{d}{dx} y(x)\right) + y(x)\right)^2}{x^2} = -\frac{y(x)^2 - x^2}{x^2} \quad (17)$$

> $DerSolTres := lhs(DerSolDos) \cdot x^2 = rhs(DerSolDos) \cdot x^2$

$$DerSolTres := \left(-x \left(\frac{d}{dx} y(x)\right) + y(x)\right)^2 = x^2 - y(x)^2 \quad (18)$$

> $DerSolCuatro := simplify(isolate(DerSolTres, diff(y(x), x)))$

$$DerSolCuatro := \frac{d}{dx} y(x) = \frac{-\sqrt{x^2 - y(x)^2} + y(x)}{x} \quad (19)$$

> $DerEcua := simplify(isolate(Ecua, diff(y(x), x)))$

$$DerEcua := \frac{d}{dx} y(x) = \frac{\sqrt{x^2 - y(x)^2} + y(x)}{x} \quad (20)$$

>

PROBLEMA DE LA BALA QUE PASA A TRAVÉS DE UNA PLACA DE MADERA

> *restart*

> $EcuaUno := diff(V(t), t) = -K \cdot V(t)^2$

$$EcuaUno := \frac{d}{dt} V(t) = -K V(t)^2 \quad (21)$$

> *with(DEtools) :*

> *odeadvisor(EcuaUno)*

[_quadrature] (22)

> $SolGralUno := dsolve(EcuaUno)$

$$SolGralUno := V(t) = \frac{1}{K t + _CI} \quad (23)$$

> $CondIni := V(0) = 200$

$$CondIni := V(0) = 200 \quad (24)$$

> $SolPartUno := dsolve(\{EcuaUno, CondIni\})$

$$SolPartUno := V(t) = \frac{200}{200 K t + 1} \quad (25)$$

> $EcuaDos := diff(x(t), t) = rhs(SolPartUno)$

$$EcuaDos := \frac{d}{dt} x(t) = \frac{200}{200 K t + 1} \quad (26)$$

> *odeadvisor(EcuaDos)*

(27)

```

|                                     [_quadrature]                                     (27)
|=
> SolGralDos := dsolve(EcuaDos)
|                                      $SolGralDos := x(t) = \frac{\ln(200 K t + 1)}{K} + \_CI$                                      (28)
|=
> CondIniDos := x(0) = 0
|                                      $CondIniDos := x(0) = 0$                                      (29)
|=
> SolPartDos := dsolve( {EcuaDos, CondIniDos} )
|                                      $SolPartDos := x(t) = \frac{\ln(200 K t + 1)}{K}$                                      (30)
|=
>
|=
> ParametroUno := isolate(subs(V(t) = 80, SolPartUno), K)
|                                      $ParametroUno := K = \frac{3}{400 t}$                                      (31)
|=
> SolPartTres := subs(K = rhs(ParametroUno), SolPartDos)
|                                      $SolPartTres := x(t) = \frac{400}{3} \ln\left(\frac{5}{2}\right) t$                                      (32)
|=
> TiempoFinal := isolate(subs(x(t) = 1/10, SolPartTres), t)
|                                      $TiempoFinal := t = \frac{3}{4000 \ln\left(\frac{5}{2}\right)}$                                      (33)
|=
> evalf(%, 5)
|                                      $t = 0.00081855$                                      (34)
>

```